ENGINEERING TRIPOS PART IIA

Friday 24 April 2009 2.30 to 4

Module 3D5

WATER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

The values of relevant parameters are listed at the end of the data sheet unless otherwise noted in the question.

Attachment: 3D5 data sheet (5 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper (3 sheets)	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) In a small catchment, the constants for the f-capacity equation of infiltration are estimated to be $f_0 = 30$ mm h^{-1}, $f_c = 0$, and $K_f = 0.2$ h^{-1}. In a two-hour rainfall event, 25 mm rain and 30 mm rain are recorded during the first and second hours, respectively. Estimate the rainfall excess during each hour. [30%]

(b) A catchment has an area of 10 km^2 and a negligible base flow. Two hours of a uniform rainfall generates a discharge at the catchment outlet with the following distribution percentages on the basis of a unit time of two hours: 5, 30, 40, 20, 5. For a one-hour excess rain of 20 mm, estimate the peak flow rate at the outlet and the time when this peak flow rate occurs. [40%]

(c) Derive the Chezy formula from first principles. Any terms involved in the derivation need to be defined fully. [10%]

(d) A steel flume, whose cross-section is in the form of an equilateral triangle with apex down and side length of 1.2 m as shown in Fig. 1, is laid on a slope of 0.01. Taking the Chezy coefficient as 65 m$^{1/2}$ s^{-1}, determine the uniform flow rate corresponding to a water depth of 0.9 m. [20%]

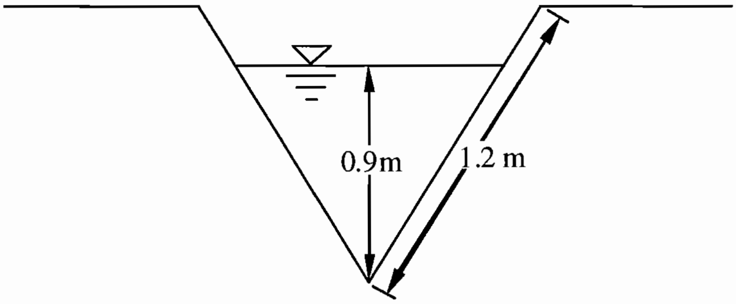

Fig. 1

2 (a) A very wide rectangular channel has a bed slope of 0.001. The Manning roughness coefficient is 0.012 s m$^{-1/3}$. The flow in the channel is uniform and the flow rate per unit width is 0.6 m^2 s^{-1}. Is the flow sub- or supercritical? [30%]

(b) For a stationary hydraulic jump as shown in Fig. 2, it is observed that the downstream water depth h_2 is twice the upstream water depth h_1. Assuming the bed is flat and frictionless, express h_1 in terms of the discharge per unit width q and the gravitational acceleration g only. [30%]

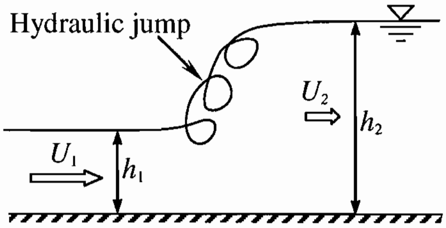

Fig. 2

(c) A long channel with rectangular cross-section enters into a reservoir. The flow velocity is 1.2 m s^{-1} and the water depth is 1.5 m. The water level at the channel mouth begins to drop at a rate of 0.25 m h^{-1} due to the emptying of the reservoir. Neglecting the bed slope and bed friction, calculate the time required for the water level 1 km upstream of the channel mouth to drop by 0.5 m. [40%]

3 (a) The upper reach of a wide stream has a flow depth of 0.8 m and a velocity of 1.0 m s^{-1}. The stream bed consists of sediments of diameter 2.0 mm. The slope of the bed is 0.0005.

(i) Calculate the gross roughness height of the bed. [10%]

(ii) Assuming the grain-related roughness height to be the same as the sediment diameter and using the Meyer-Peter and Müller formula, calculate the bedload sediment transport rate in units of kg m^{-1} s^{-1}. [30%]

(b) The lower reach of a river has flow depth of 1.6 m and velocity of 0.5 m s^{-1}. The river bed consists of sediments of diameter 0.2 mm. The gross roughness height of the bed is 0.1 m.

(i) The suspended sediment concentration at 0.6 m above the bed is measured to be 0.1 kg m^{-3}. What is the sediment concentration at 0.3 m above the bed? [30%]

(ii) Suppose that the river cross-section is rectangular with a width of 20 m. 100 kg of pollutant is released instantaneously at a cross-section. After 5 minutes, another 50 kg of pollutant is released instantaneously at the same location. Assuming that the pollutant is completely mixed over the cross-section when being released, calculate the pollutant concentration 2 km downstream and 70 minutes after the first release. [30%]

4 (a) Briefly describe the characteristics of the water surface profiles over dunes and antidunes. Explain why the shape of dunes moves downstream, while the shape of antidunes moves upstream. [20%]

(b) For the pipeline-pump system shown in Fig. 3, qualitatively sketch the position of the total energy line and the hydraulic gradient. The local energy losses at the inlet and outlet of the pipeline should be indicated. [20%]

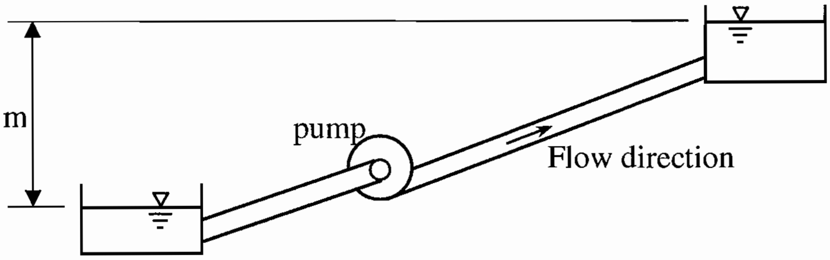

Fig. 3

(c) In the pipeline-pump system shown in Fig. 3, the pipeline has diameter 200 mm, length 2 km, and roughness height 0.03 mm. The head losses in the bends, inlet and outlet amount to $6.2\times\frac{U^2}{2g}$. The static lift is 10.0 m. Calculate the steady discharge of water between the two tanks and the power consumption by the pump. Pump characteristics are given in the table below. [50%]

Discharge (litre s^{-1})	0.0	10.0	20.0	30.0	40.0	50.0
Total head (m)	25.0	23.2	20.8	16.5	12.4	7.3
Efficiency (%)	0.0	45.0	65.0	71.0	65.0	45.0

(d) Describe how to decide the type of pump (*e.g.* axial, mix or centrifugal) to be purchased if the pump's operating speed, discharge and head are given. [10%]

END OF PAPER

UNIVERSITY OF
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Engineering Tripos Part IIA Third Year

Module 3D5: Water Engineering
Data Sheet (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity equation) $f = f_c + (f_0 - f_c)e^{-K_f t}$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flows (uniform flows in an infinitely wide channel):

Eddy viscosity coefficient $\nu_t = \kappa u_* z\left(1-\frac{z}{h}\right)$

Hydraulically smooth ($u_* k_s/\nu < 5$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$

Hydraulically rough ($u_* k_s/\nu > 70$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$

Open Channel Flow

Bed shear stress $\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$

For large Reynolds-number flows $C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

For rectangular channels $Fr = \frac{U}{\sqrt{gh}}$

Momentum equation for steady flows $\sum F = \rho Q(U_{out} - U_{in})$

Uniform flows:

Chézy formula $U = C\sqrt{R_h S_b}$

Manning formula $U = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2}$

Gradually varied flows: $\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$

Or $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$

Characteristics for unsteady flows in prismatic rectangular channels:

$$\frac{d}{dt}\left(U+2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U+\sqrt{gh}$$

$$\frac{d}{dt}\left(U-2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U-\sqrt{gh}$$

Pollutant Transport

Solutions to advective diffusion equations for the case of instantaneous release:

One-dimensional $c(x,t)=\dfrac{M/A}{\sqrt{4\pi D_x t}}\exp\left(-\dfrac{(x-x_0-u_0t)^2}{4D_x t}\right)$

Two-dimensional $c(x,y,t)=\dfrac{M/h}{4\pi t\sqrt{D_x D_y}}\exp\left(-\dfrac{(x-x_0-u_0t)^2}{4D_x t}-\dfrac{(y-y_0-v_0t)^2}{4D_y t}\right)$

Three-dimensional $c(x,y,z,t)=\dfrac{M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}\exp\left(-\dfrac{(x-x_0-u_0t)^2}{4D_x t}-\dfrac{(y-y_0-v_0t)^2}{4D_y t}-\dfrac{(z-z_0-w_0t)^2}{4D_z t}\right)$

Mixing coefficients: $D_{tx}=D_{ty}=0.15hu_*$, $D_{tz}=0.067hu_*$, $D_L=5.93hu_*$

Sediment Transport in Wide Channels

$$\theta=\frac{\tau_b}{g(\rho_s-\rho)d},\quad d_*=d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3},\quad T=\frac{\tau_b'-\tau_{bc}}{\tau_{bc}}=\frac{\theta'-\theta_c}{\theta_c}$$

Fall velocity $w_s=\dfrac{\nu}{d}\left[\sqrt{10.36^2+1.049\cdot d_*^3}-10.36\right]$

Critical Shields parameter $\theta_c=\dfrac{0.30}{1+1.2d_*}+0.055\left[1-\exp(-0.02d_*)\right]$

Shear stress partition: $C'=7.8\ln\left(\dfrac{12h}{k_s'}\right),\quad \tau_b'=\rho g\dfrac{U^2}{C'^2}$

$C=7.8\ln\left(\dfrac{12h}{k_s}\right),\quad \tau_b=\rho g\dfrac{U^2}{C^2}$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $\dfrac{q_b}{\sqrt{g(s-1)\cdot d^3}}=8\left[\left(\dfrac{C}{C'}\right)^{1.5}\theta-0.047\right]^{1.5}$

Van Rijn $\dfrac{q_b}{\sqrt{g(s-1)\cdot d^3}}=0.053\dfrac{T^{2.1}}{d_*^{0.3}}$

Rouse profile of suspended sediment concentration $\dfrac{\bar{c}(z)}{\bar{c}(a)}=\left(\dfrac{h-z}{z}\cdot\dfrac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$

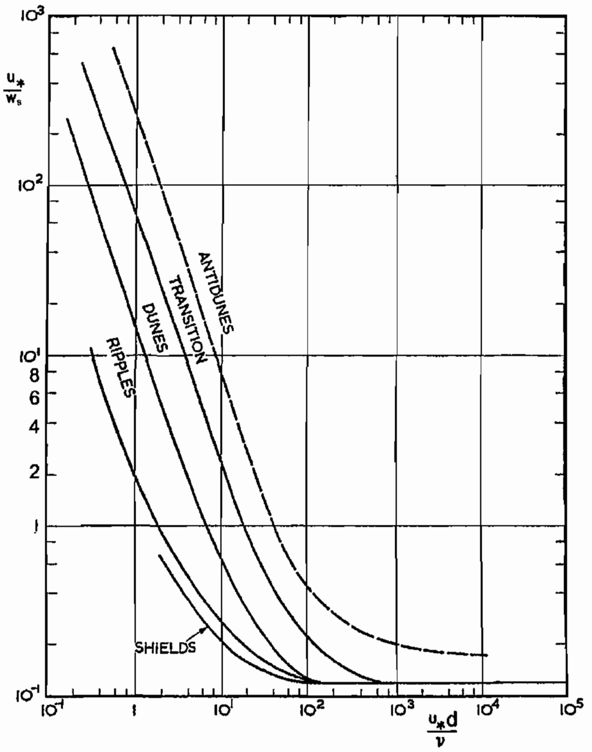
Reference concentration:

Zyserman and Fredsøe $$\bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}}$$

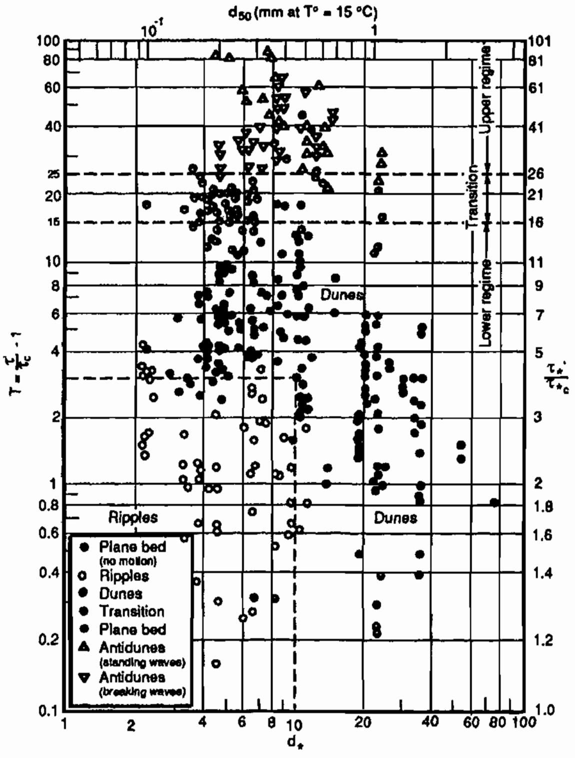
Van Rijn $$\bar{c}(a) = 0.015 \frac{d \cdot T^{1.5}}{a \cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s = \int_a^h \bar{c}(z) u(z) dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

a/h	$w_s/(\kappa u_*) = 0.2$		$w_s/(\kappa u_*) = 0.6$		$w_s/(\kappa u_*) = 1.0$		$w_s/(\kappa u_*) = 1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

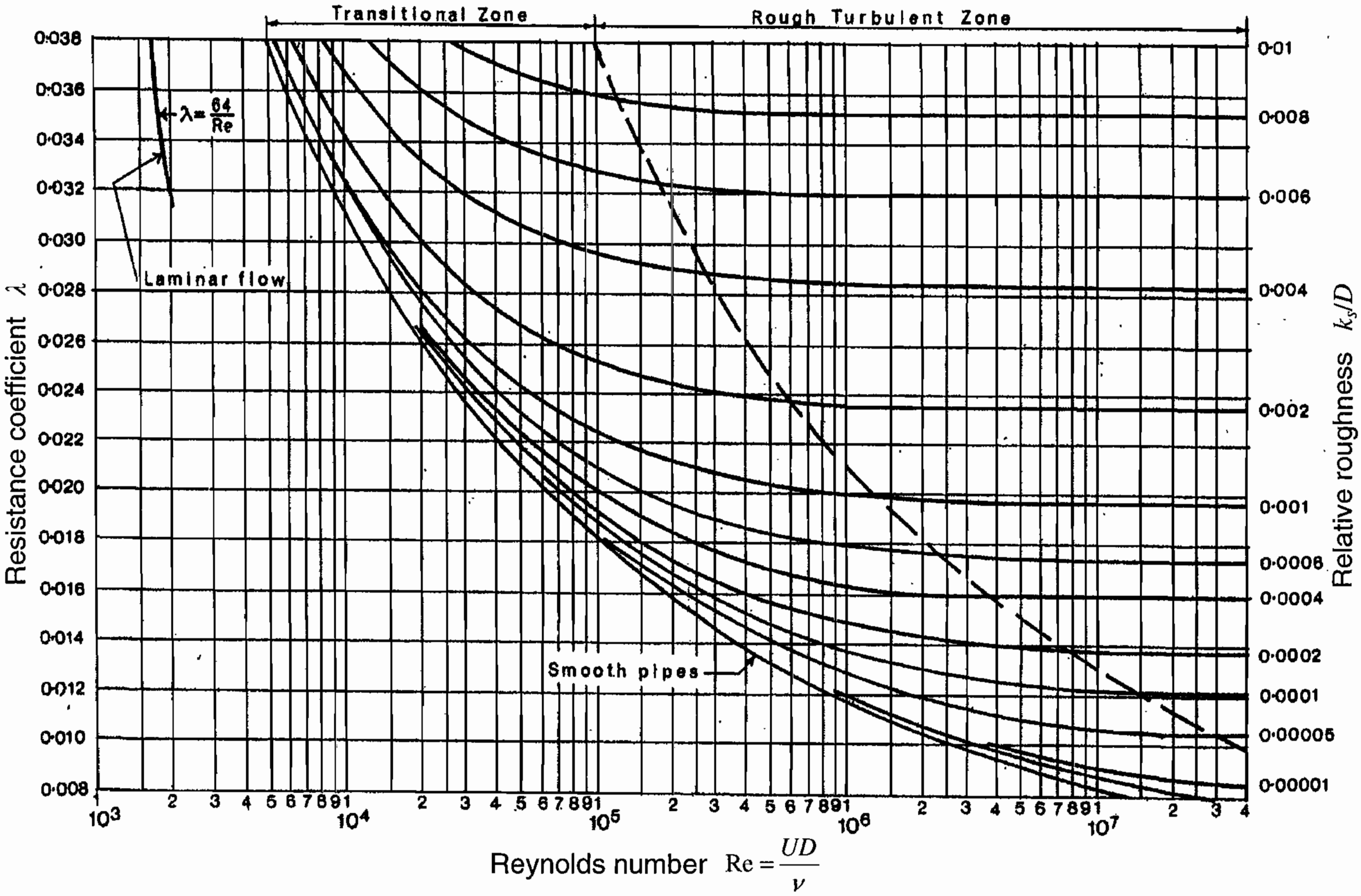
Darcy-Weisbach Equation $H_f = \lambda \frac{L}{D}\frac{U^2}{2g}$

Colebrook-White formula $\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$ with $\mathrm{Re} = \frac{UD}{\nu}$

Power of a pump $P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\frac{Q_p}{N_p \cdot D_p^{\,3}}, \quad \frac{gH_p}{N_p^{\,2} \cdot D_p^{\,2}}, \quad \frac{P_p}{\rho \cdot N_p^{\,3} \cdot D_p^{\,5}}$

Specific speed of a pump $N_S = \frac{N_p \cdot Q_p^{\,1/2}}{H_p^{\,3/4}}$ (N_p is in rev min^{-1}, Q_p is in l s^{-1}, and H_p is in m.)



Symbols

A	area
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient
c	concentration
D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x, D_y, D_z	diffusion coefficients in x, y and z directions respectively

D_{tx},D_{ty},D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.8 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	coefficient representing the rate of decrease in f capacity
k_s	equivalent roughness height, also called Nikuradse's sand roughness height
M	mass or volume of the pollutant released
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity of a cross section
u_*	shear velocity
u_0,v_0,w_0	constant velocity components in x, y and z directions respectively
w_s	fall velocity
x,y,z	spatial coordinates
x_0,y_0,z_0	release position of the pollutant in x, y and z directions respectively
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
$\overline{}$	Reynolds-averaged value
$'$	effective shear-stress contribution
$_{\mathrm{p}}$	pump

D Liang, Jan 2009

2009 IIA 3D5 WATER ENGINEERING

List of Numerical Answers

Q1. (a) 0 in the first hour, and 7.3 mm in the second hour
(b) 11.7 m^3 s^{-1} during the fifth hour
(d) 1.44 m^3 s^{-1}

Q2. (a) Subcritical
(b) $h_1 = \sqrt[3]{\dfrac{q^2}{3g}}$
(c) 2.53 hours

Q3. (a) (i) 0.016 m (ii) 0.045 kg m^{-1} s^{-1}
(b) (i) 0.512 kg m^{-3} (ii) 0.012 kg m^{-3}

Q4. (c) $H = 17.5$ m, $Q = 28.0$ l s^{-1}, $P = 6.76$ kW