ENGINEERING TRIPOS PART IIA

Thursday 30 April 2009 9 to 10.30

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain with the aid of a sketch, the meaning of the term *intersymbol interference* and describe its effect on the performance of a digital communication system. [20%]

(b) State Nyquist's pulse shaping criterion. [10%]

(c) The received pulse spectrum for a particular digital communication system is given by

$$P_R(\omega) = \frac{1}{2}\left[2\,\mathrm{sinc}\,(2T_s\omega) + \mathrm{sinc}\,(2T_s\omega - \pi) + \mathrm{sinc}\,(2T_s\omega + \pi)\right] \qquad -\infty \le \omega \le \infty$$

where T_s is the time between successive transmitted symbols.

(i) Show by using Nyquist's pulse shaping criterion that the pulse spectrum $P_R(\omega)$ gives rise to intersymbol interference. [30%]

(ii) Show that $P_R(\omega)$ decays as $1/\omega^3$. [15%]

(d) For the pulse spectrum $P_R(\omega)$ in part (c) , calculate the value of the corresponding time domain pulse $p_R(t)$ at $t = \pm T_s$. [25%]

2 (a) Define what is meant by a systematic linear binary block code. [20%]

(b) A systematic (5,2) linear binary block code C has the following parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(i) Find the generator matrix for code C. [10%]

(ii) Show that code C has a minimum Hamming distance of 3. What is the error detection and error correction capability of code C? [20%]

(iii) Construct the standard array for code C, making sure that all possible syndromes for code C are allocated to an appropriate error pattern. [30%]

(iv) Explain how the standard array constructed in part (b) (iii) can be used to implement an efficient decoder for code C, and discuss appropriate strategies to deal with syndromes that correspond to error patterns that are not uniquely correctable. [20%]

(TURN OVER

3 (a) A complex phasor waveform $p(t)$ is used to create a modulated real waveform $s(t)$ with carrier frequency ω_C . Derive an expression for $S(\omega)$, the spectrum of $s(t)$, in terms of $P(\omega)$, the spectrum of $p(t)$. [20%]

(b) A 16-level quadrature amplitude modulated (QAM) signal comprises phasors of the form

$$p_k(t) = [s_{2k} + j\, s_{2k+1}]\, g(t - kT_s)\, e^{j\phi_0}$$

where s_{2k} is the even data symbol for the k^{th} symbol interval, and s_{2k+1} is the odd data symbol for the same interval. These symbols can each take values of -3 , -1 , +1 , +3 . The symbol period is T_s and the shaping pulse $g(t)$ is rectangular and of unit amplitude with duration T_s . The phase offset ϕ_0 is a constant.

(i) Obtain an expression for the power spectrum $|P(\omega)|^2$ of the phasor waveform, assuming the four symbol values are random and equiprobable, and are uncorrelated between symbol intervals and between the odd and even symbol data streams. [40%]

(ii) Hence derive an expression for $|S(\omega)|^2$, the power spectrum of a 16-QAM signal modulated onto a carrier of frequency ω_C , where $\omega_C >> 2\pi/T_s$. [15%]

(c) Briefly discuss the relative merits of 16-QAM as a modulation scheme, when compared with QPSK and 64-QAM. [25%]

4 (a) In the analysis of communication systems, explain what is meant by the symbols E_b and N_0. What is the significance of the ratio E_b/N_0. [15%]

(b) The approximate bit error probability for a quadrature phase-shift-keying (QPSK) demodulator with differential decoding is given by

$$P_{QPSK} \approx 2\,Q\left(\sqrt{2u}\right) \quad \text{where} \quad u = E_b/N_0 \quad \text{for} \quad u \geq 2$$

and

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

Similarly the approximate bit error probability for an M-level frequency-shift-keying (MFSK) demodulator, assuming non-coherent detection, is given by

$$P_{MFSK} \approx \frac{1}{4}\left(2\ \mathrm{e}^{(-u/2)}\right)^m \quad \text{where} \quad u = E_b/N_0 \quad \text{for} \quad u \geq 2$$

and the number of states for each symbol is $M = 2^m$.

(i) Determine the minimum integer value for m such that P_{MFSK} is less than P_{QPSK} at relatively large values of u . [25%]

(ii) If $M = 32$, estimate by how much (in dB) does the signal power of QPSK need to exceed that of MFSK to achieve a bit error probability of 10^{-6} . [30%]

(c) Estimate the bandwidth (between the first zeros in their power spectra on either side of the carrier frequency) of QPSK, and of MFSK with $M = 32$. Comment on the tradeoff achieved between the two modulation schemes. [30%]

END OF PAPER

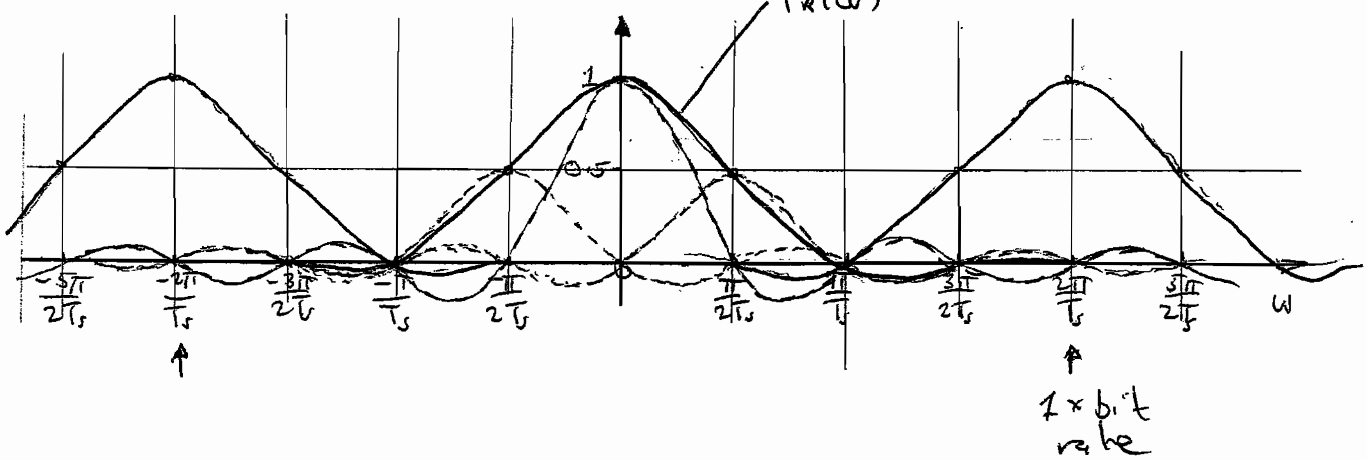
Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2009 - Answers

1.

a) See notes.

b) $\sum_{k=-\infty}^{\infty} P_R\left(f - \frac{k}{T_s}\right) = \text{constant}$

c) (i)



(ii) $\frac{1}{2}\left(\frac{\pi^2}{T_s\omega(\pi^2 - 4T_s^2\omega^2)}\right)\sin 2T_s\omega$

d) $\frac{1}{4T_s}$

2.

a) See notes.

b) (i) $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

(ii) Detect 2 errors. Correct 1 error.

(iii) See notes.

(iv) See notes.

3.

a) $S(\omega) = \frac{1}{2}\left(P(\omega - \omega_c) + P^*(-(\omega + \omega_c))\right)$

b) (i) $|P(\omega)|^2 = 10T_s \,\text{sinc}^2\left(\frac{\omega T_s}{2}\right)$

(ii) $|S(\omega)|^2 = \frac{5}{2}T_s\left[\text{sinc}^2\left(\frac{(\omega - \omega_c)T_s}{2}\right) + \text{sinc}^2\left(\frac{(\omega + \omega_c)T_s}{2}\right)\right]$

c) See notes

4.

a) See notes.

b) (i) $m = 3$.

(ii) For QPSK $u = 3$. For 32 MFSK $u = 6.358$.

$$\frac{\text{Power of QPSK}}{\text{Power of MFSK}} = \frac{11.97}{6.358} = 1.883 \approx 2.75\text{dB}$$

c) QPSK bandwidth = $\frac{1}{T_b}$ Hz. MFSK bandwidth = $\frac{33}{5T_b}$ Hz.