

ENGINEERING TRIPOS PART IIA

Thursday 4 May 2009 9.00-10.30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The historical data for a specific UK town are as follows:

Month	Sales (number of homes)	Unemployment rate (%)	Consumer Confidence Index (CCI)
1	20	7.2	72
2	41	4.0	89
3	17	7.3	85
4	35	5.5	65
5	25	6.8	80
6	31	6.0	82
7	38	5.4	90
8	50	3.6	90
9	15	8.4	63
10	19	7.0	60
11	14	9.0	65

(a)

- (i) Calculate the three-month moving average sales for months 3 to 7. Based on this method, what would be your sales forecast for month 8? [12%]
- (ii) Calculate sales forecast for month 8 using exponential smoothing with a smoothing constant of 0.2. [14%]
- (iii) Calculate the regression equation for the sales data with x as unemployment rate and y as sales. ($\sum xy = 1750.8$, $\sum x^2 = 476.3$) [16%]
- (iv) Using the regression equation in part (iii), predict home sales in town if the unemployment rate is 4.5%. [8%]
- (v) Without performing any calculations, comment on the role of CCI as a possible sales predictor. [16%]

(b)

- (i) Briefly explain CAPM and the meaning of Beta under CAPM. [12%]
- (ii) Stocks A and B have a correlation of -1. CAPM Betas for these stocks are 0.5 and -2 respectively. Correlations of stocks A and B with the market

portfolio are 0.5 and -0.6 in that order. The variance of the market portfolio is 0.25. Find the variance of an equally weighted portfolio composed of A and B. [22%]

2 (a) Briefly explain the difference between risk and uncertainty. [16%]

(b) Suppose country A's broadcasting industry is controlled by four companies: Sly TV, Nigriv Media, TB Vision and Pate. If customers subscribe to either Sly or Nigriv, they will never switch to another broadcaster. If they subscribe to TB Vision this year, the probabilities that they will switch to Sly, Nigriv and Pate next year are 0.4, 0.3 and 0.15, respectively. If they subscribe to Pate, the probabilities that they will switch to Sly, Nigriv and TB next year are 0.25, 0.2 and 0.1, respectively. Each customer subscribes to exactly one broadcaster per year.

(i) Represent this business situation using a state-transition diagram. [18%]

(ii) If TV subscribers are initially distributed as 40%, 30%, 10% and 20%, for Sly, Nigriv, TB and Pate, respectively, find the market share for each broadcaster after 2 years. [24%]

(c) After two years, Sly TV considers analysing the effect of poor handling of customers enquiries by their phone service operators on their business. Analysis over the last month has indicated that probabilities of customers switching satisfaction states every time they call Sly customer services are the following:

		To	
		Satisfied	Dissatisfied
From	Satisfied	0.65	0.35
	Dissatisfied	0.45	0.55

(i) Find the steady state probabilities of satisfaction and dissatisfaction among Sly customers. [16%]

(ii) Under the current customer service operation policy, a monthly cost of -1 is incurred if customers are dissatisfied with their calls, while a profit of 0.35 is generated under a satisfaction state. Find the average payoff of this policy per month. What long-term proportions of satisfaction and dissatisfaction are required for Sly to break even? [14%]

(d) Briefly explain the expected value of perfect information. [12%]

- 3 (a) Simple queuing systems are conventionally labelled by $U / V / s / \kappa / W$.
- (i) Briefly explain the Kendall's notation. [10%]
- (ii) What are the characteristics of an $M/G/s$ queue? [6%]
- (b) The visa section of the French Embassy in London has two phone operators arranging appointments for any potential applicant (customer) wishing to apply for a visa in person at the embassy. Let n denote the number of customers in the service system. The system is in a steady-state condition and the steady state probabilities in the system are $p_0=1/13$, $p_1=4/13$, $p_2=4/13$, $p_3=2/13$, $p_4=1/13$, $p_5=1/13$, $p_n=0$ if $n>5$.
- (i) Calculate the expected number of customers in the system and the expected number of customers in the queue. [16%]
- (ii) Calculate the expected number of customers being served. [8%]
- (iii) Suppose the call arrival rate is 3 customers per hour. Calculate the mean time in the system and the mean waiting time in the queue. [12%]
- (iv) Calculate the mean service time per customer and the utilisation factor of the system. [16%]
- (c) At the embassy, only one operator processes the initial screening stage of visa applications. He processes 10 applications every hour on average. This operator leaves his desk for 15 minutes to take a "productivity" break.
- (i) What is the probability that exactly one applicant will stand in front of the operator's desk during the 15 minute break (assume it is an $M/M/1$ queue)? [14%]
- (ii) If the inter-arrival time between applicants is exponentially distributed with a rate λ , how many customers would you expect to arrive during the 15 minute break? [4%]
- (d) Why do we need the balance equation of the birth-and-death process in queueing theory? [6%]
- (e) What are the characteristics of the queueing system described by the Pollaczek-Khitchine formula? [8%]

4 (a) Briefly explain risk aversion and risk premium. [12%]

(b) IMB Ltd. is developing a new computer system. The company is trying to decide on the manufacturing and assembly processes to be used. They are considering whether to manufacture the keyboards themselves, buy them from another local company, or buy them from a supplier in a developing country with lower labour costs. The following pay-off table has been calculated for different levels of demand:

	Sales		
	Low	Medium	High
Manufacture themselves	-15	10	55
Buy locally	5	20	40
Buy abroad	10	30	25

Determine the best policy on the basis of the maximax, maximin, and minimax regret criteria. [30%]

(c) Suppose the price of a stock is £100. The exercise prices of call and put options trading on the stock are also £100. These options mature in 24 months. Both European and American options are traded. The annualized volatility of the rate of return on the stock is 30%. The risk-free rate of return is 8% per annum.

Use the binomial option pricing approach with a time step of twelve months to price European call and put options on the stock. [30%]

(d) You are planning to scale down the operations of your manufacturing plant by 70% sometime in the next six months. Suppose the value of the plant today is £100m. Your plan is to save £70m from scaling down operations. Uncertainty in the operating environment is estimated by an annualized standard deviation of 50% in cash flows. The risk-free rate of return is 8% per annum.

(i) Use the binomial option pricing approach with a time step of three months to value the option to scale down operations. Is there an optimal time for taking such an action? [20%]

(ii) Suppose that the decision to scale down operations incurs social costs to your company. What would you recommend doing? [8%]

END OF PAPER

Numerical Answers to 3E3 Modelling Risk

- 1(a)(i) The forecast for month 8 is 31.33 or 32 houses.
1(a)(ii) The forecast for month 8 is 28.666 or 29 houses.
1(a)(iii) The regression equation is $y = 71.37 - 6.84x$.
1(a)(iv) $y = 40.59$ (41 house sales)
1(a)(v) ...

- 1(b)(i) ...
1(b)(ii) The variance of the equally weighted portfolio is 1.26.

2(a) ...

- 2(b)(i) $L = 27/13$, $L_q = 7/13$.
2(b)(ii) E (number of customers served) = $20/13$.
2(b)(iii) $W = 9/13$, $W_q = 7/39$.
2(b)(iv) $\mu = 39/20$, $\rho = 10/13$.

- 2(c)(i) $P_1(15) = 0.205$.
2(c)(ii) In 15 minutes we should expect 2.5 applicants to arrive.

2(d) ...

2(e) ...

3(a) ...

3(b) Maximax would suggest that manufacturing systems in-house will give the highest payoff (£55k) in the best case. Maximin suggests that buying abroad is a low risk strategy giving the highest payoff (£10k) in the worst case, and minimax regret suggests buying locally (maximum regret £15k).

3(c) European call option: 22.286, European put option: 7.410.

- 3(d)(i) 7.21M, 7.92M. It's better to scale down operations in 3 months.
3(d)(ii) ...

4(a) ...

- 4(b)(i) ...
4(b)(ii) $S_2 = [0.53025, 0.4015, 0.01575, 0.0525]$

- 4(c)(i) $\pi_1 = 9/16$ and $\pi_2 = 7/16$
4(c)(ii) Average payoff = -0.2406. 74.1% customer satisfaction is needed to break-even.

4(d) ...