ENGINEERING TRIPOS PART IIA

Monday 20 April 2009 2.30 to 4

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Where indicated, "ULS" denotes "Ultimate Limit State".

Attachments: Special datasheets (11 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 (a) A uniform rectangular plate has side-lengths $2L$ and L. The plate everywhere carries a transverse distributed load, w, per unit area. The plate is clamped on opposite sides of length L and is simply supported on opposite sides of length $2L$. The fully plastic moment per unit width of a given strip of plate is M_p.

(i) Using the Lower Bound Theorem, find a safe value of load at the ULS if w is assumed to be carried by a *single* set of strips running parallel to either side of plate. [25%]

(ii) It is now assumed that w is carried by two sets of orthogonal strips parallel to each side of plate. For the case when w is shared *equally* between the strips, show that the best estimate in (i) is not altered. For the case when w is not shared equally, show that the maximum ULS load increases. [25%]

(b) An extensive floor structure consists of a uniform slab carried on a symmetrical network of identical horizontal beams and upright columns, as shown in plan in Fig. 1. The self-weight per unit area of the slab is w.

(i) Using the Lower Bound Theorem and making clear any assumptions about chosen load paths, show that the force in any given column at the ULS is $4wL^2$. [15%]

(ii) For any of the beams *not* connected to any column, provide a safe estimate of the maximum bending moment at the ULS in those beams. [35%]

(cont.

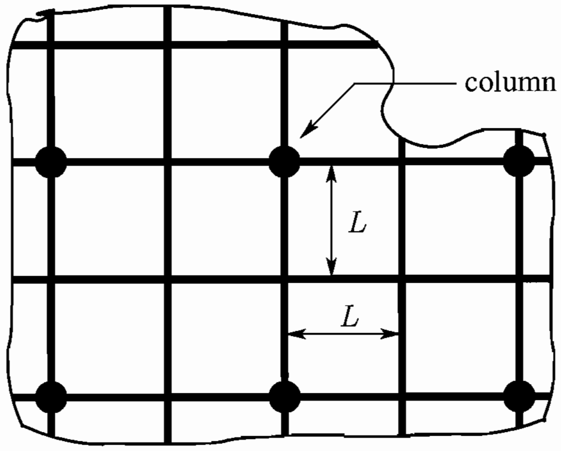


Fig. 1

(TURN OVER

2 A flat timber roof consists of a series of simply-supported C18, 75 mm wide timber beams, spanning 4500 mm and spaced at 600 mm centres. A plywood decking is screwed to the top of the beams and, together, they carry an unfactored vertical load of 2.5 kPa. Assume $k_{mod} = k_h = 1.0$, $k_{ls} = k_{c90} = 1.1$, $\gamma_m = 1.3$ and the load factor for the ULS is 1.5.

(a) Determine the minimum depth of the timber beams required to satisfy shear and flexural strength requirements. Explain the reasons for your selection of k_{crit}. [25%]

(b) Determine the minimum bearing length required at the support ends of the beams. [15%]

(c) By taking into account both shear and flexural deflections, determine the total instantaneous deflection at mid-span. Comment on whether you consider this deflection to be acceptable. [25%]

(d) Due to access restrictions on-site you have been asked to consider splicing the beams at mid-span to form a single lap joint at this location. The joint consists of two 50 mm diameter bolts in pre-drilled holes positioned as shown in Fig. 2.

(i) Assuming that the bolt is rigid (*i.e.* no hinges form in the bolt) and that all the forces due to flexure are transmitted through the bolts, use Johanson's theory and perform an Upper Bound calculation to determine the minimum distance b between the bolts. [25%]

(ii) Use your answers above to explain whether the proposed splice is an appropriate solution and without carrying out further calculations suggest possible improvements. [10%]

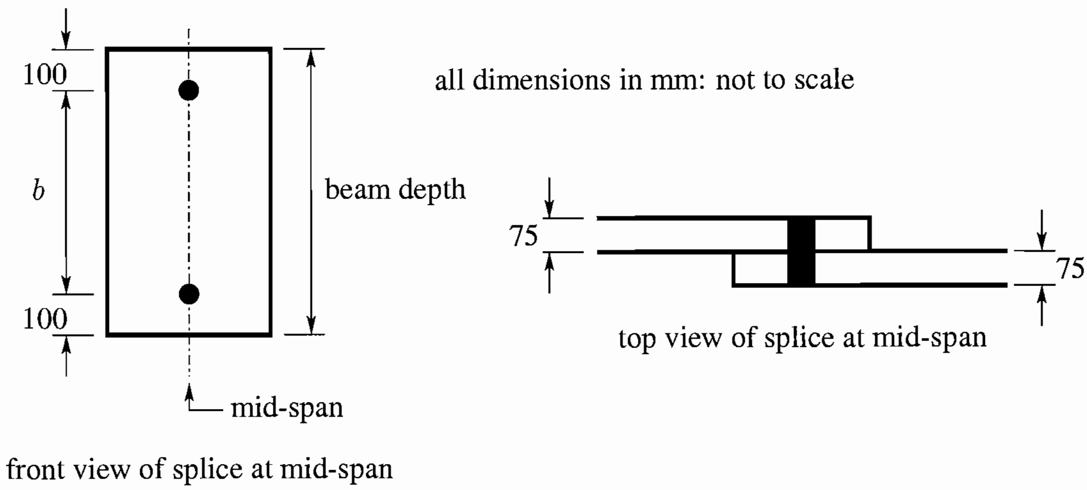


Fig. 2

(TURN OVER

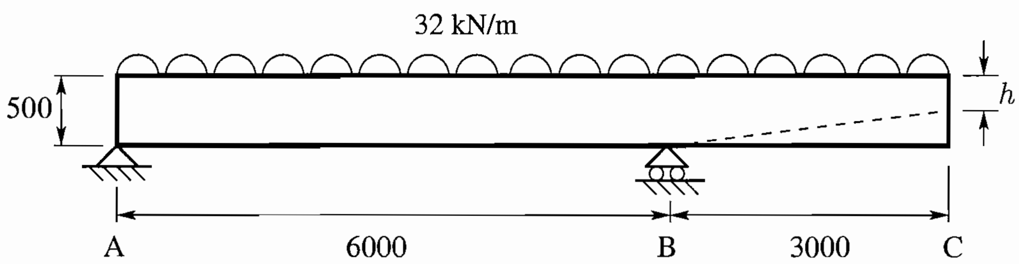
3 A 9 m long reinforced concrete beam is supported at A and B as shown in Fig. 3. The beam is required to carry a uniformly distributed working live load of 32 kN/m and its self weight. The concrete cube strength is 40 MPa and the beam cross-section is 250 mm wide by 500 mm deep with 40 mm cover. The longitudinal reinforcement bars have a diameter of 25 mm and a yield stress of 460 MPa. The shear reinforcement bars have a diameter of 8 mm and a yield stress of 250 MPa. The partial safety factors for concrete and steel are 1.5 and 1.15 respectively, and the load factors for dead and live loads are 1.4 and 1.6 respectively.

(a) By sketching the bending moment and shear force diagrams, determine the maximum shear force, sagging moment and hogging moment, and identify the locations of the critical cross sections along the beam at which they occur. [20%]

(b) Choose a layout of longitudinal reinforcement for maximum hogging, and another layout for maximum sagging. Without carrying out further calculations, indicate an efficient layout of longitudinal reinforcement along the beam. [40%]

(c) Identify the zones along the beam where the shear reinforcement is required. [25%]

(d) After completing your design for the beam, you have received a request to taper the cantilever end of the beam so that the cross-section height reduces linearly from 500 mm at B to h mm at C. The width of the beam remains unchanged. Determine the minimum depth h that may be achieved by keeping the cross sectional area of reinforcement from your original design unchanged. [15%]



all dimensions in mm: not to scale

Fig. 3

4 Figure 4 shows in elevation part of a typical portal frame of the indicated dimensions. The frame is symmetrical, has a shallow pitched roof, and is stiffened at its haunches, as shown. Its feet are pinned and the factored ULS vertical loading is 8.5 kN per horizontal metre, uniformly applied span-wise to the rafter. The frame is made from grade S275 steel and does not deflect out of plane.

(a) Neglecting the self-weight of the rafter and assuming a fixed-ended beam between the end haunches, show that the required fully plastic moment is approximately 417 kNm. Show that this moment can be safely carried by a 457 × 191 × 74 UB bending about its major axis, including its self-weight. [25%]

(b) By considering a symmetrical collapse mechanism for the actual frame under the revised design load but where negative external work is not included, show that the required fully plastic moment reduces by approximately 18%, and suggest a lighter UB for the rafter. [35%]

(c) Calculate the axial force in the rafter and the reaction at each foot and, hence, draw a bending moment diagram for the frame under the revised design load. Decide whether a heavier UB is required. [40%]

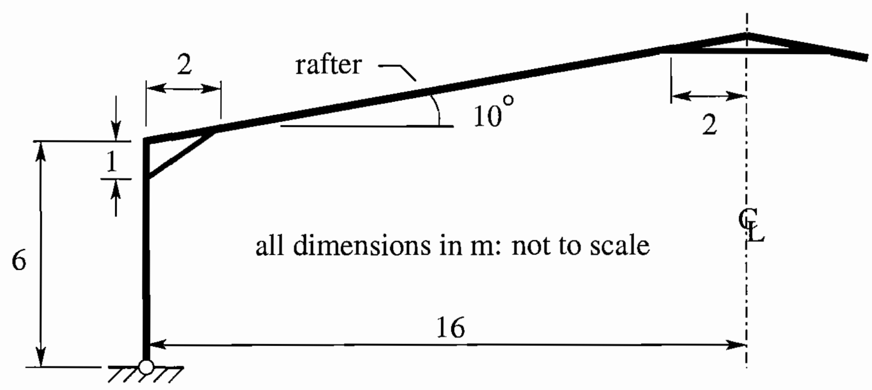


Fig. 4

END OF PAPER

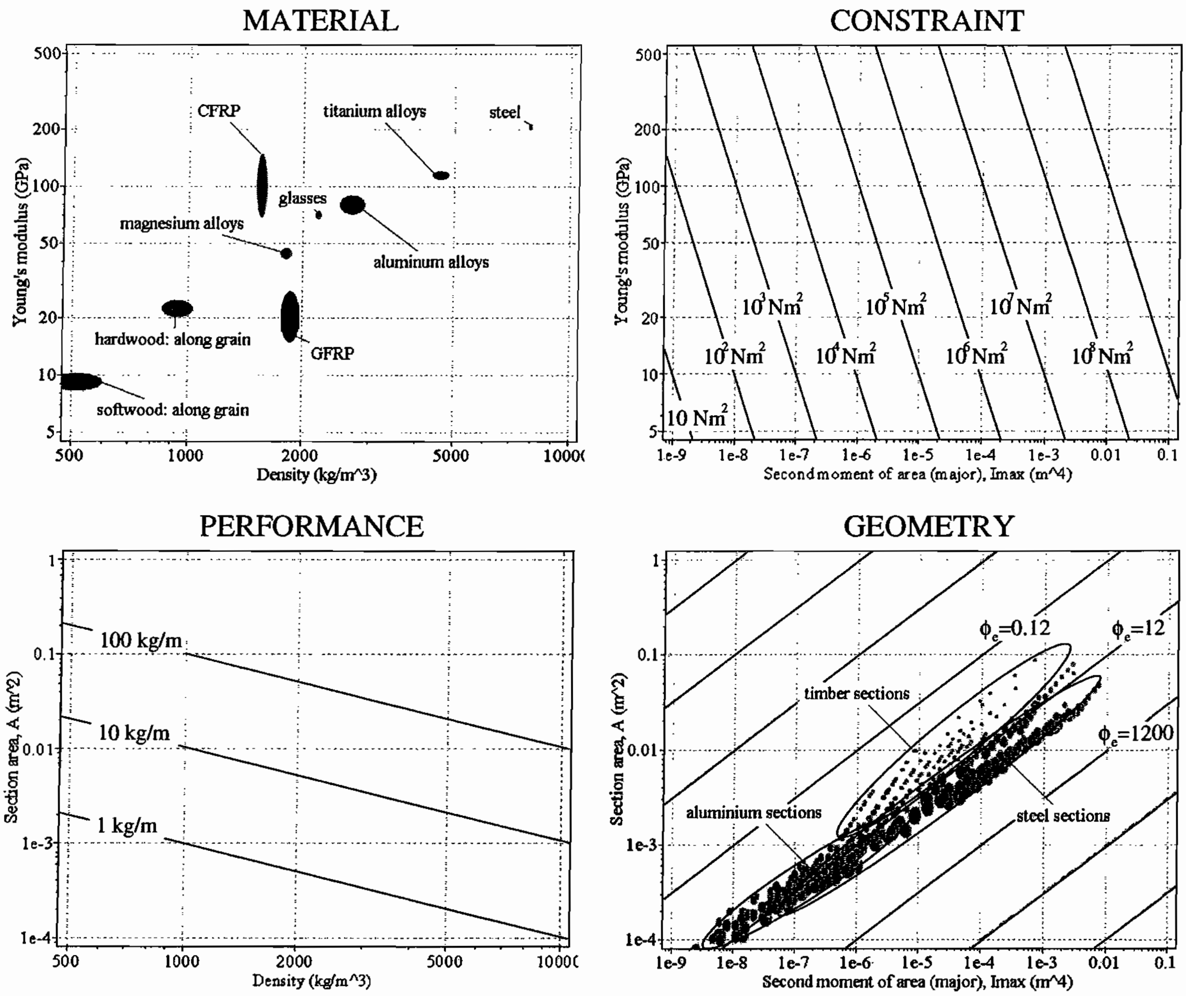
3D3: Structural Materials and Design

Data Sheets: 2008/09

K A Seffen
February 3, 2009

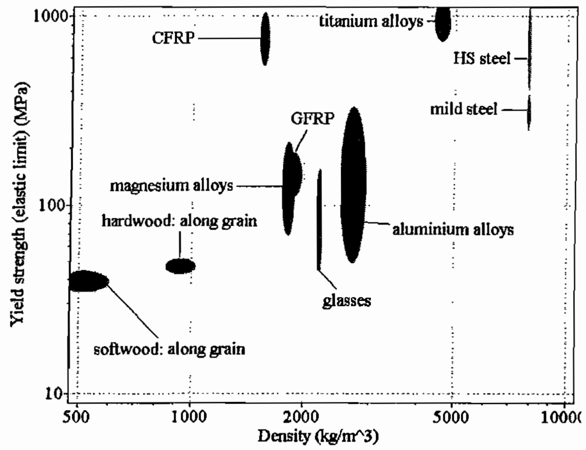
Selection of Material and Shape: Design for Bending

Stiffness. Use in conjunction with the cyclic method described in notes: note that axes are shared.

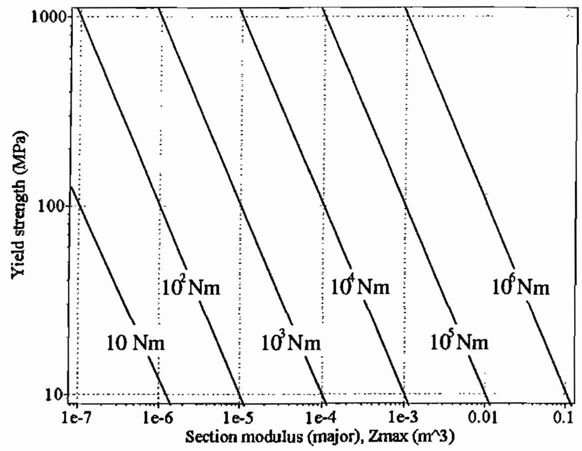


Strength. Use in conjunction with the cyclic method described in notes: note that axes are shared.

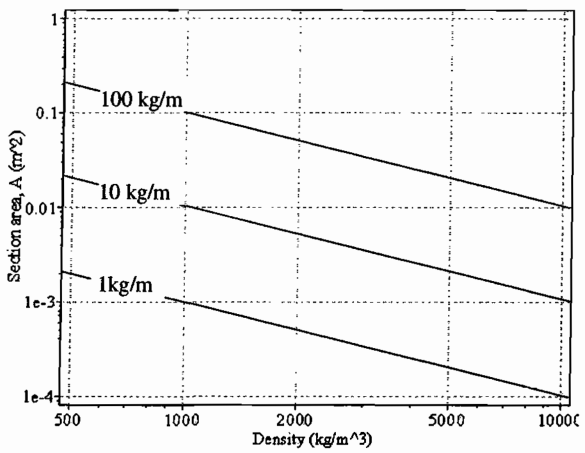
MATERIAL



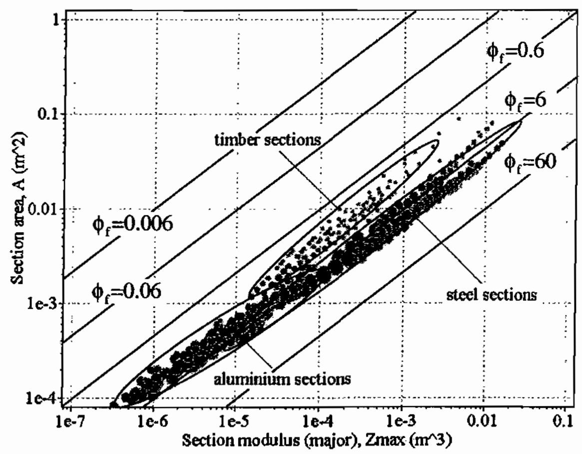
CONSTRAINT



PERFORMANCE



GEOMETRY

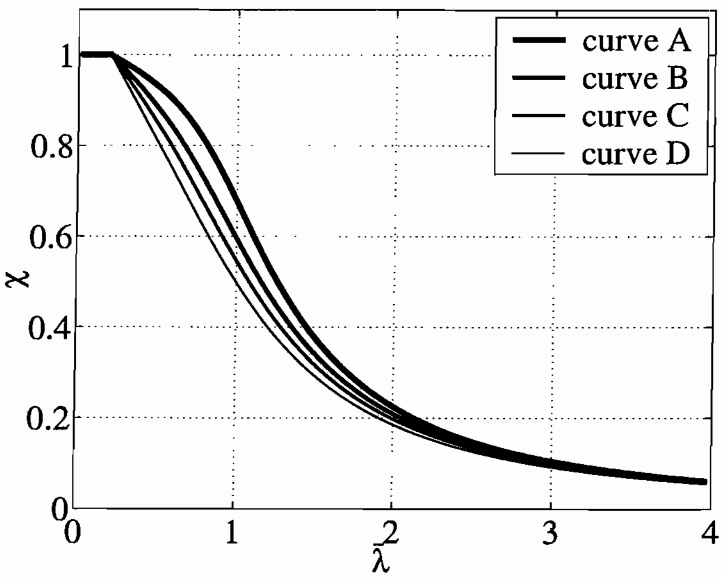


Steel Data Sheet

Material. The two most common steels are S275 and S355, with characteristic yield strengths, σ_y, of 275 MPa and 355 MPa, respectively. The design strength divides σ_y by a specified partial safety factor, γ_m. Partial safety factors for loads at ULS are often 1.4 for dead loads and 1.6 for live loads.

Tension members (axial force only). Gross cross-section area is A; net area, A_n, is A subtract hole(s). Effective section is KA_n but not greater than A, where K is 1.2 for S275 or 1.1 for S355. For eccentric connection, with area, A_{out}, not connected at the joint, the effective area is $A_e - cA_{\text{out}}$, where c is 0.5 bolts or 0.3 for welds.

Compression members (axial force only). Radius of gyration is r, extreme fibre distance is y, effective length of column is L, and $\lambda = L/r$. Define $\bar{\lambda} = \lambda/\lambda_0$ where $\lambda_0 = \pi\sqrt{E/\sigma_y}$ and a reduction factor, χ, on the full axial yield strength, equal to σ_c/σ_y where σ_c is the critical buckling stress. Buckling performance given by:



Select curve A: $r/y > 0.7$; B: $0.5 < r/y < 0.7$; C: $0.5 < r/y$; D: only when the flange thickness is larger than 40 mm.

Beams (without axial force).

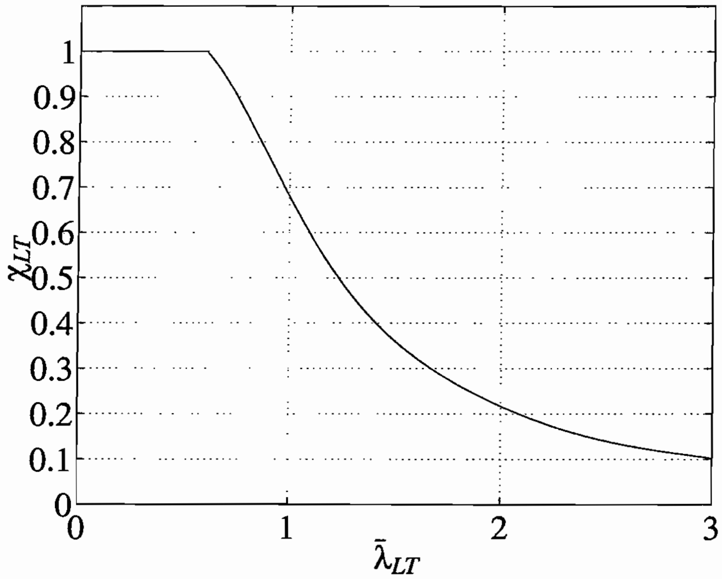
Moment: check maximum moment is less than $\sigma_y Z_p$. Beware local buckling for thin-walled sections.

Shear: yield strength, q_w, given by $0.6\sigma_y$. Check buckling stress capacity, q_b, is not exceeded in thin webs with thickness, t, and panel aspect ratio, a/b ($\leftrightarrow$ / $\updownarrow$), where $q_b = \left[3/4 + b^2/a^2\right] \cdot 1000/(b/t)^2$ in MPa.

Lateral torsional buckling, (LTB); theoretical elastic critical moment, M_c, for a beam of span L under constant moment (and supported at its ends only where lateral deflection and twist are prevented), then

$$M_c = \frac{\pi}{L}\left[EI_{yy}\left(GJ + \frac{\pi^2}{L^2}EC_w\right)\right]^{0.5}$$

where C_w is a constant due to the restraining (stiffening) effect of *warping*, equal to $D^2 I_{yy}/4$, D being the distance between flange centres. Design curve is given by



where $\bar{\lambda}_{LT} = \sqrt{M_p/M_c}$ and $\chi_{LT} = M_{cr}/M_p$. M_{cr} is the critical moment, which must be greater than the maximum moment in practice: $M_{cr} > M_{\max}$ for uniform bending moment case; $M_{cr} > 0.8M_{\max}$ for centrally loaded, simply supported case.

Joint design. Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design, often with a transmitted force uniformly distributed across the various fasteners involved. For a bolted joint in shear, a couple, C, about its centre can be taken simply by extra shear forces, F_i, on each bolt perpendicular to the line to the centre of the bolt group and proportional to the distance, d_i from the centre, so that $F_i = Cd_i/\sum d_i^2$.

Applied shear forces are commonly checked against the shear strength ($0.6\sigma_y$) of the bolt, depending on the number of active shear planes; and against bearing strength in each plate, $\sigma_y dt$ where $d \times t$ is the bolt diameter times plate thickness.

3D3 – Structural Materials and Design – Concrete Datasheet

Structural system	Span/effective depth ratio	
	EC2*	
	high	light
1. Simply supported beam, one-way or two-way spanning simply supported slab	14	20
2. End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	18	26
3. Interior span of beam or one-way or two-way spanning slab	20	30
4. Slab supported on columns without beams (flat slab), based on longer span	17	24
5. Cantilever	6	8

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 7.4N, NA.5 [10.2]

Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

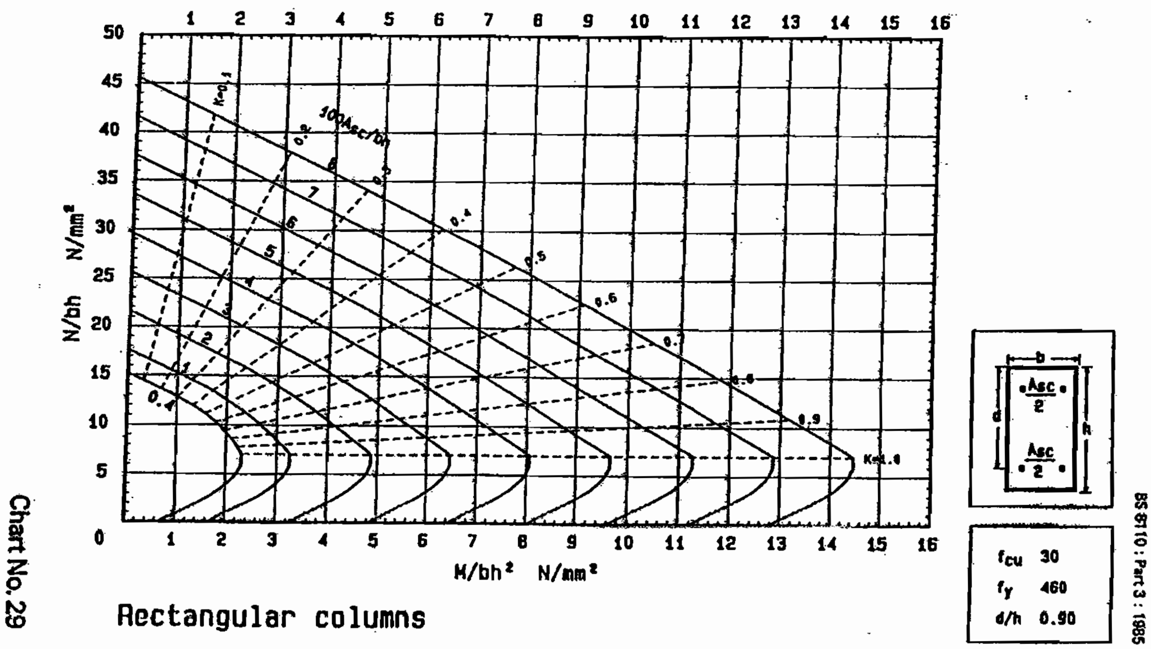


Fig 10.1 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures, EN 1992-1-1:2004, UK National Annex –NA to BS EN 1992-1-1:2004
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure
Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1-0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if x/d =0.5

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y \phi}{4(2.25 \eta_1 \eta_2 f_{ctd})}$$

where: η_1 is 1.0 for good bond, 0.7 otherwise
η_2 is 1.0 for $\phi \leq 32$

Cracking

$$w_k = s_{r,\max} (\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$$

where: k_1 is 0.8 for high bond, 1.6 for plain bars
k_2 is 1.0 for pure tension, 0.5 for bending
k_3, k_4 factors (in UK k_3=3.4 and k_4=0.425)
$\rho_{p,eff}$ is the effective steel ratio A_s/A_{ceff}

Shear
Without internal stirrups

$$V_{Rd,c} = [\frac{0.18}{\gamma_c} k(100 \rho_1 f_{ck})^{1/3}] b_w d \geq (0.035 k^{3/2} f_{ck}^{1/2}) b_w d$$

where: f_{ck} is the characteristic concrete compressive cylinder strength (MPa).
$k = 1 + \sqrt{200/d} \leq 2.0$ (d in mm)
$\rho_l = A_s/b_w d \leq 0.02$

With internal stirrups
- Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where: $f_{c,\max} = 0.6(1 - f_{ck}/250) f_{cd}$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d)(\cot\theta)/(s\gamma_s)$$

Columns – axial loading only

$$\sigma_u = 0.6 \frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm) - 6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\parallel$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\parallel$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\parallel$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	Service class 2	Service class 3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - *Design bending strength*

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – *Design shear stress*

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – *Design bearing stress*

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k}/\sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda_{rel,m}^2 & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k$ N/mm^2

For bolts up to 30 mm diameter at an angle α to the grain:

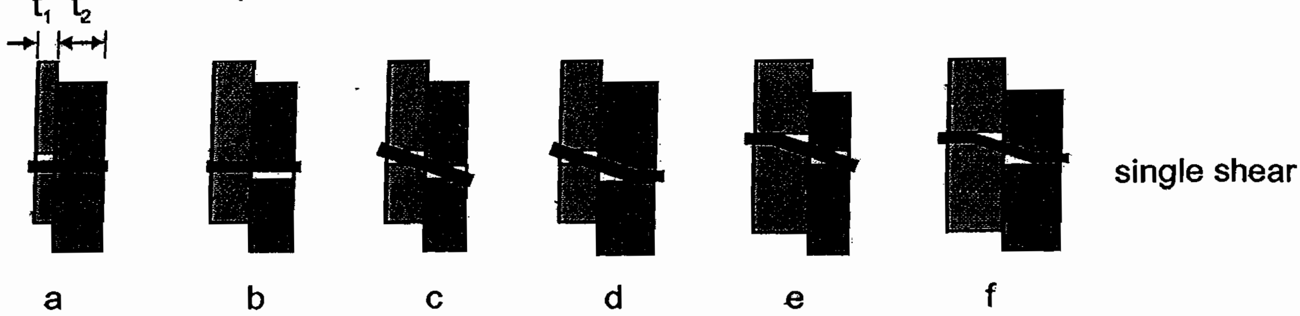
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min. \begin{cases} f_{h,1,d}\, t_1\, d & (6.2.1a) \\ f_{h,1,d}\, t_2\, d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1\, d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d\, t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2\, d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d\, t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber-to-timber and panel-to-timber joints

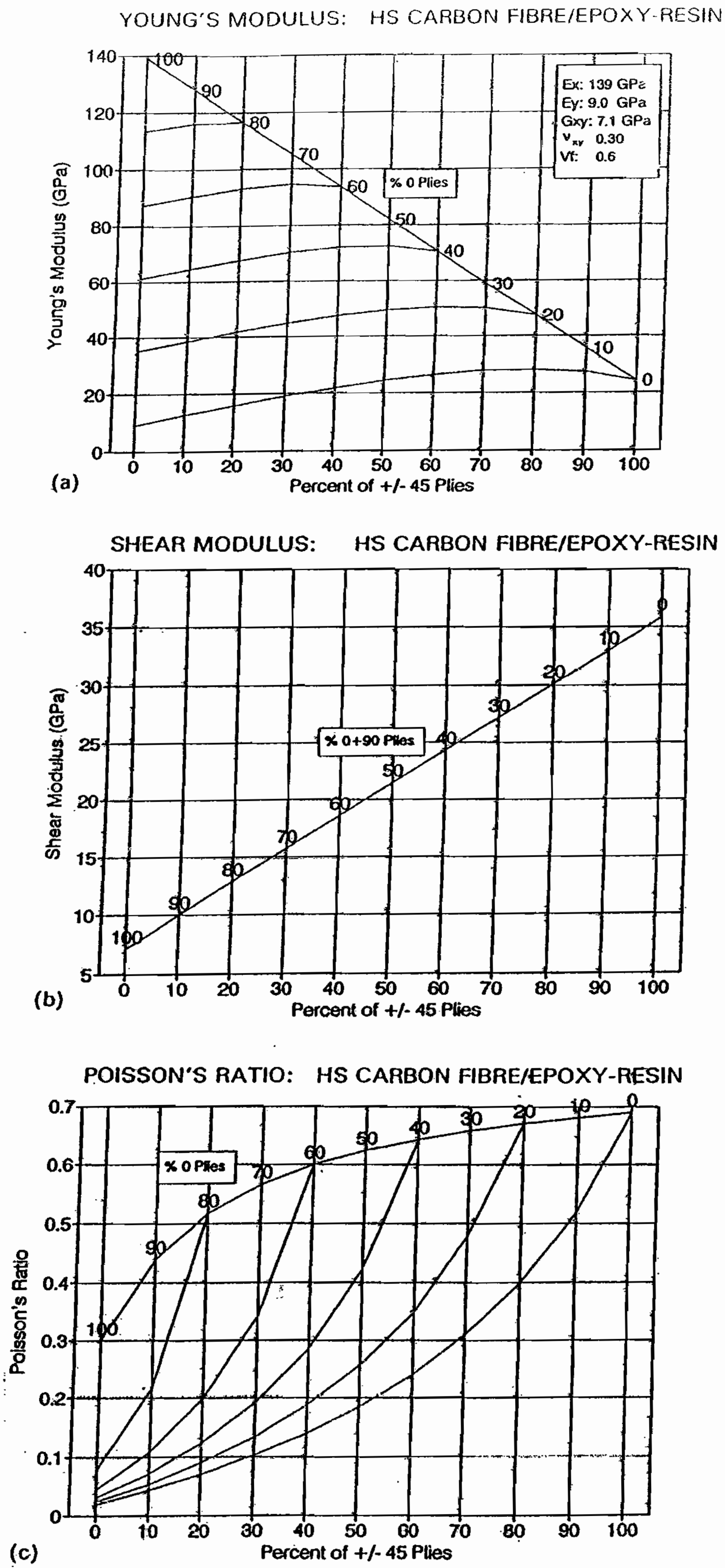


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

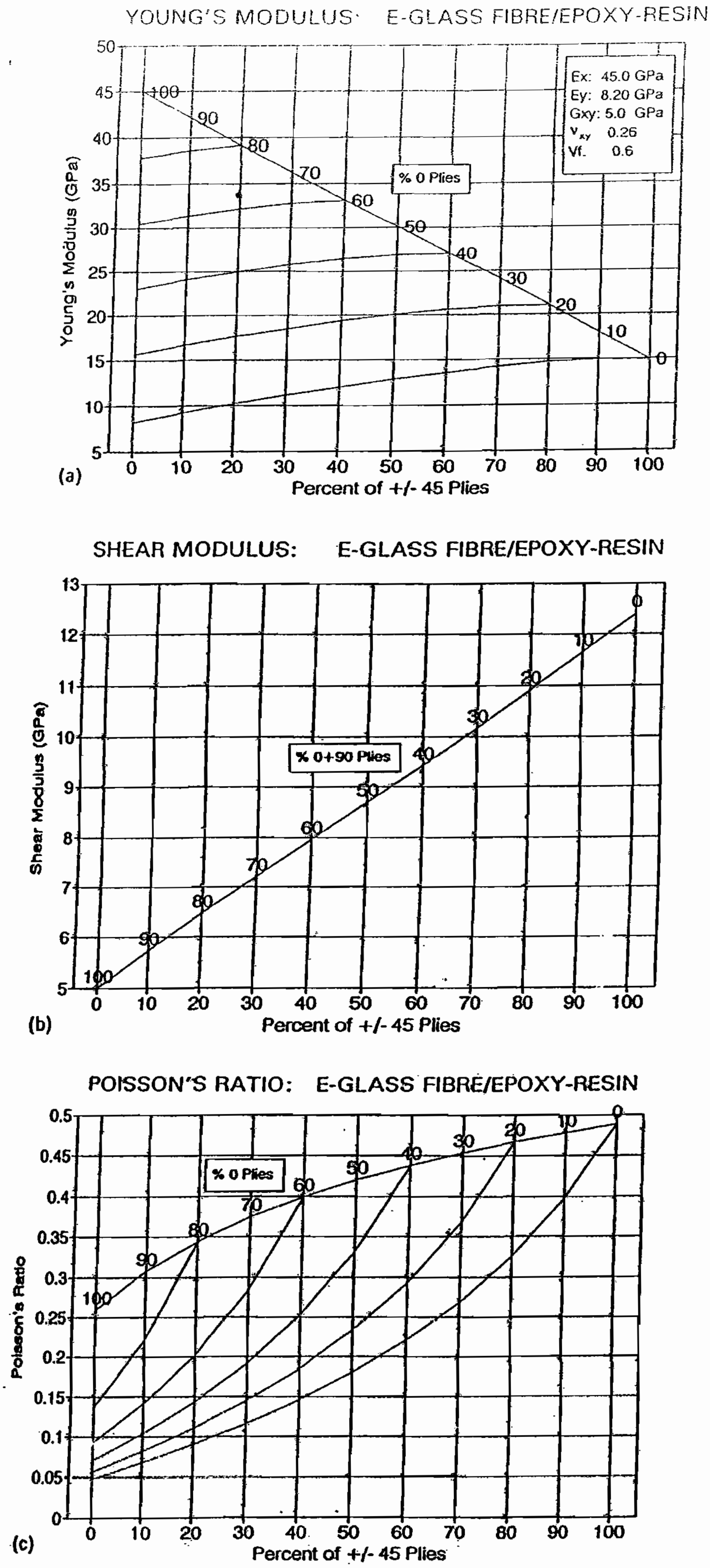


Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.)

ENGINEERING TRIPOS PART IIA 2009

MODULE 3D3, STRUCTURAL MATERIALS AND DESIGN

1 (a.i) $8\ Mp / L^2$; (a.iii) $12\ Mp / L^2$; (b.i) $4\ w\ L^2$; (b.ii) shear force = $w\ L^2$; maximum moment = $w\ L^3 / 2$

2 (a) h_{min} = 172 mm; (b) $l_b >= 16.6$ mm; (c) v_{tot} = 31.6 mm, b = 150 mm

3 (a) moment range -243 kNm to 137 kNm; (b) sagging reinforcement area = 845 mm^2; hogging reinforcement area = 1680 mm^2; (c) V_{max} = 203.2 kN; (d) h = 160 mm

4 (a) Mp = 416.5 kNm, revised Mp after self-weight included = 451.8 kNm; (b) Mp = 343.1 kNm, chose a 457 X 152 X 60 UB (Zp = 1287 cm^3); reaction components at pinned feet, 147.5 kN and 96.7 kN; (c) axial force in rafter = 120.8 kN, resulting in 0.5% loss of Mp.