ENGINEERING TRIPOS PART IIA

Monday 20 April 2009 9 to 10.30

Module 3D2

GEOTECHNICAL ENGINEERING II

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Geotechnical Engineering Data Book (19 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A retaining wall for a deep excavation is to be designed in a soft to firm overconsolidated clay for which the unit weight is 18 kN/m^3 and the critical state parameter M = 1.0. The water table is 2 m below ground level. High quality samples have been taken from a depth of 10 m. It is known that at a depth of 10 m the overconsolidation ratio (OCR) is 2.5 and the coefficient of earth pressure at rest $K_o = 0.8$.

(a) If the coefficient of earth pressure at rest for the clay when it was originally in a normally consolidated state, $K_{o,nc}$, is 0.55, calculate the previous maximum effective vertical and horizontal stresses for the samples. [15%]

(b) Two triaxial tests were undertaken on the samples. For both samples, the present day in-situ total and effective stresses were imposed prior to shearing. Calculate these stresses in terms of triaxial parameters q, p and p'. [15%]

(c) Plot the total and effective stress paths in q-p and q-p' space for each of the two tests, details of which are as follows, and answer the following questions.

(i) Test A was an undrained test to simulate rapid excavation in front of the retaining wall; the cell pressure was reduced keeping the total axial stress constant. The undrained shear strength was found to be 50 kPa. What was the measured pore pressure at failure? [30%]

(ii) Test B was a drained test to simulate very slow excavation in front of the retaining wall. It is assumed that the pore pressure in the sample remained at the original in situ value (there being an impermeable layer just below excavation level, and the retaining wall being impermeable). What was the measured strength at the end of the test? [30%]

(d) Comment briefly on the significance of the two tests to the designers. [10%]

2 A tunnel of radius R is to be constructed with its axis at a depth z below ground level in a stiff clay. The unit weight of the clay is γ, the undrained shear strength is c_u, and the elastic shear modulus is G. Isotropic conditions can be assumed.

(a) A self-boring pressuremeter test is undertaken at the level of the tunnel axis to establish values of c_u and G. Describe, with appropriate sketches, how both these parameters are determined from the test data, giving two methods by which c_u can be estimated. [30%]

(b) Assuming that the tunnel construction process can be approximated to the complete unloading of a cylindrical cavity under undrained conditions (the tunnel being temporarily unlined), show that the radial ground movement ρ_c at the tunnel boundary is given by

$$\rho_c = 0.5\, R\, (c_u/G)\, \exp(\gamma z/c_u - 1)$$ [40%]

(c) A tunnel of diameter 5 m is constructed with its axis 20 m below ground level. The unit weight of the clay is 20 kN/m^3, the undrained strength is 150 kPa and the elastic shear modulus is 30 MPa. A pipeline runs with its axis perpendicular to that of the tunnel, and at 5 m below ground level. Assuming that the tunnel is temporarily unlined, and that the pipeline is of small diameter and flexible, therefore deforming with the ground, estimate the maximum settlement of the pipeline. [30%]

(TURN OVER

3 (a) Use the SSA Cam Clay model of soil behaviour to distinguish between the phenomena of hardening and softening of clays that are sheared beyond their initial yield point, depending on their overconsolidation ratio. Make sketches on (τ, σ') and (v, σ') diagrams to illustrate your answer, and show how positive or negative excess pore pressures in undrained tests are the counterpart to volume changes that occur in drained tests. [30%]

(b) Identical samples of clay slurry are to be placed in a Simple Shear Apparatus (SSA) and consolidated to a maximum normal effective stress σ'_c. They may then be allowed to swell back to an effective normal stress σ'_o. These samples will then be sheared under a constant total normal stress σ whilst following a variety of test conditions. Use the SSA Cam Clay model to derive algebraic relationships between the normalised shear strength ($\tau_{strength}/\sigma'_o$) and the overconsolidation ratio (σ'_c/σ'_o) of the soil, taking each of the following criteria for $\tau_{strength}$:

(i) ultimate shear stress ($\tau_{u,ult}$) recorded in an undrained shear test;

(ii) maximum shear stress ($\tau_{u,max}$) recorded in an undrained shear test;

(iii) ultimate shear stress ($\tau_{d,ult}$) recorded in a drained shear test;

(iv) maximum shear stress ($\tau_{d,max}$) recorded in a drained shear test.

In each case evaluate the normalised shear strength for overconsolidation ratios of 1 and 10, using appropriate data for Weald Clay. Do not try to account for progressive failure, but mention where this phenomenon might occur. [50%]

(c) For both low over-consolidation ratio (e.g. 1) and high over-consolidation ratio (e.g. 10), discuss the significance of the relative values calculated for each of the cases (i) to (iv). Refer to the contrasting stability problems that must be anticipated when making cut slopes in soft muds and stiff clays. [20%]

4 (a) Why is knowledge of the relative density I_D of a sand insufficient to predict its strength and dilatancy? [25%]

(b) Using values given in the Geotechnical Engineering Data Book, estimate the vertical effective confining pressure σ'_{crit} ultimately developed by medium-dense Ham River Sand (relative density 0.75) tested at constant volume in a specially adapted Simple Shear Apparatus. Find the corresponding critical state shear strength τ_{crit}. Give an example of an application where these values might be relevant. [25%]

(c) Use the definition of relative dilatancy to estimate the peak angles of friction ϕ_{max} and dilation ψ_{max} of two further identical medium-dense samples, but sheared at constant vertical effective stresses of $0.1\,\sigma'_{crit}$ and $0.01\,\sigma'_{crit}$ respectively. [25%]

(d) Compare linear and power-law fittings of peak strength envelopes for the Ham River Sand at a relative density of 0.75, using your results from (b) and (c). Point to some dangers in defining a "true cohesion intercept" for dilatant soils, and explain why critical state strengths may be more appropriate in design. [25%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Geotechnical Engineering

Data Book 2007-2008

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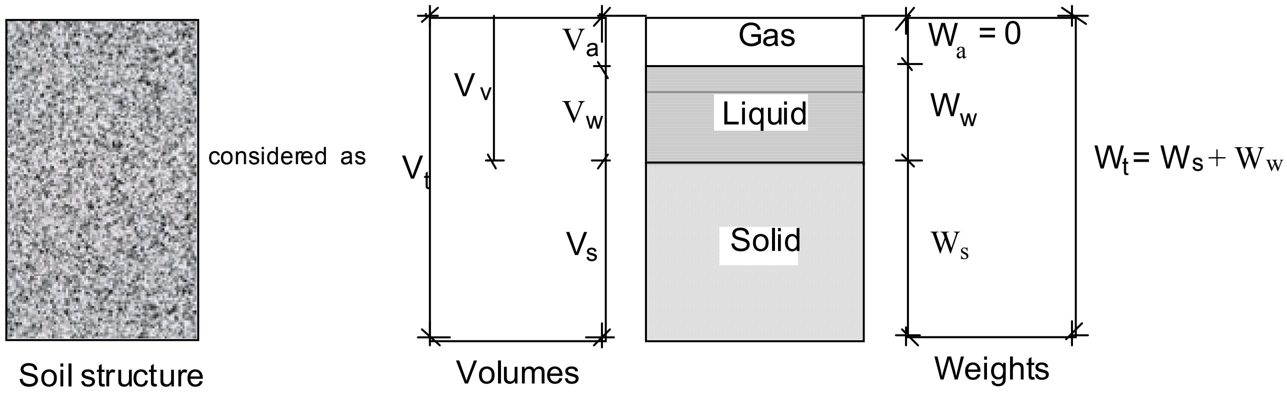
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

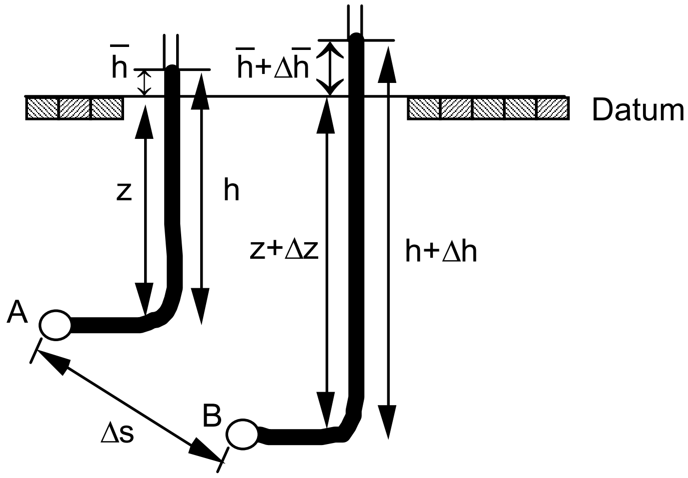
D equivalent diameter of soil particle

D_{10}, D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} >$ 1μm	:	$k \cong 0.01\ (D_{10}\text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

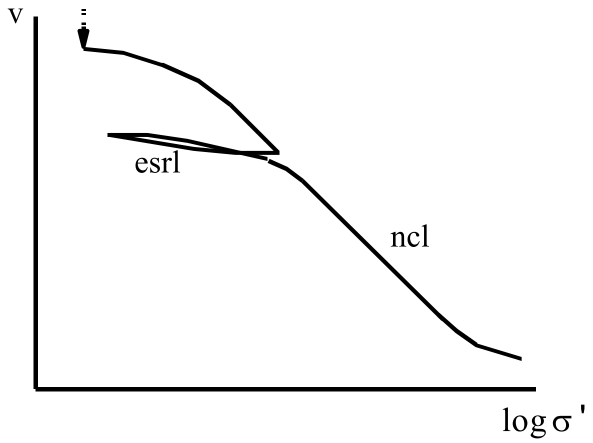
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

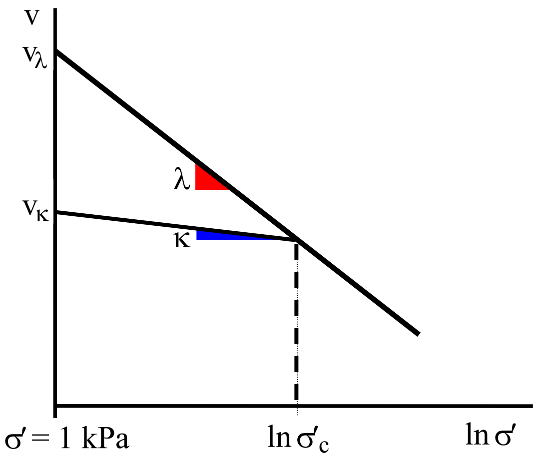
One-Dimensional Compression

• Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

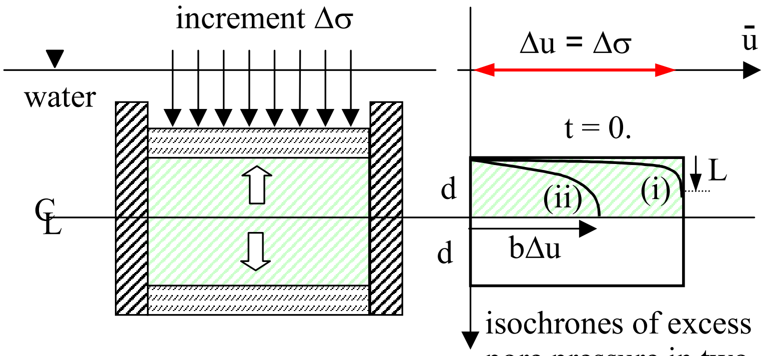
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz \quad = \int (\Delta u - \bar{u}) / E_o \, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w} \quad = \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $\quad L^2 = 12\, c_v t$

$$R_v = \sqrt{\frac{4T_v}{3}} \qquad \text{for } T_v < {}^1/_{12}$$

Phase (ii) $\quad b = \exp(\tfrac{1}{4} - 3T_v)$

$$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)] \qquad \text{for } T_v > {}^1/_{12}$$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	σ_1, σ_2, σ_3
effective stress	σ_1' , σ_2' , σ_3'
strain	ε_1, ε_2, ε_3

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2' , in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

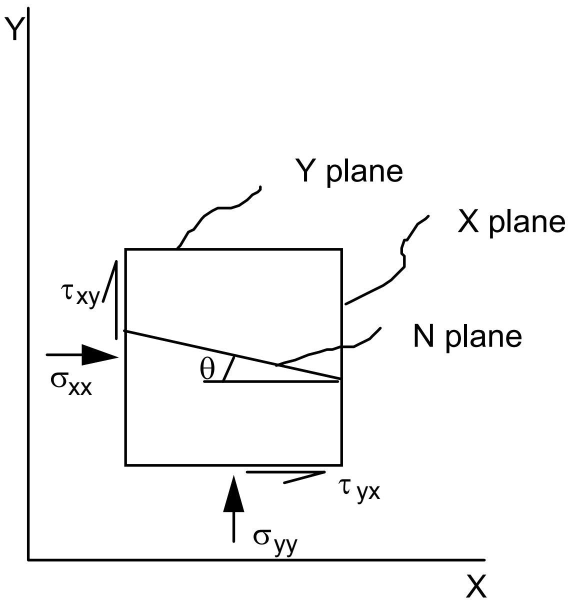
isotropic compression in which p' increases at zero q

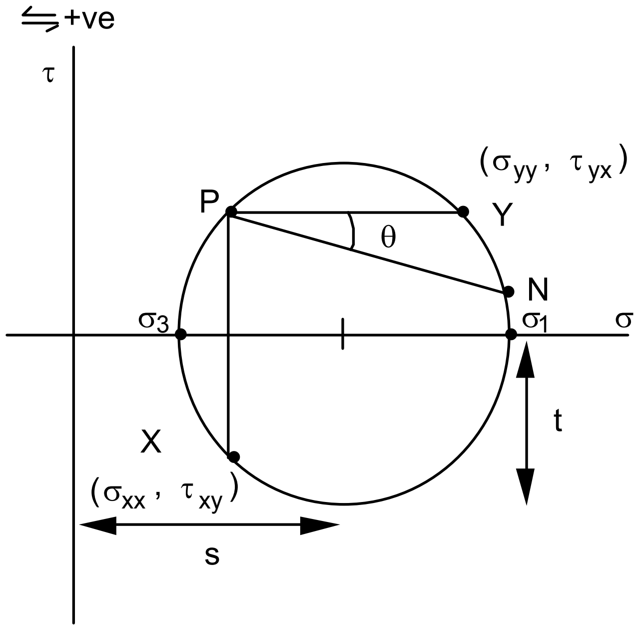
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $m_v = d\varepsilon / d\sigma'$

constrained modulus $E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $G' = dt / d\varepsilon_\gamma$

bulk modulus $K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $G_u = G'$

undrained bulk modulus $K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli E' (effective), E_u (undrained)

Poisson's ratios ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2\,(1 + \nu)}$$

$$K = \frac{E}{3\,(1 - 2\nu)}$$

$$E_o = \frac{E\,(1 - \nu)}{(1 + \nu)\,(1 - 2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

• General yield surface

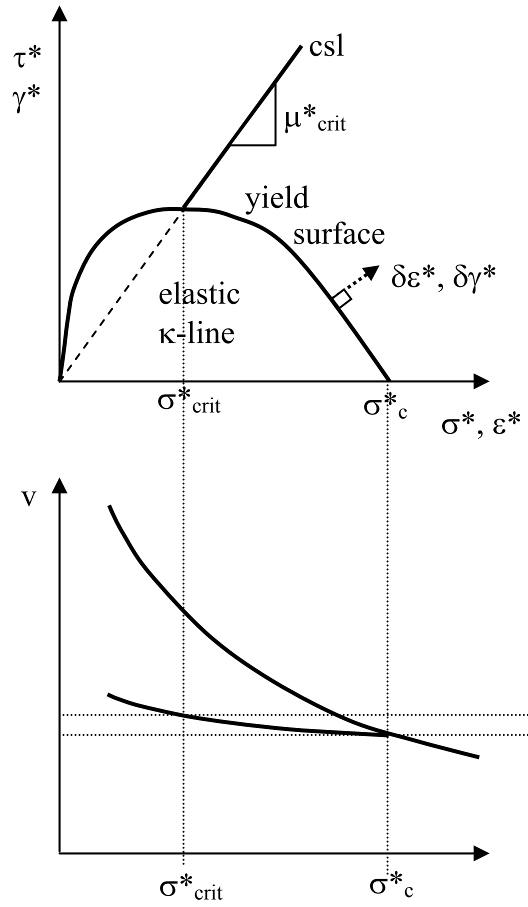
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

• Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\ virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• The yield surface in (σ*, τ*, v) space



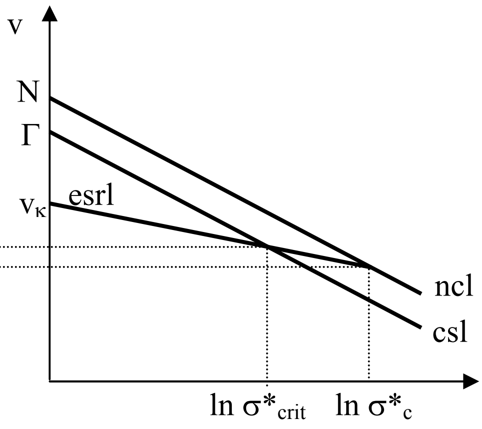
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

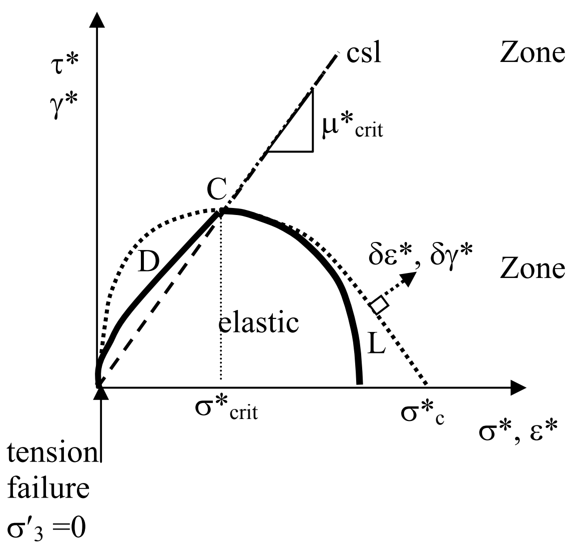
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



• Regions of limiting soil behaviour



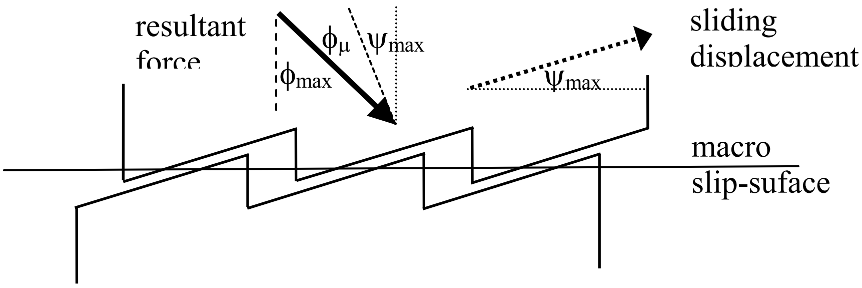
Variation of Cam Clay yield surface

Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

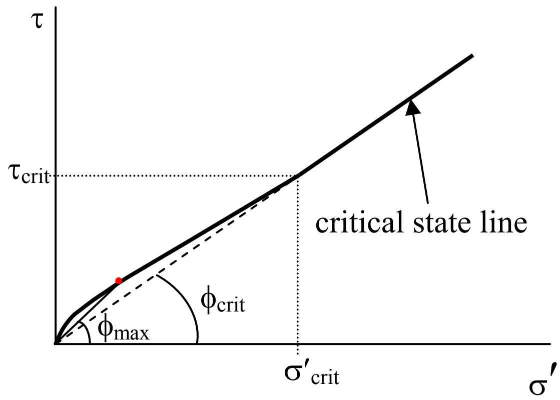


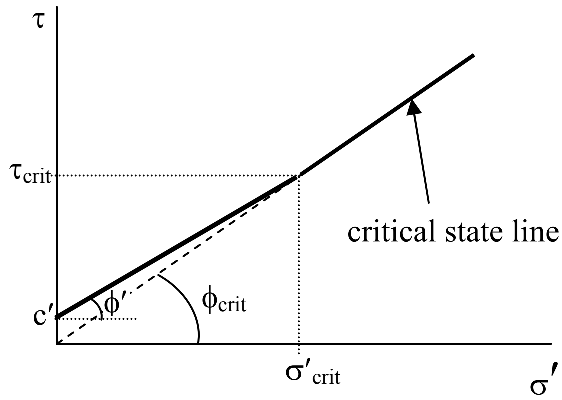
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} (± 2°) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $\quad I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ $\quad$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $\quad I_C = \ln(\sigma_c / p')$ $\quad$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

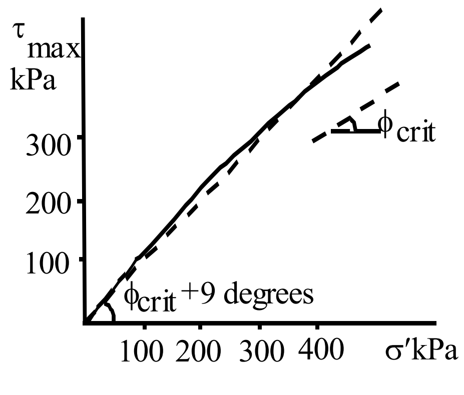
Relative dilatancy index $I_R = I_D I_C - 1$ $\quad$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

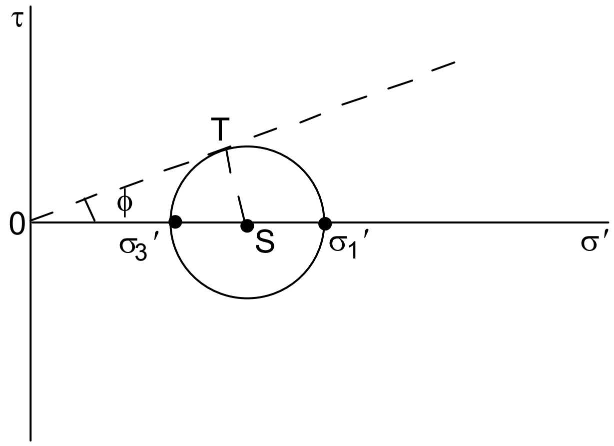
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' = 400$ kPa

- **Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane**



$$\sin\phi = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

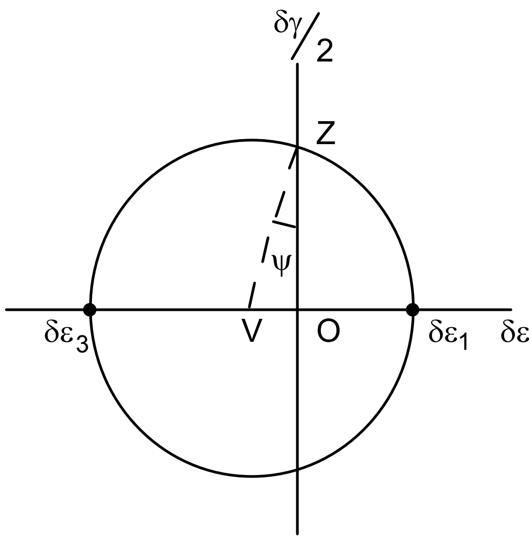
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

- **Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane**



$$\sin\psi = VO/VZ$$

$$= -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2}$$

$$= -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: **Cohesive material** $\tau_{max} = c_u$ **(or s_u)**

• **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

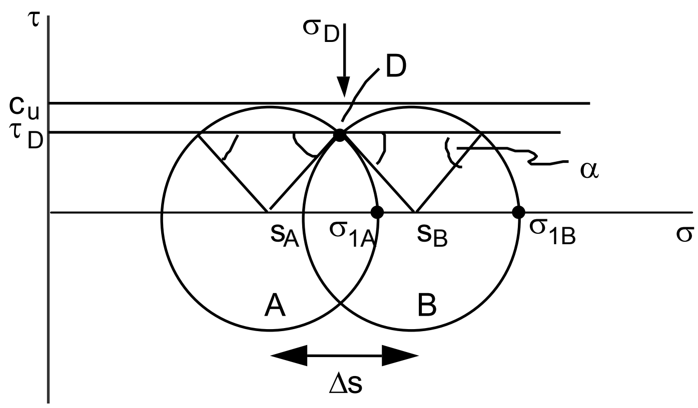
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

• **Stress conditions across a discontinuity**

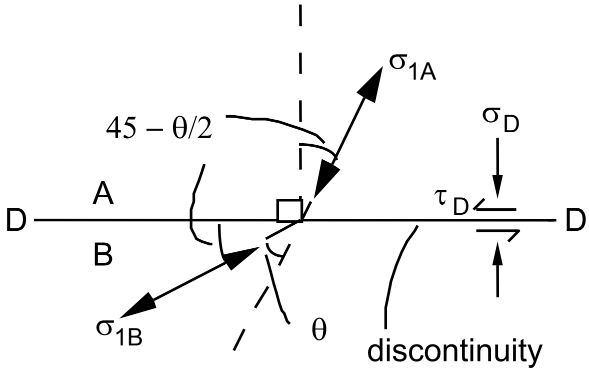


Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$
$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \rightarrow 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$\theta = 30^o$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

• Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

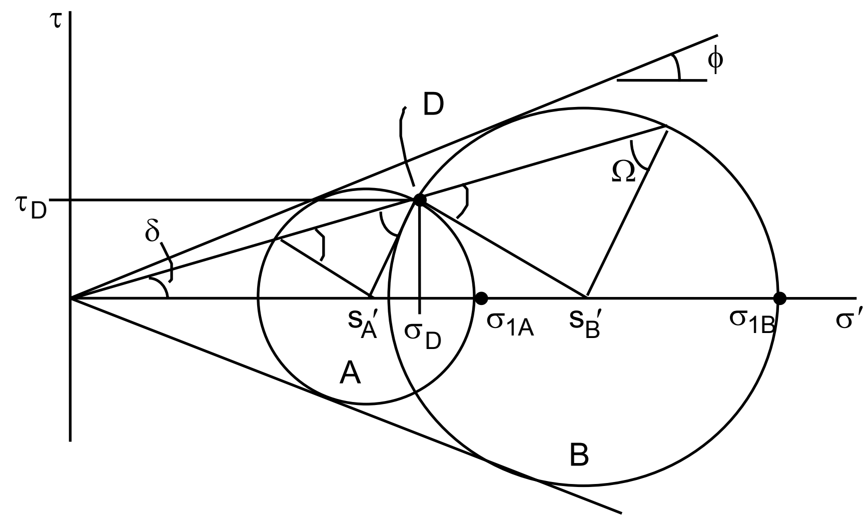
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

• Stress conditions across a discontinuity



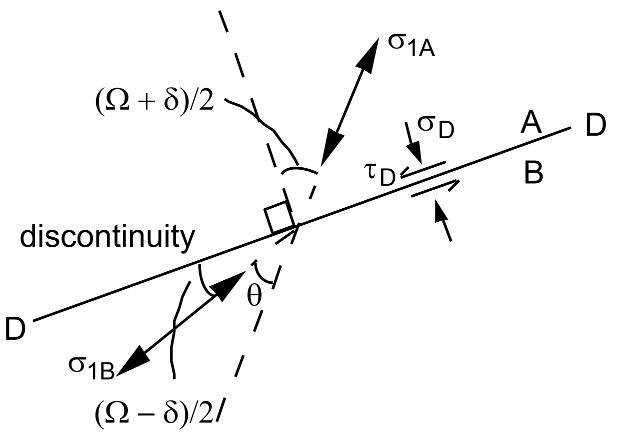
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \frac{\delta A}{2\pi r}$

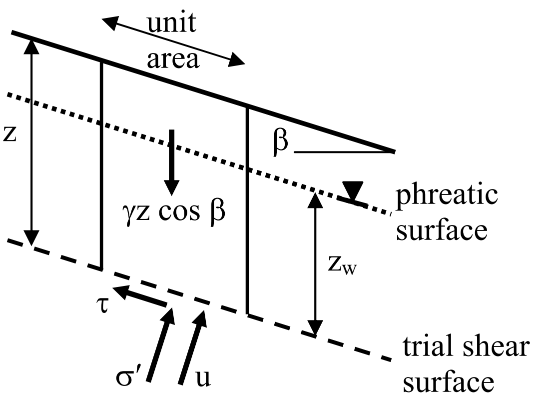
small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r\frac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$

Infinite slope analysis



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\ B / L$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1}(h/B)$ (or h/D for a circular foundation)

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}}$ or $\frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$

If $V/V_{ult} < 0.5$: $H = H_{ult} = B s_u$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$ (Taiebet & Carter 2000)

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

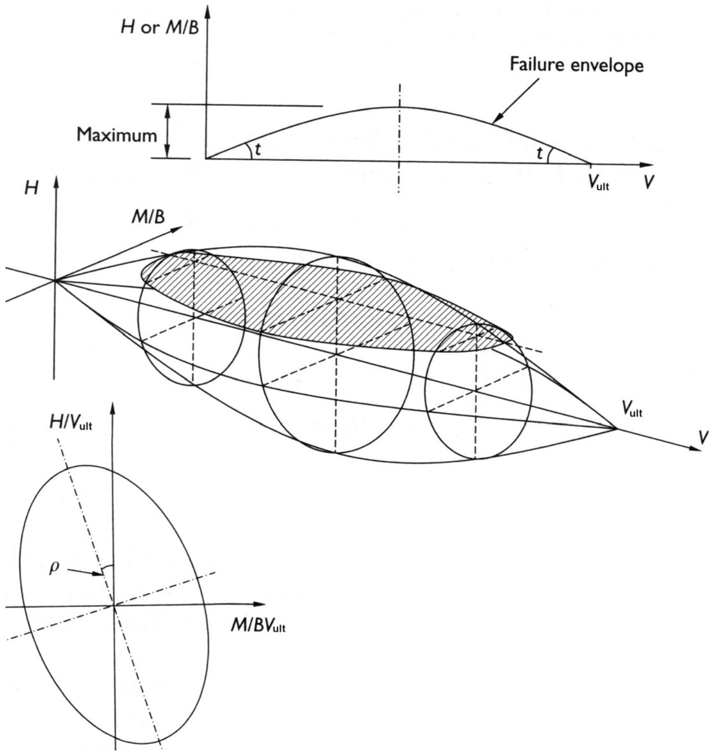
For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.



Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. Note that t_h is the friction coefficient, H/V= tanϕ, during sliding.

Engineering Tripos Part IIA 2009 Module 3D2 Answers

1 (a) 250 kPa and 138 kPa (taking $\gamma_w = 10$ kN/m^3)

(b) $q = 20$ kPa, $p = 166$ kPa, $p' = 86$ kPa

(c) (i) 13 kPa (ii) 60 kPa

(d) the clay in the excavation will soften as time passes, and drainage takes place

2 (c) 5.5 mm

3 (b) (i) $\frac{\tau_{u,ult}}{\sigma'_o} = \tan\phi_{crit}\left[\frac{\sigma'_c}{2.72\,\sigma'_o}\right]^{1-\kappa/\lambda}$

(ii) Find $\frac{\tau_y}{\sigma'_o} = \tan\phi_{crit}\,\ln\left[\frac{\sigma'_c}{\sigma'_o}\right]$. Then $\frac{\tau_{u,\max}}{\sigma'_o} =$ the greater of $\frac{\tau_y}{\sigma'_o}$ and $\frac{\tau_{u,ult}}{\sigma'_o}$.

(iii) $\frac{\tau_{d,ult}}{\sigma'_o} = \tan\phi_{crit}$

(iv) $\frac{\tau_{d,\max}}{\sigma'_o} =$ the greater of $\frac{\tau_y}{\sigma'_o}$ and $\frac{\tau_{d,ult}}{\sigma'_o}$

For OCR = 1: 0.24, 0.24, 0.45, 0.45

For OCR = 10: 1.00, 1.02, 0.45, 1.02

(c) For OCR = 1, mud slumps to far below its critical state angle.

For OCR = 10, stiff clay initially stands steeply, but it can soften to its residual friction angle, about half its critical state angle, on slip surfaces.

4 (b) 3950 kPa and 2470 kPa (taking $\sigma'_c = 15000$ kPa)

(c) 40.6° and 10.8°; 49.3° and 21.6°

(d) Linear fit: $c' = 13$ kPa, $\phi' = 39.5°$

Power law fit: $\beta = 0.86$ correctly passes through the origin and the critical state.