3C5 Dynamics: Answers to Tripos Paper 2010

1. (a) $\mathbf{I} = \dfrac{m}{12}\begin{pmatrix} 6R^2+L^2 & 0 & 0 \\ 0 & 6R^2+L^2 & -6RL/\pi \\ 0 & -6RL/\pi & 12R^2 \end{pmatrix}$.

(b) $\mathbf{I} = \dfrac{5mR^2}{6}\begin{pmatrix} 53 & 0 & 0 \\ 0 & 53 & -6/\pi \\ 0 & -6/\pi & 6 \end{pmatrix}$.

(c) $\mathbf{Q}=(5mR^2\Omega^2/\pi)\mathbf{i}$.

2. (a) $\dot{\phi} = mgL/(C\omega_3)$.

(b) Stable for $\omega_3^2 > (4AmgL/C^2)\cos\theta$.

(c) Always stable because $\cos\theta < 0$.

(d) For $A \approx mL^2$ the two solutions are $\dot{\phi} = \dfrac{C\omega_3}{2A} \pm \sqrt{\dfrac{g}{L}}$.

3. (b) $Q = maV^2/R$ in the direction of the velocity V.

(c) $\alpha = 2/7$.

4. (a) $\begin{pmatrix} m_1+m_2 & m_2\cos\theta \\ m_2\cos\theta & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} P\cos\theta \\ P + m_2 g\sin\theta \end{pmatrix}$.

(b) $\omega_1 = 0$, $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\omega_2^2 = \dfrac{k(m_1+m_2)}{m_2(m_1+m_2\sin^2\theta)}$, $\mathbf{u}_2 = \begin{pmatrix} \dfrac{-m_2\cos\theta}{m_1+m_2} \\ 1 \end{pmatrix}$.

(c) $p_1 = (m_1+m_2)\dot{x}_1 + m_2\dot{x}_2\cos\theta$ (conserved)
$p_2 = m_2\dot{x}_2 + m_2\dot{x}_1\cos\theta$ (not conserved)

5. (b) $M\ddot{x} + \lambda\dot{x} + Kx = Mg$.

(c) (i) $T = (1/2)\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{q}}$, $V = (1/2)\mathbf{q}^{\mathrm{T}}\mathbf{K}\mathbf{q}$.

(ii) $F = (1/2)\dot{\mathbf{q}}^{\mathrm{T}}\mathbf{C}\dot{\mathbf{q}}$, (iii) not possible.