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Tuesday 28 April 2009 2.30 to 4.00

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Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Attachments:*

*3C8 Data sheet (9 pages)*

*Duplicate Fig. 1*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 (a) Draw a block diagram to show the concept of a simple parallel hybrid drive comprising an internal combustion engine and an electric motor-generator and battery. Briefly state the operating principle. List the important advantages and disadvantages of the parallel hybrid when compared to the internal combustion engine alone. [20%]

(b) The torque-speed characteristics of the internal combustion engine of a racing car for maximum and minimum throttle openings is shown in Fig. 1. Contours of output power are spaced at 50 kW. The maximum engine speed is 19,000 rpm (revolutions per minute) and the minimum (idle) speed is 4,000 rpm. The road wheel radius is 0.3 m and the total mass of the car is 700 kg. The tractive force  $F$  in N required to overcome rolling and aerodynamic resistance is given by

$$F = 200 + 0.5V^2$$

where  $V$  is the speed in  $\text{m s}^{-1}$ . If the engine:wheels speed ratio is 6:1, find the maximum vehicle speed and explain why a different speed ratio would only decrease the maximum vehicle speed. [35%]

(c) To reduce the cost of the engine it is proposed to reduce the maximum speed of the engine. To compensate for the reduction in maximum power it is also proposed to connect an electric motor-generator directly to the engine crankshaft to form a simple parallel hybrid. The maximum mechanical power allowed in, or out, of the motor-generator is 50 kW.

(i) Find the speed to which the engine should be limited if the maximum combined power output of the engine and the motor-generator is to be the same as the maximum power of the original engine. What value of speed ratio is now needed to achieve maximum vehicle speed? [15%]

(ii) With the speed ratio at the original value of 6:1, find the maximum acceleration of the vehicle if the motor-generator is delivering maximum power to the wheels, and state the engine speed at which it occurs. [20%]

(iii) If the motor-generator is absorbing maximum power, find the maximum vehicle deceleration achievable without use of the car's brakes at the speed found in (ii). [10%]

(cont.)

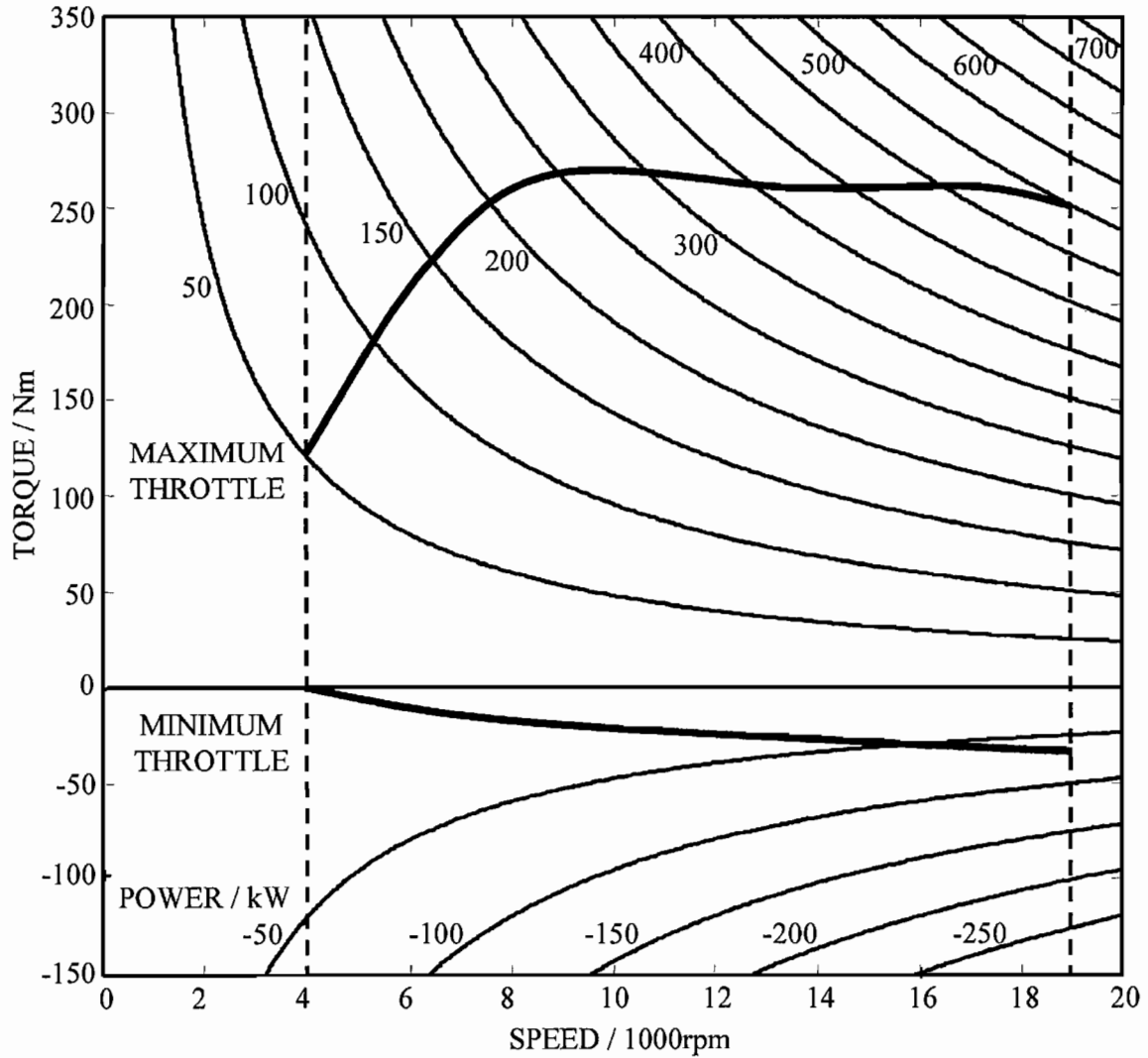


Fig. 1

A duplicate of this figure is attached after the data sheet.

(TURN OVER

2 Figure 2 shows schematically a planetary gear system used to transmit power from an input shaft at speed  $\omega_a$  to two output shafts at speeds  $\omega_e$  and  $\omega_f$  with corresponding torques on the shafts of  $T_a$ ,  $T_e$  and  $T_f$ . The input shaft carries a bevel gear A which engages with a freely running ring gear B onto which two sets of planetary bevel gears C and D are mounted via frictionless bearings. These planetary gears are connected together so that they rotate at the same speed and in turn drive each of the two output shafts via gears E and F. Tooth numbers for each of the gears are denoted by the corresponding letters A to F. Losses in the gearbox can be neglected.

(a) Find an expression for the ratio of the input to output speeds  $\omega_a / \omega_e$  when the two output shafts rotate at the same speed. [15%]

(b) Derive the following equation relating the speeds of the three shafts

$$\omega_f \frac{F}{D} + \omega_e \frac{E}{C} + \omega_a \frac{A}{B} \left\{ \frac{E}{C} + \frac{F}{D} \right\} = 0 . \quad [40\%]$$

(c) The output speeds are fixed so that  $\omega_e = 2\omega_f$  and tooth numbers are chosen as  $A = 20$ ,  $B = 100$ ,  $C = 20$ ,  $D = 15$ ,  $E = 30$  and  $F = 15$ .

(i) Find an expression for the speed ratio  $\omega_a / \omega_e$ . [15%]

(ii) Find expressions for the torques  $T_e$  and  $T_f$  transmitted by the two output shafts in terms of the torque  $T_a$  on the input shaft. [15%]

(iii) Find the power split between the two output shafts. [15%]

(cont.)

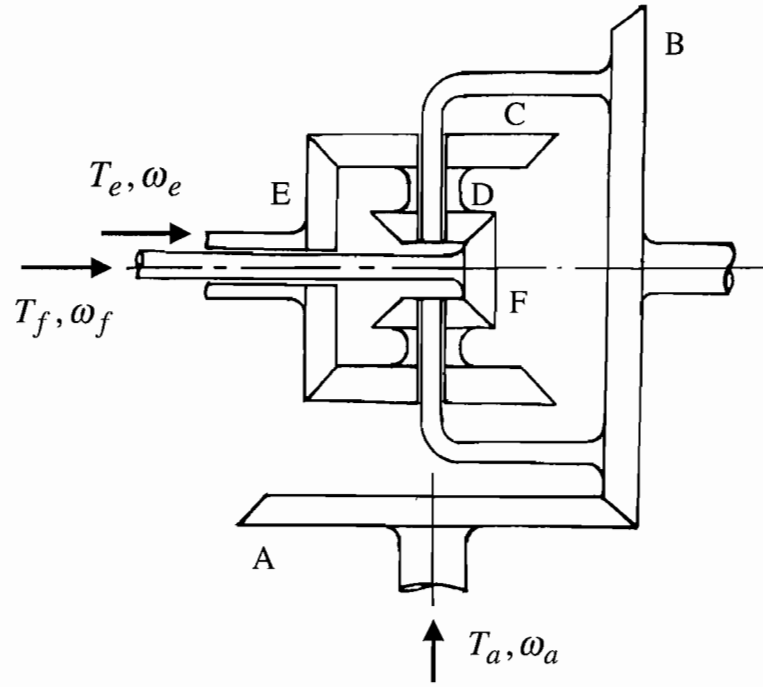


Fig. 2

3 (a) A material has compressive yield stress  $Y$ . Explain why contact pressures numerically greater than  $Y$  can be applied to the surface of a specimen of this material without causing yield. [20%]

(b) A sphere of radius  $R$  consists of material with elastic modulus  $E$  and Poisson ratio  $\nu$ . It is pressed into contact with a rigid plane surface by a normal load  $P$  which is sufficient to give a Hertz pressure of magnitude  $p_0$ . Show that the stored elastic energy  $U$  is given by the expression

$$U = \frac{\pi^5}{60} \frac{p_0^5}{E^{*4}} R^3$$

where  $E^*$  is equal to  $E/(1-\nu^2)$ . [40%]

(c) Figure 3 shows a simple Newton's cradle which consists of two pendula: each of the masses is a sphere of radius  $R$  of material with elastic properties  $E$  and  $\nu$ . The yield stress of the material in pure shear is  $k$  and it has density  $\rho$ . Each suspension is of length  $\ell$  and  $\ell \gg R$ . One pendulum is released from a position in which the angle  $\theta$  is  $90^\circ$  while the other hangs vertically at rest. Derive a relationship between  $k$ ,  $\rho$ ,  $g$ ,  $E^*$  and  $\ell$  if the subsequent collision between the two masses just initiates yield. [40%]

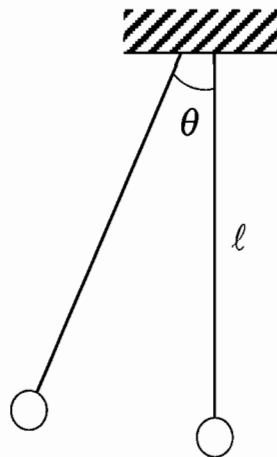


Fig. 3

4 A pair of spur gears with standard involute teeth have tooth numbers 20 and 60. The addendum is equal to the module and the nominal pressure angle  $\phi$  is  $20^\circ$ . The module, based on this nominal pressure angle, is 3 mm.

(a) Show that, to achieve the nominal pressure angle, the distance between gear centres should be 120 mm. Calculate the corresponding contact ratio and minimum effective radius of curvature for single and double pair contact. [45%]

(b) As a result of errors in the casing geometry, the distance between gear centres is 120.5 mm.

(i) Discuss qualitatively how this will affect the operation of the gears. [15%]

(ii) Explain why the base pitch is unaffected by the change in gear centres while the circumferential pitch is altered. [10%]

(iii) Calculate the new contact ratio. [30%]

**END OF PAPER**





# ENGINEERING TRIPOS Part IIA

## Module 3C8 Data Sheet

### ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

$$\text{Effective curvature } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{Contact modulus } \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

where  $R_1, R_2$  are the radii of curvature of the two bodies (convex positive).

where  $E_1, E_2$  and  $\nu_1, \nu_2$  are Young's moduli and Poisson's ratios

	<u>Line contact</u> width $2b$ ; load $P'$ per unit length	<u>Circular contact</u> diameter $2a$ ; load $P$
Semi contact width or contact radius	$b = 2 \left\{ \frac{P'R}{\pi E^*} \right\}^{1/2}$	$a = \left\{ \frac{3PR}{4E^*} \right\}^{1/3}$
Maximum contact pressure ('Hertz stress')	$p_0 = \left\{ \frac{P'E^*}{\pi R} \right\}^{1/2}$	$p_0 = \frac{1}{\pi} \left\{ \frac{6PE^{*2}}{R^2} \right\}^{1/3}$
Approach of centres	$\delta = \frac{2P'}{\pi} \left[ \frac{1-\nu_1^2}{E_1} \left\{ \ln \left( \frac{4R_1}{b} \right) - \frac{1}{2} \right\} + \frac{1-\nu_2^2}{E_2} \left\{ \ln \left( \frac{4R_2}{b} \right) - \frac{1}{2} \right\} \right]$	$\delta = \frac{a^2}{R} = \frac{1}{2} \left\{ \frac{9}{2} \frac{P^2}{E^{*2} R} \right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{P'}{2b} = \frac{\pi}{4} p_0$	$\bar{p} = \frac{P}{\pi a^2} = \frac{2}{3} p_0$
	$\tau_{\max} = 0.300 p_0$ at $x = 0, z = 0.79b$	$\tau_{\max} = 0.310 p_0$ at $r = 0, z = 0.48a$ for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0.79b$

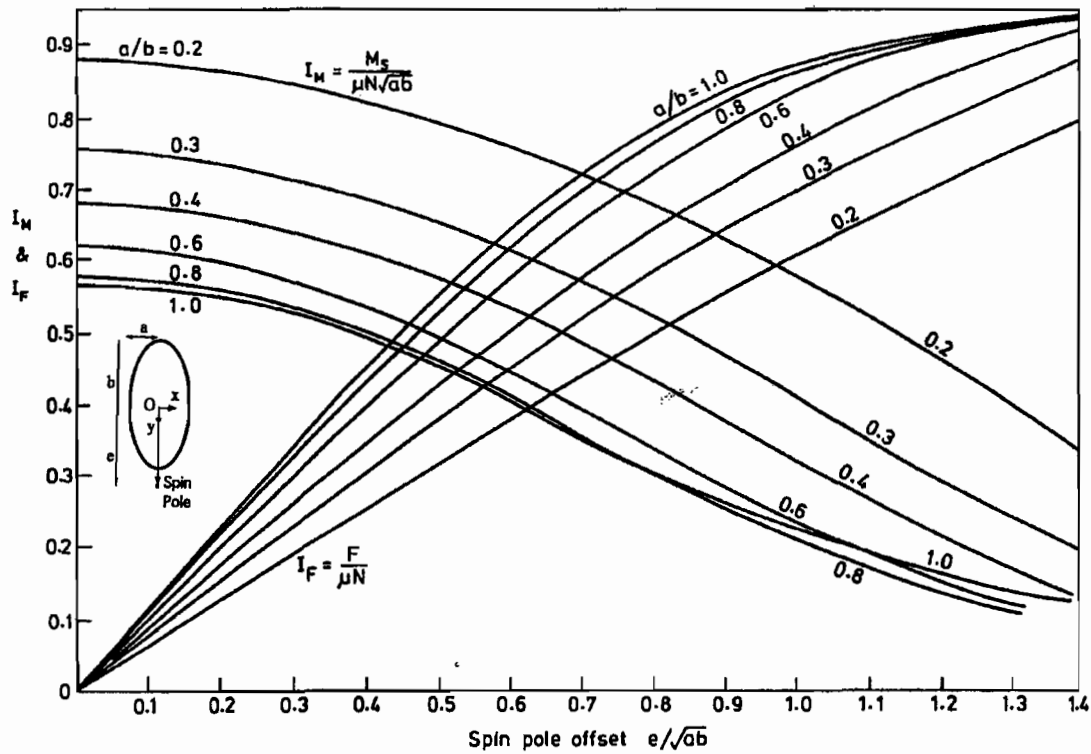
#### Mildly elliptical contacts

If the gap at zero load is  $h = \frac{1}{2}Ax^2 + \frac{1}{2}By^2$  and  $0.2 < A/B < 5$  then ratio of semi-axes  $b/a \equiv (A/B)^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with  $R = (AB)^{-1/2}$  or better  $R_e = [AB(A+B)/2]^{-1/3}$ .

For **approach** use circular contact equation with  $R = (AB)^{-1/2}$  (**not**  $R_e$ )

## Hertzian contact frictional losses

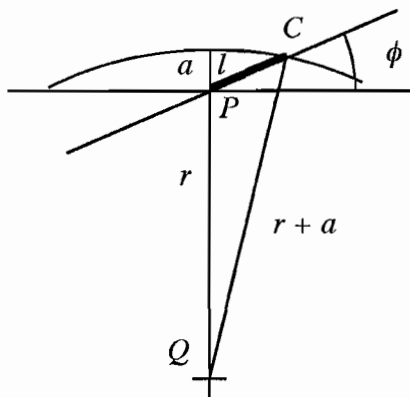


## INVOLUTE GEARING

### Spur gears

pitch cylinder radii	$r$	} with suffix 1 or 2	circumferential pitch	$p = 2\pi r/N$
base cylinder radii	$r_b$		base pitch	$p_b = p \cos \phi$
addendum cylinder radii	$r_a$		module	$m = p/\pi = 2r/N$
number of teeth	$N$		ratio of contact	$r_c$
addendum	$a = r_a - r$		radius of curvature at pitch point	$\rho = r \sin \phi$
pressure angle	$\phi$			

### Path of contact



$$l = \left\{ r^2 \sin^2 \phi + a(2r + a) \right\}^{1/2} - r \sin \phi$$

For a standard  $20^\circ$  spur wheel with  $N$  teeth of module  $m$  this becomes

$$\frac{l}{m} = \left( 0.02924N^2 + N + 1 \right)^{1/2} - 0.1710N$$

Standard tooth forms

Addendum  $a = m$ , Dedendum  $= \frac{7}{6}m$ , pressure angle  $= 20^\circ$ .

Modules: 0.3 – 1.0 mm in 0.1 mm steps  
 1.0 – 4.0 mm in 0.25 mm steps 4.0 – 7.0 mm in 0.5 mm steps  
 7.0 – 16.0 mm in 1.0 mm steps 16.0 – 24.0 mm in 2.0 mm steps  
 24.0 – 45.0 mm in 3.0 mm steps 45.0 – 75.0 mm in 5.0 mm steps

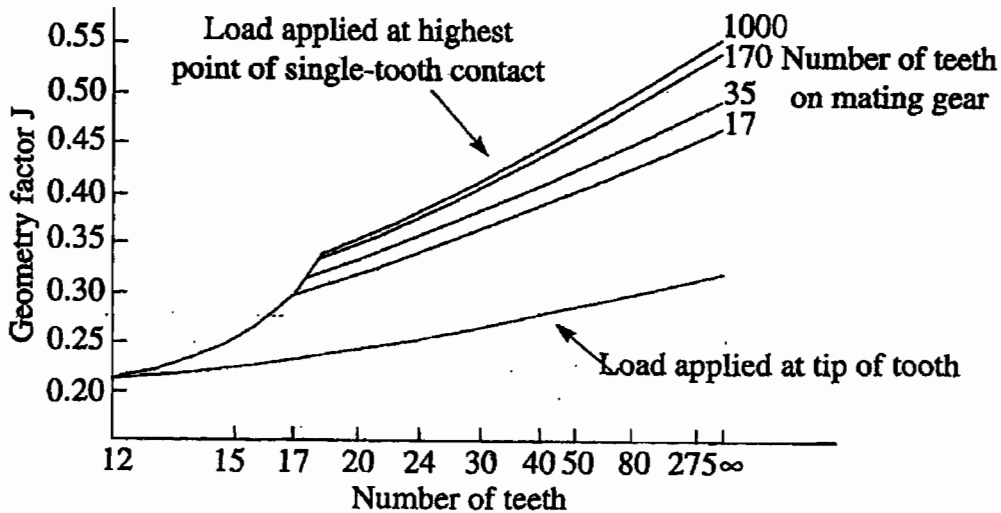
Friction in spur gears

$$\frac{\text{average friction loss}}{\text{power transmitted}} \approx \mu\pi \left\{ \frac{1}{N_1} + \frac{1}{N_2} \right\}$$

Tooth failure

Allowable bending stress  $\sigma_b$  according to AGMA guidelines given by  $\sigma_b = \frac{P_T'}{Jm}$

where  $P_T'$  is force per unit face-width acting tangentially to pitch circle and  $J$  given in the figure below for  $20^\circ$  spur gears. Typical values of  $\sigma_b$  shown in table.



Typical allowable tooth stresses (AGMA)

Material	Condition	Bending fatigue strength $\sigma_b$ (MPa)	Surface fatigue strength $\sigma_s$ (MPa)
Steel	Through hardened and tempered	170-390	590-1200
	Carburised and case hardened	380-480	1250-1550
Cast iron	As cast	69-90	450-590
Nodular iron	Quenched, annealed and tempered	150-300	500-800
Malleable iron	Pearlitic	70-145	500-650

## EPICYCLIC SPEED RULE

$$\omega_s = (1 + R)\omega_c - R\omega_a \quad \text{where } R = \frac{A}{S}$$

## ROLLING ELEMENT BEARINGS

### Fatigue life

$$L = a_1 a_2 a_3 (C/P)^p \quad p = 3 \text{ for ball and } 10/3 \text{ for roller bearings}$$

Fatigue probability %	10	5	4	3	2	1
Life adjust factor $a_1$	1	0.62	0.53	0.44	0.33	0.21

### Bearing choice

The information on the following pages concerning loads, viscosities and standard bearing sizes and ratings is extracted from the SKF General Bearing Catalogue and is copied with permission. It is SKF copyright and is not to be further reproduced.

MPFS, DJC, JAW  
November 07

Required viscosities and the effect of viscosity ratio on  $a_{23}$

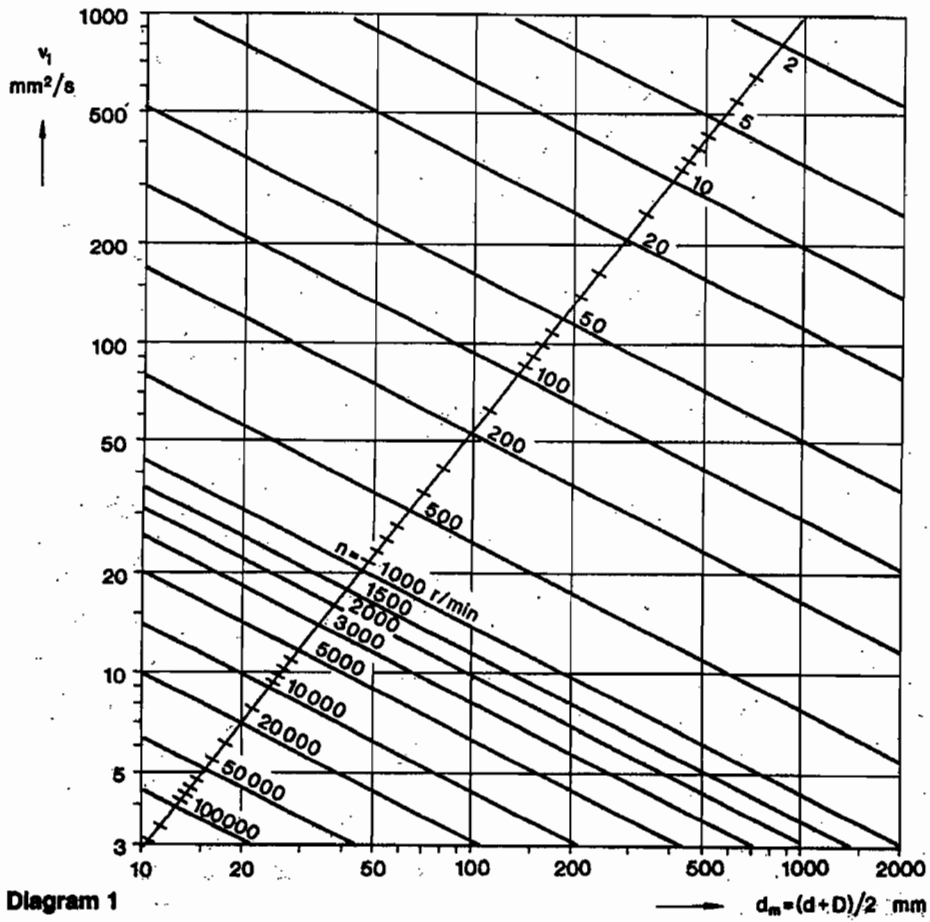


Diagram 1

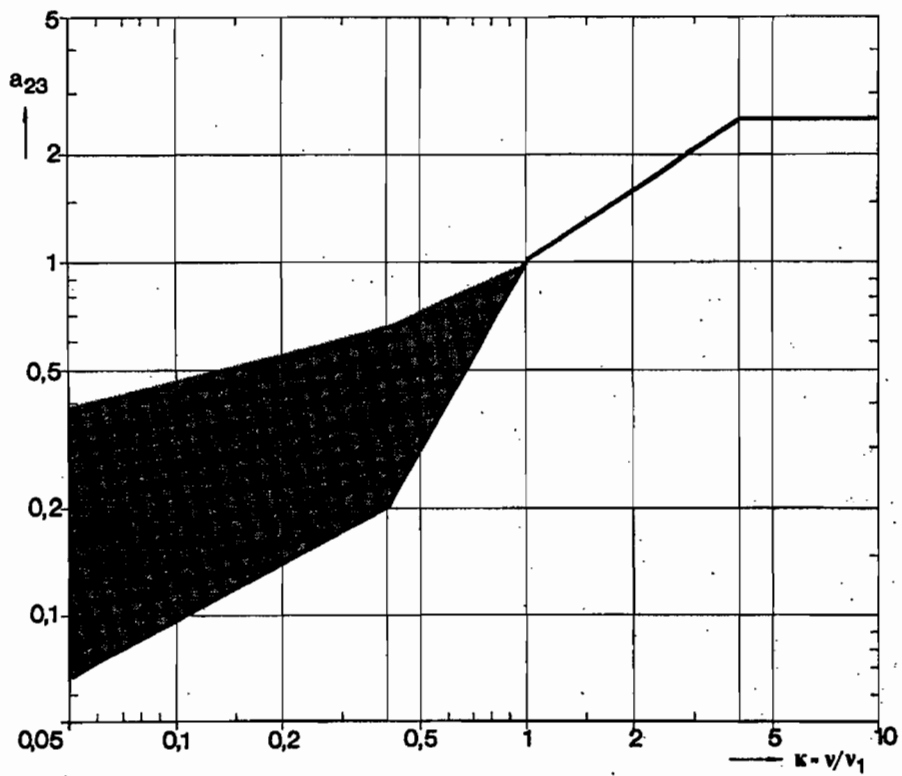
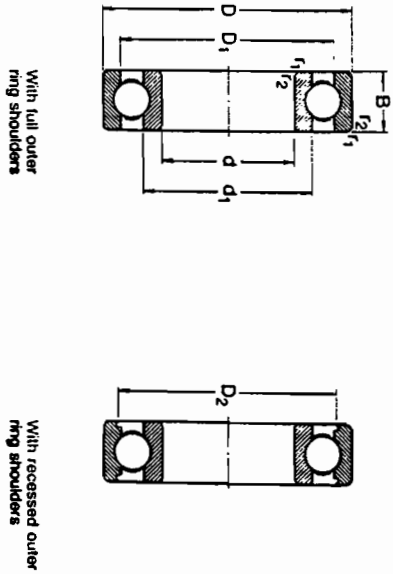


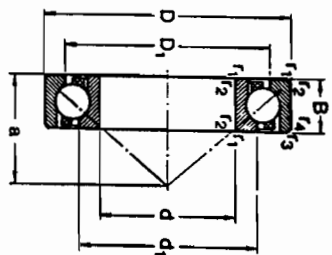
Diagram 3

**Deep groove ball bearings  
single row  
d 35–55 mm**



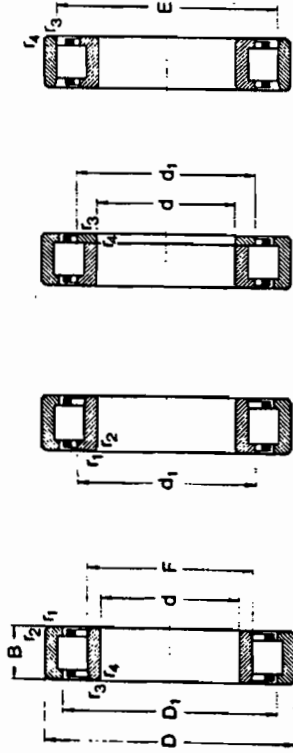
Principal dimensions	Basic load ratings			Fatigue load limit $P_u$	Speed ratings		Mass	Designation
	dynamic	static	$C_0$		Lubrication grease	oil		
d	D	B	C	$C_0$	N	r/min	kg	-
35	47	7	4 750	3 200	186	13 000	0,030	61807
	55	10	9 560	6 200	290	11 000	0,080	61907
	62	9	12 400	8 150	375	10 000	0,11	16007
	62	14	15 900	10 200	440	10 000	0,16	6007
	72	17	25 500	15 300	655	9 000	0,29	6207
	80	21	33 200	19 000	815	8 500	0,46	6307
	100	25	55 300	31 000	1 290	7 000	0,95	6407
40	52	7	4 940	3 450	186	11 000	0,034	61808
	62	12	13 800	9 300	425	10 000	0,12	61908
	68	9	13 300	9 150	440	9 500	0,13	16008
	68	15	16 800	11 600	490	9 500	0,18	6008
	80	16	30 700	19 000	800	8 500	0,37	6208
	90	23	41 000	24 000	1 020	7 500	0,63	6308
	110	27	63 700	36 500	1 530	6 700	1,25	6408
45	58	7	6 050	4 300	228	9 500	0,040	61809
	68	12	10 100	6 700	285	9 000	0,14	61909
	75	10	15 600	10 800	520	9 000	0,17	16009
	75	16	20 800	14 600	640	9 000	0,25	6009
	85	19	33 200	21 600	915	7 500	0,41	6209
	100	25	52 700	31 500	1 340	6 700	0,83	6309
	120	29	78 100	45 000	1 900	6 000	1,55	6409
50	65	7	6 240	4 750	250	9 000	0,052	61810
	72	12	14 600	10 400	500	8 500	0,14	61910
	80	10	16 300	11 400	560	8 500	0,18	16010
	80	16	21 600	16 000	710	8 000	0,26	6010
	90	20	35 100	23 200	980	7 000	0,46	6210
	110	27	61 800	38 000	1 600	6 300	1,05	6310
	130	31	87 100	52 000	2 200	5 300	1,90	6410
55	72	9	8 320	6 200	325	8 500	0,083	61811
	80	13	15 800	11 400	560	8 000	0,19	61911
	90	11	19 500	14 000	695	7 500	0,26	16011
	90	16	28 100	21 200	900	7 500	0,39	6011
	100	21	43 600	29 000	1 250	6 300	0,61	6211
	120	29	71 500	45 000	1 800	5 600	1,35	6311
	140	33	99 500	62 000	2 600	5 000	2,30	6411

**Angular contact ball bearings  
single row  
d 10–65 mm**



Principal dimensions	Basic load ratings			Fatigue load limit $P_u$	Speed ratings		Mass	Designation
	dynamic	static	$C_0$		Lubrication grease	oil		
d	D	B	C	$C_0$	N	r/min	kg	-
10	30	9	7 020	3 350	140	19 000	0,030	7200 BE
	12	32	7 610	3 800	160	18 000	0,036	7201 BE
	37	12	10 600	5 000	208	17 000	0,060	7301 BE
	35	11	8 840	4 800	204	17 000	0,045	7202 BE
	42	13	13 000	6 700	280	15 000	0,080	7302 BE
17	40	12	11 100	6 100	260	15 000	0,065	7203 BE
	47	14	15 900	8 300	355	13 000	0,11	7303 BE
20	47	14	14 000	8 300	355	12 000	0,11	7204 BE
	52	15	19 000	10 400	440	11 000	0,14	7304 BE
25	52	15	15 600	10 200	430	10 000	0,13	7205 BE
	62	17	26 000	15 600	655	9 000	0,23	7305 BE
30	62	16	23 800	15 600	655	8 500	0,20	7206 BE
	72	19	34 500	21 200	800	8 000	0,34	7306 BE
35	72	17	30 700	20 800	880	8 000	0,28	7207 BE
	80	21	39 000	24 500	1 040	7 500	0,45	7307 BE
40	80	18	36 400	26 000	1 100	7 000	0,37	7208 BE
	90	23	49 400	33 500	1 400	6 700	0,63	7308 BE
45	85	19	37 700	28 000	1 200	6 700	0,42	7209 BE
	100	25	60 500	41 500	1 730	6 000	0,85	7309 BE
60	90	20	39 000	30 500	1 280	6 000	0,47	7210 BE
	110	27	74 100	51 000	2 200	5 300	1,10	7310 BE
65	100	21	48 800	38 000	1 630	5 600	0,62	7211 BE
	120	29	85 200	60 000	2 550	4 800	1,40	7311 BE
60	110	22	57 200	45 500	1 930	5 000	0,60	7212 BE
	130	31	95 600	69 600	3 000	4 500	1,75	7312 BE
65	120	23	66 300	54 000	2 280	4 500	1,00	7213 BE
	140	33	106 000	80 000	3 350	4 300	2,15	7313 BE

**Cylindrical roller bearings  
single row  
d 50–55 mm**



Type NU

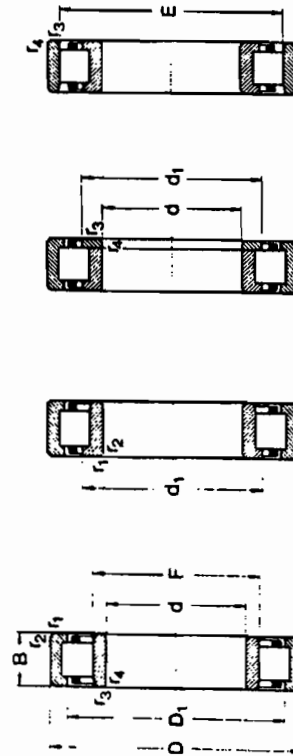
Type NUJ

Type NUP

Type N

Principal dimensions	Basic load ratings			Fatigue load limit $P_u$	Speed ratings Lubrication grease oil	Mass	Designation
	dynamic	static	$C_0$				
d	D	B	C	N	r/min	kg	-
mm	mm	mm	mm	N	r/min	kg	-
50	80	16	30 800	34 500	4 000	0,31	NU 1010
	90	20	64 400	69 500	8 800	0,48	NU 210 EC
	90	20	64 400	69 500	8 800	0,49	NJ 210 EC
	90	20	64 400	69 500	8 800	0,51	NUP 210 EC
	90	20	64 400	69 500	8 800	0,48	N 210 EC
	90	23	78 100	88 000	11 400	0,58	NU 2210 EC
	90	23	78 100	88 000	11 400	0,58	NJ 2210 EC
	90	23	78 100	88 000	11 400	0,59	NUP 2210 EC
	110	27	110 000	112 000	15 000	1,15	NU 310 EC
	110	27	110 000	112 000	15 000	1,15	NJ 310 EC
	110	27	110 000	112 000	15 000	1,20	NUP 310 EC
	110	27	110 000	112 000	15 000	1,15	N 310 EC
	110	40	161 000	186 000	24 500	1,70	NU 2310 EC
	110	40	161 000	186 000	24 500	1,75	NJ 2310 EC
	110	40	161 000	186 000	24 500	1,80	NUP 2310 EC
	130	31	130 000	127 000	16 600	2,00	NU 410
	130	31	130 000	127 000	16 600	2,05	NJ 410
	85	18	57 200	69 500	8 300	0,40	NU 1011 EC
	100	21	84 200	95 000	12 200	0,66	NU 211 EC
	100	21	84 200	95 000	12 200	0,67	NJ 211 EC
	100	21	84 200	95 000	12 200	0,69	NUP 211 EC
	100	21	84 200	95 000	12 200	0,66	N 211 EC
	100	25	99 000	118 000	15 300	0,79	NU 2211 EC
	100	25	99 000	118 000	15 300	0,81	NJ 2211 EC
	100	25	99 000	118 000	15 300	0,82	NUP 2211 EC
	100	25	99 000	118 000	15 300	0,79	N 2211 EC
	120	29	138 000	143 000	18 600	1,45	NU 311 EC
	120	29	138 000	143 000	18 600	1,50	NJ 311 EC
	120	29	138 000	143 000	18 600	1,55	NUP 311 EC
	120	29	138 000	143 000	18 600	1,45	N 311 EC

**Cylindrical roller bearings  
single row  
d 40–45 mm**



Type NU

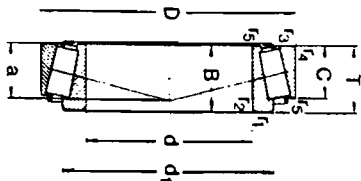
Type NUJ

Type NUP

Type N

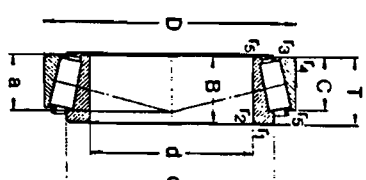
Principal dimensions	Basic load ratings			Fatigue load limit $P_u$	Speed ratings Lubrication grease oil	Mass	Designation
	dynamic	static	$C_0$				
d	D	B	C	N	r/min	kg	-
mm	mm	mm	mm	N	r/min	kg	-
40	90	23	80 900	78 000	10 200	0,65	NU 308 EC
(cont.)	90	23	80 900	78 000	10 200	0,67	NJ 308 EC
	90	23	80 900	78 000	10 200	0,68	NUP 308 EC
	90	23	80 900	78 000	10 200	0,64	N 308 EC
	90	33	112 000	120 000	15 300	0,94	NU 2308 EC
	90	33	112 000	120 000	15 300	0,96	NJ 2308 EC
	90	33	112 000	120 000	15 300	0,98	NUP 2308 EC
	110	27	96 800	90 000	11 600	1,30	NU 408
	110	27	96 800	90 000	11 600	1,30	NJ 408
	110	27	96 800	90 000	11 600	1,35	NUP 408
	45	16	44 600	52 000	6 300	0,26	NU 1009 EC
	85	19	60 500	64 000	8 150	0,43	NU 209 EC
	85	19	60 500	64 000	8 150	0,44	NJ 209 EC
	85	19	60 500	64 000	8 150	0,45	NUP 209 EC
	85	19	60 500	64 000	8 150	0,43	N 209 EC
	85	23	73 700	81 500	10 600	0,52	NU 2209 EC
	85	23	73 700	81 500	10 600	0,54	NJ 2209 EC
	85	23	73 700	81 500	10 600	0,55	NUP 2209 EC
	85	23	73 700	81 500	10 600	0,52	N 2209 EC
	100	25	99 000	100 000	12 900	0,90	NU 309 EC
	100	25	99 000	100 000	12 900	0,92	NJ 309 EC
	100	25	99 000	100 000	12 900	0,95	NUP 309 EC
	100	25	99 000	100 000	12 900	0,88	N 309 EC
	100	36	138 000	153 000	20 000	1,30	NU 2309 EC
	100	36	138 000	153 000	20 000	1,30	NJ 2309 EC
	100	36	138 000	153 000	20 000	1,35	NUP 2309 EC
	120	29	106 000	102 000	13 400	1,65	NU 409
	120	29	106 000	102 000	13 400	1,65	NJ 409
	120	29	106 000	102 000	13 400	1,70	NUP 409

Taper roller bearings  
single row  
d 35-50 mm



Principal dimensions	Basic load ratings		Fatigue load limit $P_u$	Speed ratings		Mass	Designation	Dimension Series to ISO 355
	dynamic	static		$C_0$	$C_g$			
d	D	T	C	$C_0$	$C_g$	$P_u$		
mm			N	N	N	r/min	kg	
35 (cont.)	80	22,75	72 100	73 500	8 500	5 000	30307	2FB
	80	22,75	61 600	67 000	7 600	4 500	31307	7FB
	80	32,75	95 200	106 000	12 200	4 800	32307	2FE
	80	32,75	93 500	114 000	13 200	4 500	32307 B	5FE
40	68	19	52 800	71 000	7 800	5 300	32008 X	3CD
	75	26	79 200	104 000	11 600	5 000	33108	2CE
	80	18,75	61 600	68 000	7 650	4 800	30208	3DB
	80	24,75	74 800	86 500	9 800	4 800	32208	3DC
	80	32,75	105 000	132 000	15 300	4 300	33208	2DE
	85	32	121 000	150 000	17 200	4 500	33208	2EE
	90	28,25	95 000	95 000	11 000	4 500	30308	2FB
	90	28,25	73 700	81 500	9 650	4 000	31308	2FE
	90	35,25	117 000	140 000	16 300	4 000	32308	2FD
	90	35,25	108 000	140 000	16 300	4 000	32308 B	5FD
45	75	20	58 300	80 000	8 800	4 800	32009 X	3CC
	80	26	84 200	114 000	12 900	4 500	33109	3DB
	85	20,75	66 000	76 500	8 650	4 500	30209	3DC
	85	24,75	80 900	98 000	11 200	4 500	32209	3DC
	85	24,75	73 700	93 000	11 000	4 300	32209 B	3DE
	85	32	108 000	143 000	16 300	4 000	33209	2EE
	85	29	89 700	112 000	12 900	3 600	33209	2ED
	95	36	147 000	186 000	21 200	4 000	33009	2FB
	100	27,25	108 000	120 000	14 600	3 400	31309	2FD
	100	27,25	91 300	102 000	12 500	3 400	31309	2FD
	100	36,25	140 000	170 000	20 400	3 600	32309	5FD
	100	36,25	134 000	176 000	20 000	3 600	32309 B	5FD
50	80	20	60 500	88 000	9 650	4 500	32010 X	3CC
	80	24	69 300	102 000	11 400	4 500	33010	2CE
	82	21,5	72 100	100 000	11 000	4 500	33010	3CE
	85	26	85 800	122 000	13 700	4 300	33110	3DB
	90	21,75	76 500	91 500	10 400	4 300	30210	3DC
	90	24,75	82 500	104 000	11 600	4 300	32210	3DC
	90	24,75	82 500	104 000	11 600	4 300	32210 B	5DC
	90	28	106 000	140 000	16 300	4 000	33210	2EE
	90	28	116 000	140 000	16 300	4 000	33210 A	3DE
	90	32	114 000	160 000	18 300	3 800	33210	2ED
	100	36	154 000	200 000	22 800	3 800	33210	2ED
	100	36	148 000	190 000	22 800	3 800	33210	2ED
	100	32	108 000	137 000	16 000	4 300	32310	5FD

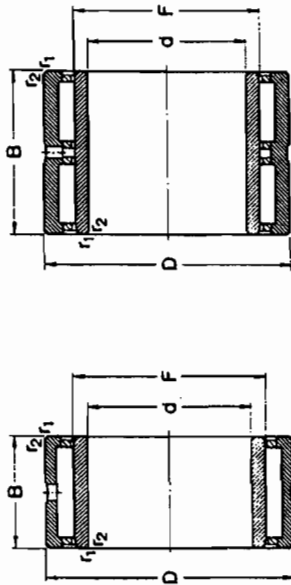
Taper roller bearings  
single row  
d 50-65 mm



Principal dimensions	Basic load ratings		Fatigue load limit $P_u$	Speed ratings		Mass	Designation	Dimension Series to ISO 355
	dynamic	static		$C_0$	$C_g$			
d	D	T	C	$C_0$	$C_g$	$P_u$		
mm			N	N	N	r/min	kg	
50 (cont.)	110	28,25	125 000	140 000	17 000	3 600	30310	2FB
	110	28,25	106 000	120 000	14 500	3 200	31310	7FB
	110	42,25	172 000	212 000	24 500	3 200	32310	2FD
	110	42,25	161 000	216 000	25 000	3 200	32310 B	5FD
55	80	23	78 100	112 000	12 500	4 000	32011 X	3CC
	80	23	80 900	116 000	13 200	4 000	33011	2CE
	80	27	89 700	137 000	15 900	3 800	33111	3CE
	85	30	110 000	156 000	18 200	3 800	33111	3DB
	100	22,75	89 700	108 000	12 200	3 800	30211	3DC
	100	26,75	106 000	129 000	15 000	3 600	32211	2DE
	100	35	138 000	180 000	22 000	3 400	33211	2EE
	100	39	179 000	232 000	26 500	3 000	33211	2ED
	110	39	178 000	232 000	26 500	3 000	33211	2ED
	115	34	125 000	163 000	19 600	3 200	30311	2FB
	120	31,5	142 000	197 000	23 000	2 800	31311	2FE
	120	31,5	121 000	137 000	17 000	2 800	31311	2FE
	120	45,5	196 000	250 000	29 000	2 300	32311	5FD
	120	45,5	180 000	260 000	30 000	2 800	32311 B	5FD
60	85	23	82 500	122 000	13 700	3 800	32012 X	4CC
	85	24	84 200	132 000	15 000	3 800	33012	2CE
	85	27	91 300	143 000	16 000	3 800	33112	3CE
	100	23,75	117 000	170 000	19 600	3 400	33112	3DB
	100	27,75	99 000	114 000	13 400	3 400	30212	3DC
	110	38	125 000	160 000	20 000	3 000	33212	2EE
	110	38	168 000	236 000	27 000	2 700	33212	2ED
	110	38	168 000	236 000	27 000	2 700	33212	2ED
	115	39	168 000	250 000	27 500	3 200	30312	2FB
	115	40	194 000	280 000	30 000	3 000	31312	2FE
	125	37	154 000	204 000	24 500	2 600	32312	5FD
	130	33,5	168 000	196 000	23 600	3 000	30312	2FB
	130	33,5	145 000	166 000	20 400	2 800	31312	2FE
	130	48,5	228 000	280 000	34 000	2 600	32312	5FD
	130	48,5	220 000	305 000	35 500	2 600	32312 B	5FD
65	100	23	64 200	127 000	14 900	3 400	32013 X	4CC
	100	27	66 800	156 000	17 600	3 200	33013	2CE
	110	28	123 000	183 000	21 200	3 000	33113	3CE
	110	28	142 000	208 000	24 500	3 000	33113	3DB
	120	24,75	114 000	134 000	16 500	3 000	30213	3DC
	120	32,75	151 000	193 000	23 500	2 800	32213	2EE
	120	39	161 000	240 000	27 500	3 000	32213	2ED
	120	39	161 000	240 000	27 500	3 000	32213	2ED



**Needle roller bearings with flanges  
with inner ring  
d 40–65 mm**



Series NK(IS), NA 48

Series NA 69

Principal dimensions	Basic load ratings			Fatigue load limit $P_u$	Speed ratings Lubrication grease oil	Mass	Designation
	d	D	B				
	mm			N	r/min	kg	
40	55	20	27 500	57 000	7 200	0.14	NKI 40/20
	55	30	40 200	82 000	12 000	0.22	NKI 40/30
	62	22	42 900	71 000	9 150	0.23	NA 4908
	62	40	67 100	125 000	16 000	0.43	NA 6908
	65	22	42 900	72 000	9 150	0.28	NKIS 40
42	57	20	29 200	61 000	7 650	0.15	NKI 42/20
	57	30	41 800	98 000	12 900	0.22	NKI 42/30
45	62	25	38 000	78 000	10 000	0.23	NKI 45/25
	62	35	49 500	110 000	14 300	0.32	NKI 45/35
	68	22	45 700	76 000	10 000	0.27	NA 4909
	68	40	70 400	137 000	17 300	0.50	NA 6909
	72	22	44 600	78 000	10 000	0.34	NKIS 45
50	68	25	40 200	88 000	11 200	0.27	NKI 50/25
	68	35	52 300	122 000	16 000	0.38	NKI 50/35
	72	22	47 300	85 000	11 000	0.27	NA 4910
	72	40	73 700	150 000	19 000	0.52	NA 6910
	80	28	62 700	104 000	13 700	0.52	NKIS 50
55	72	25	41 800	98 500	12 200	0.27	NKI 55/25
	72	35	55 000	134 000	17 600	0.38	NKI 55/35
	80	25	57 200	106 000	13 700	0.40	NA 4911
	80	45	89 700	190 000	24 000	0.78	NA 6911
	85	28	66 000	114 000	15 000	0.56	NKIS 55
60	82	25	44 000	95 000	12 000	0.40	NKI 60/25
	82	35	60 500	146 000	19 000	0.55	NKI 60/35
	85	25	60 500	114 000	14 600	0.43	NA 4912
	85	45	93 500	204 000	28 000	0.81	NA 6912
	90	28	68 200	120 000	15 600	0.56	NKIS 60
65	90	25	61 600	120 000	15 300	0.46	NA 4913
	90	35	82 600	163 000	21 600	0.47	NKI 65/25
	90	45	95 200	212 000	27 000	0.86	NKI 65/35
	95	28	70 400	132 000	17 000	0.83	NA 6913
					3 800	5.300	NKIS 65



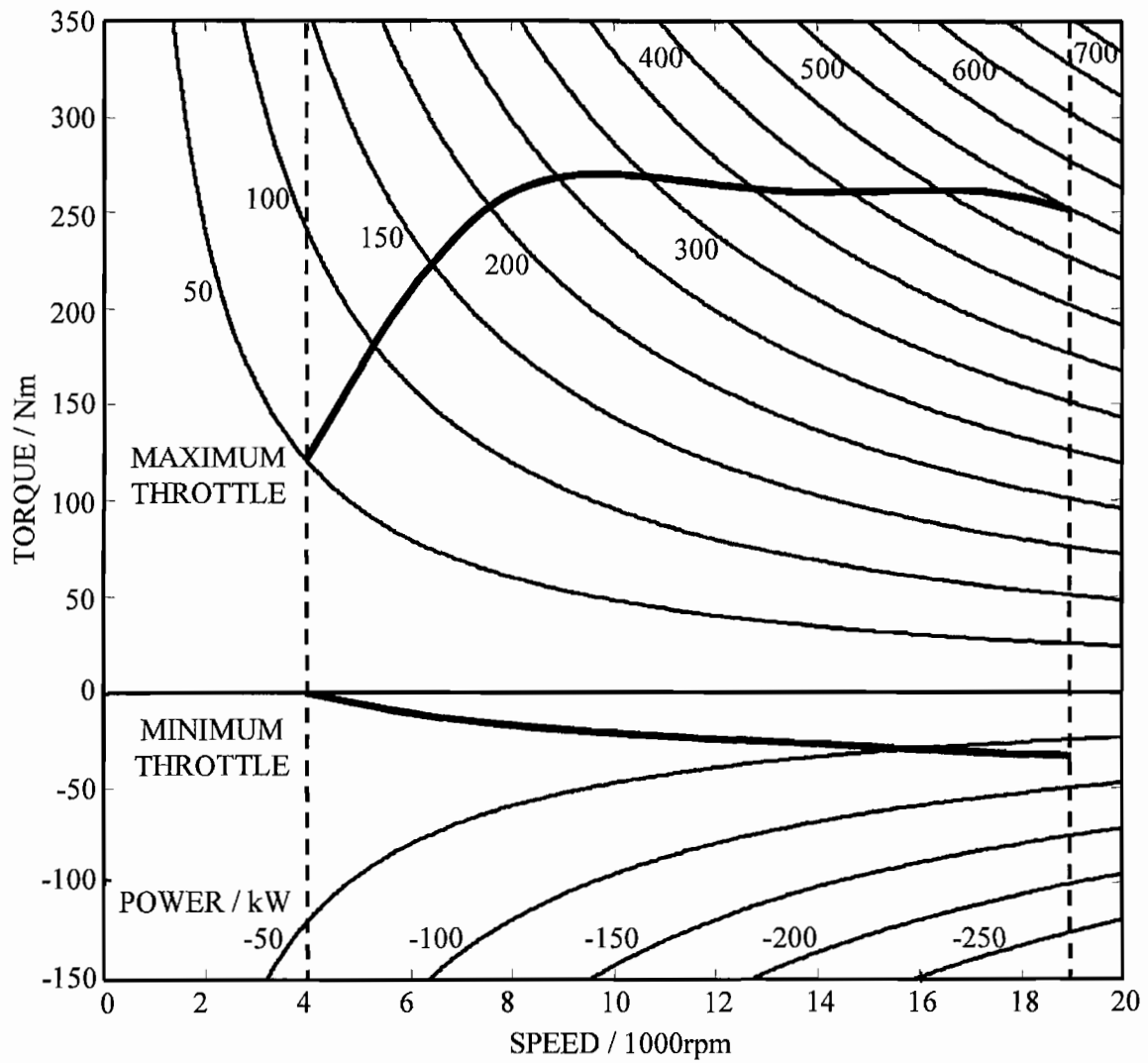


Fig. 1

Working sheet for Q 1  
(may be handed in with your script)

Answers

1 (b)  $99.5 \text{ ms}^{-1}$  (c)(i)  $5.15 : 1$  (ii)  $\sim 7.6 \text{ ms}^{-2}$  (iii)  $\sim 3.9 \text{ ms}^{-2}$

2 (c)(i)  $\frac{\omega_a}{\omega_e} = -\frac{B}{A}$  (ii)  $T_e = 3T_a$  ;  $T_f = 2T_a$  (iii)  $P_e : P_f = 3 : 1$

3 (c)  $k \geq 0.281 \sqrt[5]{\rho g l E} *^4$

4 (a) contact ratio = 1.67 single  $R_{\min} = 6.62 \text{ mm}$ ; double  $R_{\min} = 2.22 \text{ mm}$   
 $\phi' = 20.64^\circ$ , contact ratio = 1.51