ENGINEERING TRIPOS PART IIA

Monday 27 April 2009 9 to 10.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: None

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Figure 1 shows two sections A and B. They are identical except that B has been stretched vertically by a factor λ. From first principles, show how the location of the centroid and the second moments of area of B are related to those of A. [10%]

(b) Figure 2(a) shows a section with a circular quadrant.

(i) Find its centroid with respect to the (x, y) axes indicated. [20%]

(ii) Determine the second moments of area I_{XX}, I_{YY} and I_{XY} with respect to the (X, Y) axes shown based at its centroid. [30%]

(c) Determine the major and minor principal second moments of area, and the orientation of the principal axes, for the quarter ellipse shown in Fig. 2(b). [40%]

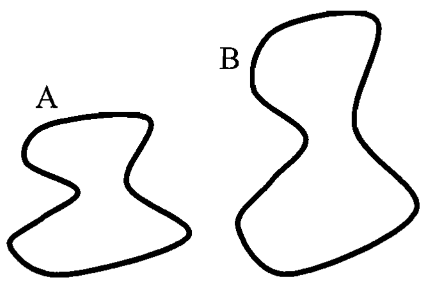

Fig. 1

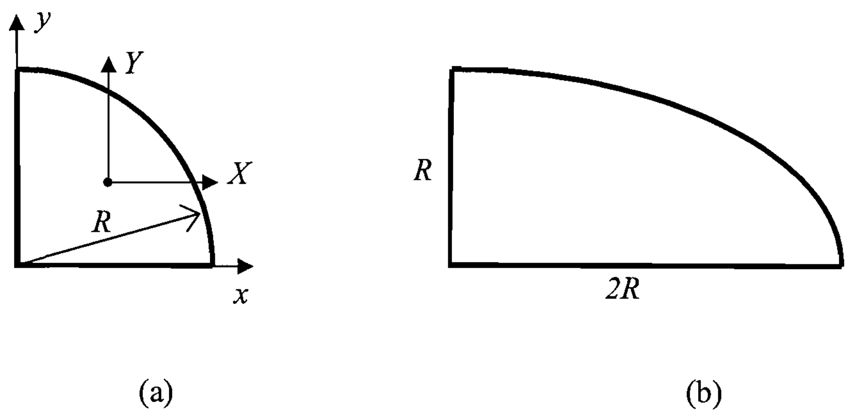

Fig. 2

2 Figure 3 shows a rod AB that forms a quarter circle lying in the x-y plane. It is completely clamped at A. The other end of the rod, at B, is prevented from twisting about the y-axis but is otherwise free to deflect and rotate. The rod has a flexural stiffness EI about any axis and a torsional stiffness GJ.

It is loaded at B by a force P in the negative z direction.

(a) Determine the restraining torque in the y direction at B. [80%]

(b) Without further calculation, describe how you would calculate the deflection at B in the z direction. [20%]

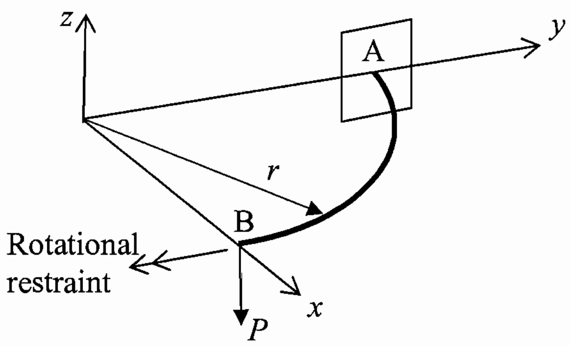

Fig. 3

3 (a) Figure 4 shows a rigid rod AB connected to a wall by a spring AC with stiffness k. The spring is constrained to remain always horizontal, and is unstressed when the rod AB is vertical. A vertical load P is applied at joint A, and a couple M is applied to the base of the column. The couple M is work conjugate to the rotation θ of the column.

(i) For $M = 0$, sketch all equilibrium solution paths on a P versus θ diagram and determine the critical value of P at which the system will buckle. [30%]

(ii) Without need for a formal stability calculation, indicate the stability of each equilibrium path, and describe what happens as P is increased above the critical load. [20%]

(iii) Add to your sketch the equilibrium paths for the case when M is small and nonzero, again indicating the expected stability along each path. [20%]

(b) A simple toggle device consists of two light rigid rods each of length L connected by a spring of stiffness k in the tied-arch configuration shown in Fig. 5. The unstressed length of the spring is $\sqrt{3}L$. The supported weight mg is gradually increased from zero. What angle will the rods make with the horizontal when the toggle is at the point of snapthrough? [30%]

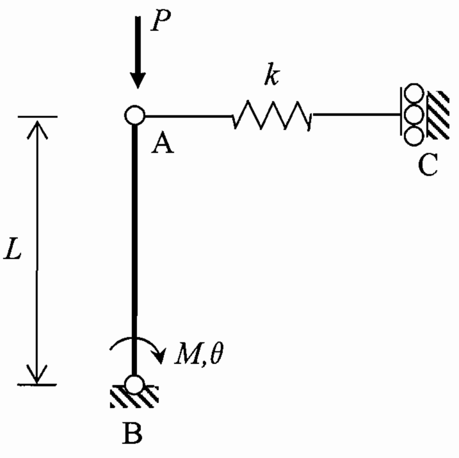

Fig. 4

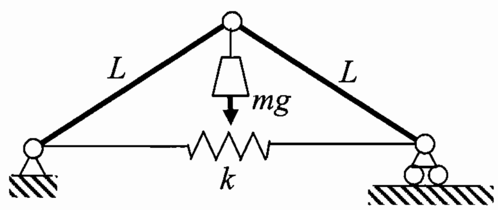

Fig 5.

4 (a) Explain briefly Shanley's insight which resolved the "column paradox" of inelastic buckling. [20%]

(b) All members in the frame shown in Fig. 6 have the same basic stiffness $k = EI/L$. All connections are rigid-jointed except for the pin-connection at C. Vertical point loads, each of magnitude P, are applied simultaneously at A and B as shown.

A table of s and c stability functions is provided in Fig. 7 overleaf.

(i) Determine the lowest critical value of P/P_E, where $P_E = \pi^2 EI/L^2$. [30%]

(ii) Determine the relative proportions of the rotations at A, B, C, D and E during buckling and illustrate these on a clear sketch of the buckling mode. [20%]

(iii) Repeat parts (i) and (ii) for the case when the connections at the feet D and E are both pinned. [30%]

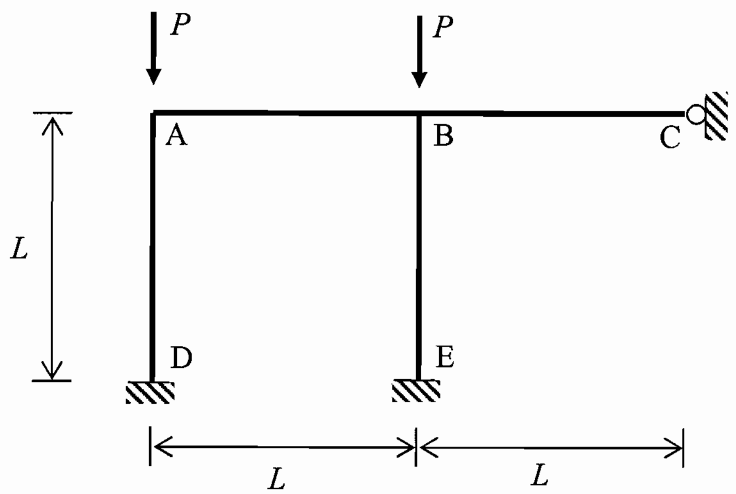

Fig. 6

(TURN OVER

P / P_E	s	c	$s(1-c^2)$
1.0	2.47	1.00	0
1.2	2.09	1.25	-1.18
1.4	1.68	1.66	-2.95
1.6	1.22	2.43	-5.98
1.8	0.72	3.17	-6.52
2.0	0.14	24.68	-85.13
2.2	-0.52	-7.51	28.80
2.4	-1.30	-3.37	13.46
2.6	-2.25	-2.23	8.94
2.8	-3.44	-1.71	6.62
3.0	-5.03	-1.42	5.11

Table of stability functions

Fig. 7

END OF PAPER