ENGINEERING TRIPOS PART IIA

Wednesday 29 April 2009 9 to 10.30

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) For a linear system

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx + Du$$

explain the role of the *state transition matrix* e^{At}. [15%]

(b) Verify that

$$(sI - A)^{-1} = I\frac{1}{s} + A\frac{1}{s^2} + A^2\frac{1}{s^3} + \cdots$$

(assuming convergence of the series) and explain how this result can be used to evaluate the state transition matrix. [30%]

(c) A linear controller $K(s)$ is connected to a linear plant $G(s)$ in series, as shown in Fig. 1. The controller has a state-space realisation:

$$K(s): \qquad A_K = 0, \quad B_K = 1, \quad C_K = 1, \quad D_K = 1$$

and the plant has a state-space realisation:

$$G(s): \begin{cases} A_G = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, & B_G = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ C_G = \begin{bmatrix} 1 & 0 \end{bmatrix}, & D_G = 0 \end{cases}$$

Find a state-space realisation of the series connection $G(s)K(s)$. [25%]

(d) A feedback loop is now closed around the systems of part (c), as shown in Fig. 2.

(i) Find a state-space model of the closed loop, with input r and output y. [15%]

(ii) Verify that one of the closed-loop poles lies at -1 and find the locations of the other closed-loop poles. [15%]

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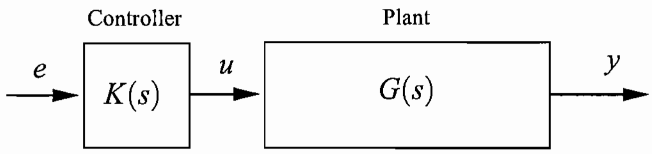

Fig. 1

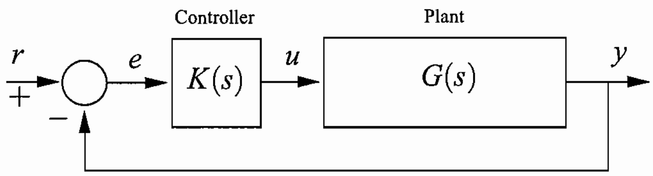

Fig. 2

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2 The open-loop transfer function of an engine, for which a speed control system is to be designed, is given by

$$\frac{8}{(s+2)(s+4)^2}$$

It is proposed to apply negative feedback with proportional gain k to it, as shown in Fig. 3.

(a) It is required that the closed loop should exhibit a damping factor no smaller than $1/\sqrt{2}$. What is a specification on closed-loop pole locations that will ensure this requirement is satisfied? [10%]

(b) Show that the root-locus plot for this system, with proportional feedback, passes through the points $-2 \pm 2j$. [20%]

(c) Sketch the complete root-locus plot. [30%]

(d) Determine the gain k that is required to obtain a pair of closed-loop poles at $-2 \pm 2j$. [20%]

(e) The steady-state error is required to be no bigger than 5%. Show that it is not possible to obtain this performance with proportional control, while meeting the specification of part (a). [10%]

(f) Suggest, without any detailed calculations, how the controller might be modified to meet both the damping and the steady-state error specifications. [10%]

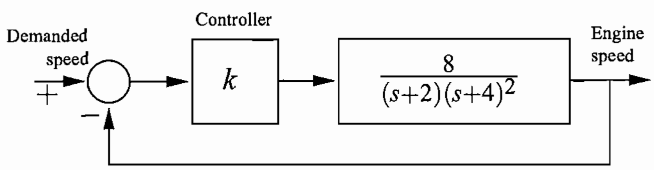

Fig. 3

3 Figure 4 shows the cross-section of a magnetically-levitated train. γ denotes the air gap between the vertical train position and the vertical track position. Magnets produce a vertical force which depends on the magnetising current I and on the air gap γ. The magnetising current I depends on a voltage u applied to the magnet's coils.

For small perturbations about the equilibrium values, the linearised equations of motion of the magnetically-levitated train can be written as

$$M\frac{d^2\gamma}{dt^2} = S\gamma - GI \qquad (1)$$

$$\frac{dI}{dt} = \frac{S}{G}\frac{d\gamma}{dt} - \frac{R}{L}I + \frac{1}{L}u \qquad (2)$$

where M is the mass of the train, and G, L, R and S are positive constants.

(a) Write the equations (1)–(2) in state-space form, taking the state variables as $x_1 = \gamma$, $x_2 = d\gamma/dt$, and $x_3 = I$, and assuming that only the current I is measured. [20%]

(b) Show that this system is not asymptotically stable. [20%]

(c) Show that the linearised system is observable from measurements of the current I alone. [20%]

(d) Draw a block diagram of an observer for this system, and show how the observer could be used together with state feedback to control the train. [20%]

(e) Explain why, in practice, it would be advantageous to measure the air-gap γ as well as the current I. [20%]

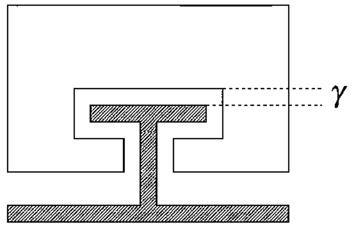

Fig. 4

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4 (a) Define what is meant by saying that a linear system

$$\frac{dx}{dt} = Ax + Bu \tag{3}$$

is *controllable*, where x is the state vector and u is the input vector, and state a criterion for testing controllability. [20%]

(b) Define the *reachable states* of (3), and explain how they can be determined from the controllability matrix. [20%]

(c) Two spacecraft are in orbit around a planet in the same orbital plane, as shown in Fig.5. The 'chaser' craft is manoeuvreing to approach the 'target' craft, by firing thrusters which exert forces F_y and F_z, as shown in Fig.6. (Bidirectional thrusters are used, so F_y and F_z may be negative.) When the two craft are close together, the target craft may be considered to be travelling in a straight line; the tangential and radial distances from the chaser craft to the target craft, y and z respectively, can then be defined as shown in Fig.6.

If the state and input vectors are defined as

$$x = [y, \dot{y}, z, \dot{z}]^T, \qquad u = [F_y, F_z]^T,$$

respectively, then the dynamics are given approximately by equation (3), with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 3\omega^2 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}$$

where m is the mass of the chaser craft and ω is a constant related to the orbital period.

Show that the system is controllable. [30%]

(d) Suppose that the thrusters in the y direction fail. Show that the system is then not controllable. [20%]

What are the reachable states in this case? [10%]

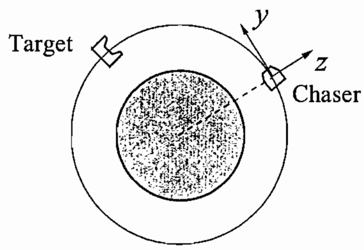

Fig. 5

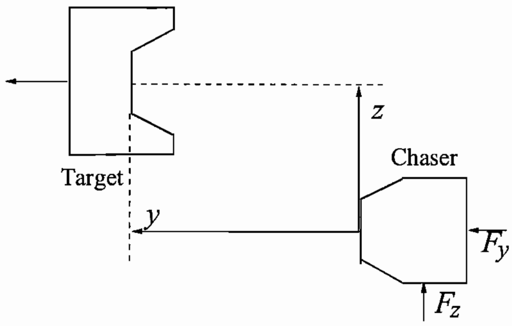

Fig. 6

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3F2 Systems and Control: 2009 Numerical answers

Prof. J.M. Maciejowski

21 May 2009

1. (d)(ii): $-1 \pm j$
2. (d): $k = 2$
3. —
4. —