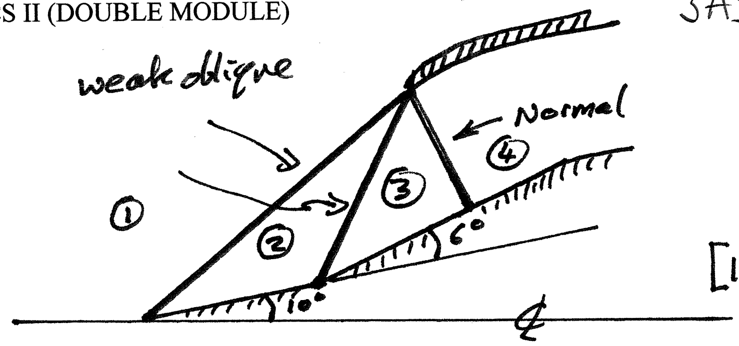
3A3

Q1 (a)(i)



[10%]

(a)(ii) $M_1 = 1.80 \quad \theta = 10^\circ$ WEAK $\Rightarrow$ $\boxed{\beta = 44.057^\circ \quad M_2 = 1.4494 \quad p_{02}/p_{01} = 0.98683}$ [5%]

$M_2 = 1.4494 \quad \theta = 6^\circ$ WEAK $\Rightarrow$

$M_2 = 1.40 \begin{cases} \beta = 54.633 \\ M_3 = 1.1737 \\ p_{03}/p_{02} = 0.99717 \end{cases}$

$M_2 = 1.45 \begin{cases} \beta = 51.755 \\ M_3 = 1.2325 \\ p_{03}/p_{02} = 0.99733 \end{cases}$

$$\frac{1.4494 - 1.40}{1.45 - 1.40} = 0.988$$

@ $M_2 = 1.4494 \Rightarrow \boxed{\beta = 51.788^\circ \quad M_3 = 1.2318 \quad p_{03}/p_{02} = 0.99733}$ [10%]

NORMAL $M_3 = 1.2318$

$M_3 = 1.230 \begin{cases} M_4 = 0.8241 \\ p_{04}/p_{03} = 0.9896 \end{cases}$

$M_3 = 1.240 \begin{cases} M_4 = 0.8183 \\ p_{04}/p_{03} = 0.9884 \end{cases}$

@ $M = 1.2318 \quad \dfrac{1.2318 - 1.230}{1.240 - 1.230} = 0.180$

$\Rightarrow \boxed{M_4 = 0.8231 \quad p_{04}/p_{03} = 0.9894}$ [5%]

$$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}} \times \frac{p_{03}}{p_{02}} \times \frac{p_{02}}{p_{01}} = 0.9894 \times 0.99733 \times 0.98683 = \underline{\underline{0.9738}}$$ [5%]

$$\underline{\underline{M_4 = 0.8231}}$$

Q1 cont (b) (i) $M_1 = 1.80 \quad \theta = 8^\circ$ WEAK $\Rightarrow$ $\boxed{\beta = 41.673^\circ \quad M_2 = 1.5225 \quad p_{02}/p_{01} = 0.99310}$

[5%]

$M = 1.50, \theta = 6^\circ$ WEAK $\begin{cases} \beta = 49.326^\circ \\ M_3 = 1.2879 \\ p_{03}/p_0 = 0.99739 \end{cases}$

$M = 1.55 \quad \theta = 6^\circ$ WEAK $\begin{cases} \beta = 47.214 \\ M_3 = 1.3414 \\ p_{03}/p_0 = 0.99739 \end{cases}$

$$\frac{1.5225 - 1.50}{1.55 - 1.50} = 0.45$$

@ $M_2 = 1.5225$, $\theta = 6^\circ$ WEAK $\Rightarrow$ $\boxed{\beta = 48.376 \quad M_3 = 1.3120 \quad p_{03}/p_{02} = 0.99739}$

[10%]

NORMAL $M = 1.3120$

$M_3 = 1.31 \begin{cases} M_4 = 0.7809 \\ p_{04}/p_{03} = 0.9776 \end{cases}$

$M_3 = 1.32 \begin{cases} M_4 = 0.7760 \\ p_{04}/p_{03} = 0.9758 \end{cases}$

$$\Rightarrow \frac{1.3120 - 1.31}{1.32 - 1.31} = 0.20$$

$\Rightarrow$ $\boxed{M_4 = 0.7799 \quad p_{04}/p_{03} = 0.9772}$

[5%]

$$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}} \times \frac{p_{03}}{p_{02}} \times \frac{p_{02}}{p_{01}} = 0.9772 \times 0.99739 \times 0.99310 = \underline{\underline{0.9679}}$$

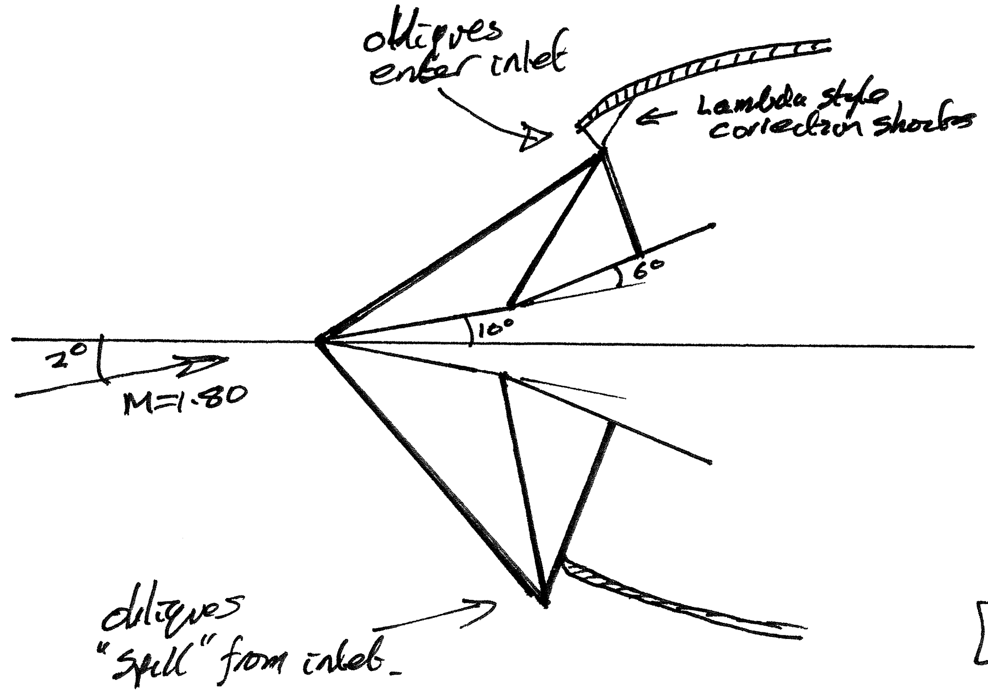
$$\underline{\underline{M_4 = 0.7799}}$$

[5%]

Estimate is less reliable as the shocks are not focused on the cowl lip so extra "shocks" & flow corrections are required

[10%]

(Q1 cont) (b)(ii)



[20%]

(b)(iii) Yaw will cause asymmetric shocks on aircraft but also different losses in the two intakes.
[Both '1' & '2' are worse than design case].
May incur engine surge (exceed stability range) or just get different thrust from two engines

[10%]

(Q2)(a) Isentropic (No shocks)
Irrotational (steady)

(b) When M_∞ = free stream $\phi(x,y)$ $u = \frac{\partial \phi}{\partial x}$ $v = \frac{\partial \phi}{\partial y}$

$$(1-M_\infty^2)\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (M_\infty < 1)$$

Put $X = x$ $Y = y\sqrt{1-M_\infty^2}$ airfoil still $0<X<c$

$\Phi(X,Y) = \phi(x,y)$

$$\frac{\partial \Phi}{\partial X} = \frac{\partial \phi}{\partial x} \qquad \frac{\partial^2 \Phi}{\partial X^2} = \frac{\partial^2 \phi}{\partial x^2}$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial Y}{\partial y}\frac{\partial \Phi}{\partial Y} = \sqrt{1-M_\infty^2}\,\frac{\partial \Phi}{\partial Y}$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial y^2} = (1-M_\infty^2)\frac{\partial^2 \Phi}{\partial Y^2}$$

$$\Rightarrow (1-M_\infty^2)\frac{\partial^2 \Phi}{\partial X^2} + (1-M_\infty^2)\frac{\partial^2 \Phi}{\partial Y^2} = 0$$

$$\Rightarrow \underline{\underline{\frac{\partial^2 \Phi}{\partial X^2} + \frac{\partial^2 \Phi}{\partial Y^2} = 0}} \qquad \text{LAPLACE (ie } M=0)$$

$$\frac{\partial \phi}{\partial y} = \frac{\tau}{c} g'(x/c) U_\infty \qquad y=0 \quad 0<x<c$$

$$\Rightarrow \sqrt{1-M_\infty^2}\,\frac{\partial \Phi}{\partial Y} = \frac{\tau}{c} g'(X/c) U_\infty \quad @\, y=0 \quad 0<X<c$$

$$\Rightarrow \underline{\underline{\frac{\partial \Phi}{\partial Y} = \frac{(\tau/\sqrt{1-M_\infty^2})}{c} g'(X/c) U_\infty}} \quad @\, Y=0 \quad 0<X<c$$

(ie airfoil get thicker @ low Mach)

2(c) $M_\infty > 1 \Rightarrow (1-M_\infty^2) < 0$ so HYPERBOLIC rather than ELLIPTIC (LAPLACE). There are similarity solutions for $M_\infty > 1$ family, in same way as $M_\infty < 1$ family.

2(d) Elliptic ⇒ complete knowledge of all boundaries.
Hyperbolic ⇒ characteristics ⇒ limited knowledge.
So, CFD scheme has to reflect "domain of dependence" that matches the physical elliptic or hyperbolic problem. Solver has to be different depending on $M > 1$ or $M < 1$. Time integration keeps problem Hyperbolic.

(2e) In supersonic flow can have shocks. These generate entropy so flowfield need not remain isentropic. However, if the shock is curved it is possible to generate vorticity so flow ceases to be irrotational.

3A3 Draft crib 2010

1) (a) SFEE $\dot{Q} - \dot{W}_x = \dot{m}\left[\left(h_2 + \frac{1}{2}V_2^2 + gz_2\right) - \left(h_1 + \frac{1}{2}V_1^2 + gz_1\right)\right]$

No heat or work transfer, no gravitational PE, ∴

$$h_2 + \frac{1}{2}V_2^2 = h_1 + \frac{1}{2}V_1^2 = \underline{h + \frac{1}{2}V^2 = \text{constant}}.$$

(b) $$h + \frac{1}{2}V^2 = c_pT + \frac{1}{2}V^2 = c_pT + \frac{1}{2}M^2(\gamma RT)$$

$$\therefore \frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2 \quad ; \quad \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \text{ isentropic}$$

$$\therefore \underline{\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}$$

Binomial Theorem : $\frac{P_0}{P} = 1 + \frac{\gamma}{\gamma-1}\frac{\gamma-1}{2}M^2 + \ldots$

$$= 1 + \frac{\gamma}{2}M^2 = 1 + \frac{\gamma}{2}\frac{V^2}{\gamma RT}$$

$$= 1 + \frac{\rho V^2}{2P}$$

$$\therefore \underline{P_0 = P + \frac{1}{2}\rho V^2} \quad \text{as } M \to 0. \quad \text{Bernoulli}$$

c) Standard atmosphere tables at 12000m :

$$P/P_{SL} = 0.1915 \; ; \quad T/T_{SL} = 0.7519$$

$$P_{SL} = 1.01325 \text{ bar} \; ; \quad T_{SL} = 288.15K$$

$$\therefore P = 19404 \text{ Pa} = 19.4 \text{kPa}$$

$$T = 216.7K.$$

Have $P_0 = 50\text{kPa}$ $\therefore \frac{P_0}{P} = 2.577$

CUED tables $\underline{M = 1.26}$ (tables have $\frac{P_0}{P} = 2.5875$)

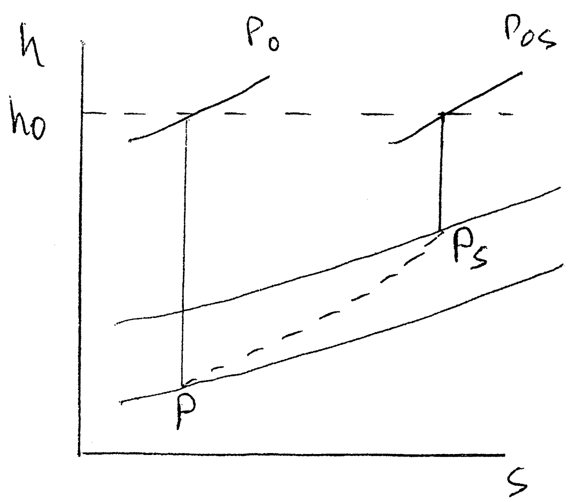
Speed of sound is $\sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 216.7} = \underline{295 m/s}$

[or use $a/a_{SL} = (T/T_{SL})^{1/2}$; $a_{SL} = \sqrt{1.4 \times 287 \times 288.15}$
$= 340 m/s$]

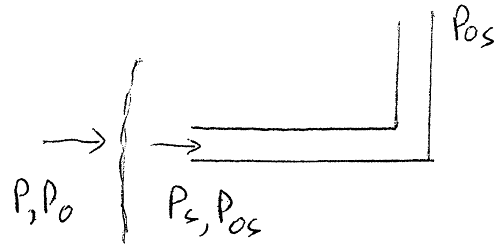
$\therefore$ velocity (airspeed) $= Ma = 1.26 \times 295 = \underline{371.7 m/s}$

c) upstream stagnation pressure (CUED tables) $\frac{P_{0s}}{P_0} = 0.9857$

$$\therefore P_0 = 50/0.9857 = \underline{50.72 \text{kPa}}$$

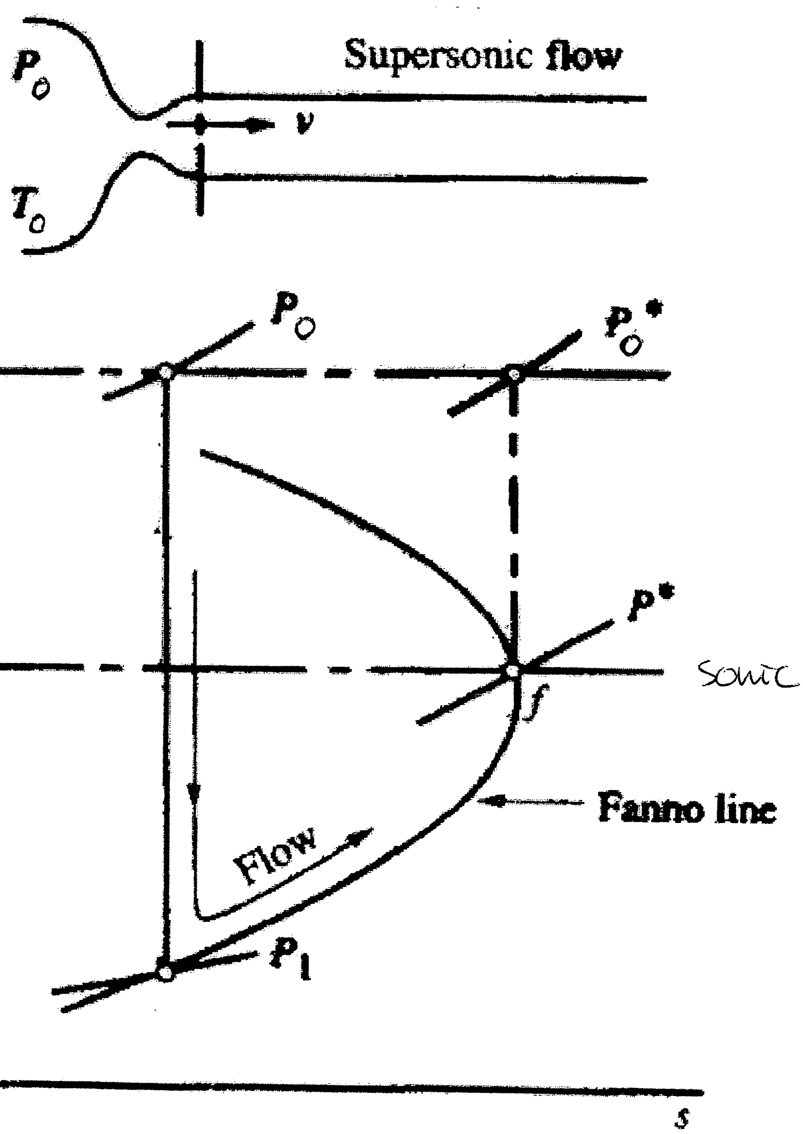


h
ho
Po
Pos
Ps
P
s



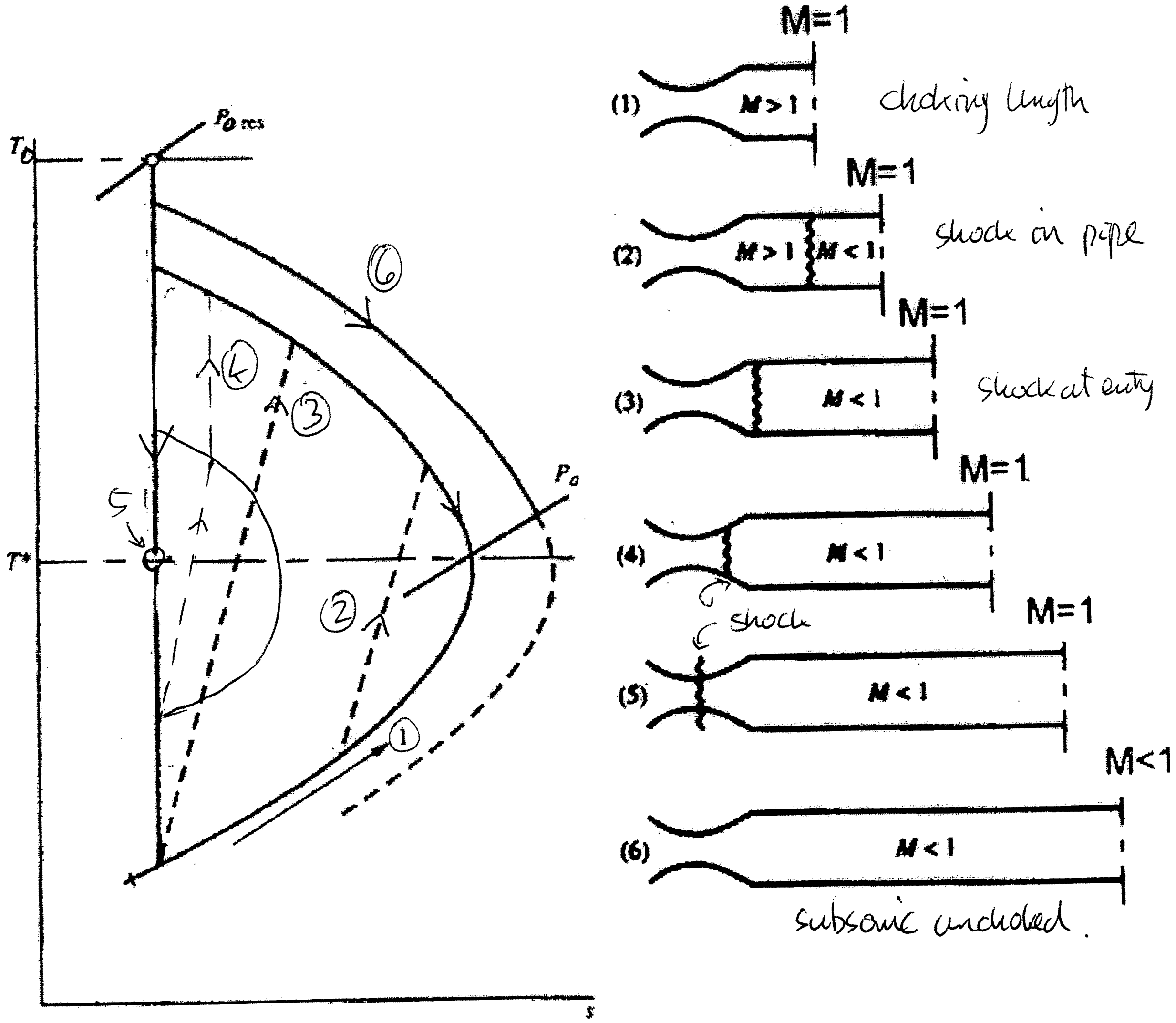
Pos
P, Po
Ps, Pos

2) (a)



Supersonic flow
P_0
T_0
v
T
T^*
P_0
P_0^*
P^*
sonic
f
Fanno line
Flow
P_1
s

(b)



(c) $M = 1{\cdot}2$; CUED tables give $\dfrac{4c_fL_1}{D} = 0{\cdot}0336$

Shock strength $M = 1{\cdot}18$; $\dfrac{4c_fL_2}{D} = 0{\cdot}0281$

$$\frac{4c_fL_{12}}{D} = \frac{4c_fL_1}{D} - \frac{4c_fL_2}{D} = 0{\cdot}0055$$

Now behind the shock have $M_s = 0{\cdot}8549$

$\therefore \dfrac{4c_fL_{2'}}{D} = 0{\cdot}03365$ (interpolated from CUED tables)

$\therefore$ total from inlet is $\dfrac{4c_fL_{12}}{D} + \dfrac{4c_fL_{2'}}{D} = 0{\cdot}0055 + 0{\cdot}03365$

$= 0.03915$

But total length is 0.35m and pipe diameter $D = 0.1m$.

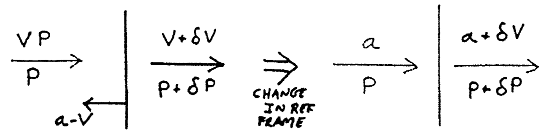
$$\therefore \quad C_f = \frac{0.03915}{4 \times 0.35} \times 0.1 = \underline{0.0028}$$

$$\therefore \quad \text{length } L_2 = \frac{0.0055 \times 0.1}{4 \times 0.0028} = \underline{0.049m}$$

3) (a)

Riemann Invariants for Sound Waves

Left-running waves



Momentum: $-\delta P = \rho a \delta V$

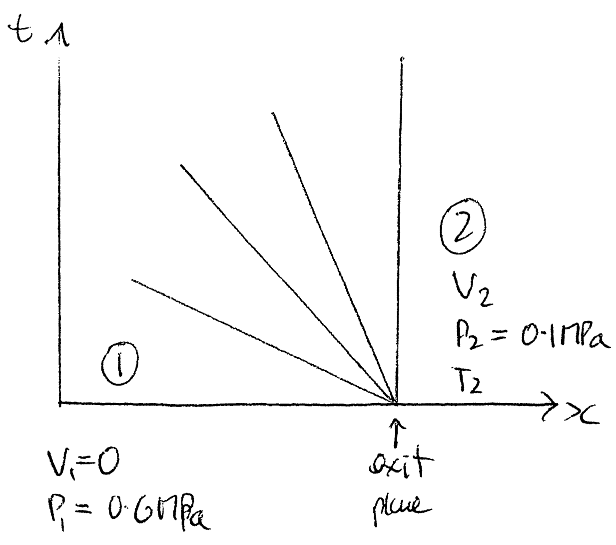
Isentropic relation: $P \propto T^{\gamma/\gamma-1} \propto a^{2\gamma/\gamma-1}$ therefore $\frac{\delta P}{P} = \frac{2\gamma}{\gamma-1}\frac{\delta a}{a}$

Speed of sound $a^2 = \frac{\gamma P}{\rho}$ so that $\delta P = \frac{2}{\gamma-1}\rho a \delta a$

Substitute into momentum eqn: $\delta V + \frac{2}{\gamma-1}\delta a = 0$

Hence across a <u>left-running</u> wave $d\left(V + \frac{2}{\gamma-1}a\right) = 0$ and $V + \frac{2a}{\gamma-1} = \text{const.}$

(b) (i)



choked

exit: last wave is fixed on exit plane.

Pressure ratio $P_1/P_2 = 6$: temp. ratio $T_1/T_2 = (P_1/P_2)^{\frac{\gamma-1}{\gamma}}$ (isentropic)

$= 1.6685$

$\therefore T_2 = T_1/1.6685 = 169.7K$

$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 283.15} = 337.3 \, m/s$

$a_2 = \sqrt{1.4 \times 287 \times 169.7} = 261.1 \, m/s$

Riemann invariant $V + \frac{2a}{\gamma-1} = \text{const}$

$$\cancel{V_1}^{0} + \frac{2a_1}{\gamma-1} = V_2 + \frac{2a_2}{\gamma-1}$$

$$V_2 = \frac{2}{\gamma-1}(a_1 - a_2) = \frac{2}{0.4}(337.3 - 261.1) = 381 m/s$$

This is $> a_2 \therefore$ exit of valve is choked.

Recalculate exit sound speed using Riemann invariants!

$$\therefore a^* + \frac{2a^*}{\gamma-1} = \frac{2a_1}{\gamma-1} \Rightarrow \frac{\gamma+1}{\gamma-1} a^* = \frac{2a_1}{\gamma-1}$$

$$\therefore a^* = \frac{2a_1}{\gamma+1} = 281.1 m/s$$

$= \text{exit velocity}$

Density at exit (isentropic) $\frac{\rho^*}{\rho_1} = \left(\frac{T}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{a}{a_1}\right)^{\frac{2}{\gamma-1}}$

$$= \left(\frac{281.1}{337.3}\right)^5 = 0.4019$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{6\times10^5}{287\times283.15} = 7.383\,kg/m^3$$

$$\therefore \rho^* = 7.383\times0.499 = \underline{2.97\,kg/m^3}$$

Area $A = \frac{\pi}{4}\times(0.5)^2 = 0.1963\,m^2$

$\therefore$ flow rate $\dot{m}^* = \rho^* V A = 2.97\times281.1\times0.1963$
$= 163.9\,kg/s$

$\therefore$ time to vent 1000 kg is $\underline{6.1 \text{ seconds}}$

6) (a) $$\frac{d^2y}{dt^2} + \omega^2 y = 0; \quad y(0) = 0; \quad \frac{dy}{dt}(0) = \omega$$

Discretise: $$\frac{y^{(n+1)} - 2y^{(n)} + y^{(n-1)}}{\Delta t^2} + \omega^2 y^{(n)} = 0$$

second order in time (no proof required)

$$y^{(0)} = 0; \quad \frac{y^{(1)} - y^{(0)}}{\Delta t} = \omega \Rightarrow y^{(1)} = \omega \Delta t$$

(b) $$y^{(n+1)} = 2y^{(n)} - y^{(n-1)} - \omega^2 y^{(n)} \Delta t^2$$

$$y^{(n+1)} = (2 - \omega^2 \Delta t^2) y^{(n)} - y^{(n-1)}$$

substitute $y^{(n)} = \lambda^n$:

$$\lambda^2 - (2 - \omega^2 \Delta t^2)\lambda + 1 = 0$$

quadratic formula

$$\lambda = \frac{2 - \omega^2 \Delta t^2 \pm \sqrt{(2 - \omega^2 \Delta t^2)^2 - 4}}{2}$$

$$\lambda = 1 - \frac{\omega^2 \Delta t^2}{2} \pm \sqrt{\left(\frac{\omega^2 \Delta t^2}{4} - 1\right)}\,\omega \Delta t$$

if $\frac{\omega^2 \Delta t^2}{4} > 1$ have real solution

$\frac{\omega^2 \Delta t^2}{4} < 1$ have complex (oscillatory) solution

Look at the oscillatory solution: $\omega \Delta t < 2$

$$\lambda = 1 - \frac{\omega^2 \Delta t^2}{2} \pm i\omega\Delta t\left(1 - \frac{\omega^2\Delta t^2}{4}\right)^{1/2}$$

and $y^{(n)} = A\lambda_1^n + B\lambda_2^n$

set $\lambda_1 = e^{i\alpha}$; $\lambda_2 = e^{-i\alpha}$; $\left[\begin{array}{l} \cos\alpha = 1 - \frac{\omega^2\Delta t^2}{2} \\ \sin\alpha = \omega\Delta t\left(1 - \frac{\omega^2\Delta t^2}{4}\right)^{1/2} \end{array}\right.$

$$y^{(n)} = Ae^{i\alpha n} + Be^{-i\alpha n}$$

at $n=0$ have $0 = A + B$

$n=1$ have $\omega\Delta t = Ae^{i\alpha} + Be^{-i\alpha}$

$$\Rightarrow A = -B = \frac{\omega\Delta t}{e^{i\alpha} - e^{-i\alpha}}$$

$$\therefore \quad y^{(n)} = \omega\Delta t \frac{\sin n\alpha}{\sin\alpha}$$

(c) Solution of the original ODE is $y(t) = \sin\omega t$

$\therefore$ as $\omega\Delta t \to 0$ have $\sin\alpha \to \omega\Delta t$

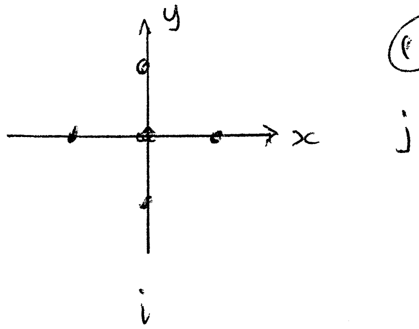
$\alpha \to \omega\Delta t$

then $y^{(n)} \to \sin n\omega\Delta t = \sin\omega t$

(d) Solution becomes inaccurate as $\sin\alpha \neq \omega\Delta t$, then diverges for $\omega\Delta t > 2$

7) (a) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

(10)



(i) $$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} = 0$$

(ii) $$u_{i+1,j} = u_{i,j} + \left.\frac{\partial u}{\partial x}\right|_{i,j} \Delta x + \left.\frac{\partial^2 u}{\partial x^2}\right|_{i,j} \frac{\Delta x^2}{2!}$$

Taylor series

$$+ \left.\frac{\partial^3 u}{\partial x^3}\right|_{i,j} \frac{\Delta x^3}{3!} + \frac{\partial^4 u}{\partial x^4} \frac{\Delta x^4}{4!} + \ldots$$

$$u_{i-1,j} = u_{i,j} - \left.\frac{\partial u}{\partial x}\right|_{i,j} \Delta x + \left.\frac{\partial^2 u}{\partial x^2}\right|_{i,j} \frac{\Delta x^2}{2!}$$

$$- \left.\frac{\partial^3 u}{\partial x^3}\right|_{i,j} \frac{\Delta x^3}{3!} + \left.\frac{\partial^4 u}{\partial x^4}\right|_{ij} \frac{\Delta x^4}{4!} + \ldots$$

Add $$u_{i+1,j} + u_{i-1,j} = 2u_{i,j} + \left.\frac{\partial^2 u}{\partial x^2}\right|_{i,j} \Delta x^2 + \left.\frac{\partial^4 u}{\partial x^4}\right|_{ij} \frac{\Delta x^4}{12} + \ldots$$

$$\therefore \left.\frac{\partial^2 u}{\partial x^2}\right|_{ij} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} - \left.\frac{\partial^4 u}{\partial x^4}\right|_{ij} \frac{\Delta x^2}{12} + \ldots$$

ie, second order accurate.

→ and similarly for y-direction.

iii) Discrete Laplace equation may be solved using

pseudo time-stepping $\frac{\partial u}{\partial t} = \nabla^2 u$

matrix methods (tridiagonal matrix using Gaussian elimination).

direct methods, e.g. Fourier.

Q7b)(i) $\psi = \frac{\Delta h_0}{U^2}$ measure of work input, typical ≈0.4, ratioed to blade KE.

$\phi = \frac{u}{U}$ relates to shape of velocity Δ higher ϕ means greater axial KE - easier to put work in - smaller flow deflection.

$\Lambda = \frac{\Delta h|_{rotor}}{\Delta h|_{stage}}$ Reaction, for isentropic flow, can be interpreted as fraction of pressure rise across rotor compared to stage.

(ii) $$\Delta h|_{rotor} = C_p(T_2 - T_1)$$

$$= C_p\left[\left(T_{02} - \frac{u^2+v_2^2}{2C_p}\right) - \left(T_{01} - \frac{u^2+v_1^2}{2C_p}\right)\right]$$

$$= C_p(T_{02} - T_{01}) + \frac{1}{2}(v_2^2 - v_1^2)$$

$$\underline{\underline{\Delta h|_{rotor} = \Delta h_0 - \frac{1}{2}(v_2 - v_1^2)}}$$

(iii) $\Delta h_0 = \Delta h|_{stage}$ since $u = \text{const}$ & $\alpha_1 = \alpha_3$

also $\Delta h_0 = U(v_2 - v_1)$

$$\Rightarrow \Delta h|_{rotor} = \Lambda \Delta h|_{stage} = \Lambda \Delta h_0 = \Delta h_0 - \frac{1}{2}(v_2 - v_1)(v_2 + v_1)$$

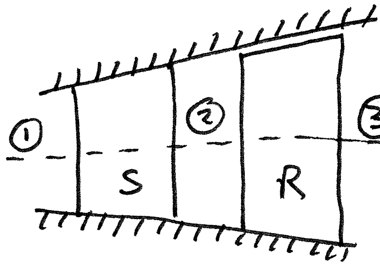
$$\Lambda \Delta h_0 = \Delta h_0 - \frac{1}{2}\frac{\Delta h_0}{U}\left((v_2 - v_1) + 2v_1\right)$$

$$\Lambda = 1 - \frac{1}{2U}\left(\frac{\Delta h_0}{U} + 2u\tan\alpha_1\right)$$

$$\Lambda = 1 - \frac{1}{2}\frac{\Delta h_0}{U^2} - \phi\tan\alpha_1 \Rightarrow \underline{\underline{\psi = 2(1 - \Lambda - \phi\tan\alpha_1)}}$$

Q8)

a) $\dfrac{P_1}{P_{01}} = \dfrac{572.4}{600.0} = 0.9540 \quad \underset{\gamma=1.4}{\text{TABLES}} \Rightarrow \quad \underline{M_1 = 0.260}$



$\Rightarrow \dfrac{\dot{m}\sqrt{C_p T_{01}}}{P_{01} A_1 \cos\alpha_1} = Q_t(M_1) = 0.5528$

$\Rightarrow \dot{m} = \dfrac{0.5528 \times P_{01} \times A_1 \times \cos\alpha_1}{\sqrt{C_p T_{01}}} = \dfrac{0.5528 \times 600 \times 10^3 \times 0.075 \times \cos 0^\circ}{\sqrt{1005 \times 900}} = \underline{\underline{26.16 \text{ kg/s}}}$

b) $\dfrac{P_2}{P_{02}} = \dfrac{381.8}{582.0} = 0.6560 \qquad \underset{\gamma=1.4}{\text{TABLES}} \Rightarrow \underline{M_2 = 0.800}$

$T_2 = \dfrac{T_{02}}{1 + \frac{\gamma-1}{2} M_2^2} = \dfrac{900}{1 + 0.2 \times 0.8^2} = \underline{797.9 \text{ K}}$

$V_2 = M_2 \sqrt{\gamma R T_2} = 0.8\sqrt{1.4 \times 287 \times 797.9} = 452.97 \text{ m/s}$

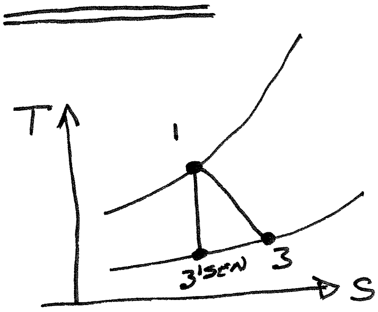
$u_2 = V_2 \cos\alpha_2 = 452.97 \cos 70^\circ = \underline{\underline{154.9 \text{ m/s}}}$

$v_2 = V_2 \sin\alpha_2 = 452.97 \sin 70^\circ = \underline{\underline{425.7 \text{ m/s}}}$

c) EULER WORK: $C_p(T_{02} - T_{03}) = U_2 v_2 - U_3 v_3 \qquad U_2 = U_3 = 285$
$v_3 = 0 \; (\alpha_3 = 0)$

$\Rightarrow T_{03} = T_{02} - \dfrac{U v_2}{C_p} = 900 - \dfrac{285 \times 425.7}{1005} = \underline{\underline{779.3 \text{ K}}}$

d) $\eta_{tt} = \dfrac{W_{ACT}}{W_{IDEAL}} = \dfrac{T_{01} - T_{03}}{T_{01} - T_{03}^{ISEN}}$



$T_{03}^{ISEN} = T_{01}\left(\dfrac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 900\left(\dfrac{345.7}{600.0}\right)^{\frac{1}{3.5}} = 768.8 \text{ K}$

$\eta_{tt} = \dfrac{900 - 779.3}{900 - 768.8} = 0.920 \qquad \underline{\underline{\eta_{tt} = 92.0\,\%}}$

(Q8 cont)

e) $$Y_p\big|_{STATOR} = \frac{P_{01} - P_{02}}{P_{02} - P_2}$$ N.B. ABSOLUTE VALUES (STATOR)
EXIT DYN. HEAD (TURBINE)

$$Y_p\big|_{STATOR} = \frac{600.0 - 582.0}{582.0 - 381.8} = \underline{\underline{0.090}}$$

f) $$Y_p\big|_{ROTOR} = \frac{P_{02}^{REL} - P_{03}^{REL}}{P_{03}^{REL} - P_3}$$ N.B. RELATIVE VALUES (ROTOR)

$$v_2^{REL} = v_2 - U = 425.7 - 285 = 140.7 \text{ m/s}$$

$$T_{02}^{REL} = T_2 + \frac{u_2^2 + v_2^{REL^2}}{2c_p} = 797.9 + \frac{(154.9^2 + 140.7^2)}{2\times 1005} = \underline{819.7 \text{ K}}$$

$$P_{02}^{REL} = P_2\left(\frac{T_{02}^{REL}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = 381.8\left(\frac{819.7}{797.9}\right)^{3.5} = \underline{419.6 \text{ kPa}}$$

ROTOR EXIT: $\frac{P_3}{P_{03}} = \frac{335.2}{345.7} = 0.9696$ TABLES $\gamma=1.4$ $\Rightarrow$ $\underline{M_3 = 0.210}$

$$T_3 = \frac{T_{03}}{1+\frac{\gamma-1}{2}M_3^2} = \frac{779.3}{1+0.2\times 0.21^2} = \underline{772.5 \text{ K}} \quad \Rightarrow u_3 = 0.21\sqrt{\gamma R T_3}$$

$$\underline{u_3 = 117.0 \text{ m/s}}$$

$$T_{03}^{REL} = T_3 + \frac{(u_3^2 + v_3^{REL^2})}{2c_p} = 772.5 + \frac{117.0^2 + 285^2}{2\times 1005} = \underline{819.7 \text{ K}}$$ (AS EXPECTED $T_{03}^{REL} = T_{02}^{REL}$)

$$P_{03}^{REL} = P_3\left(\frac{T_{03}^{REL}}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = 335.2\left(\frac{819.7}{772.5}\right)^{3.5} = \underline{412.5 \text{ kPa}}$$

$$Y_p\big|_{ROTOR} = \frac{419.6 - 412.5}{412.5 - 335.2} = \underline{\underline{0.092}}$$

g) BOTH STATOR & ROTOR HAVE LOSS COEFF. ≈ 9%
STATOR $P_2/P_1 = \frac{381.8}{572.4} = 0.667$ ROTOR $\frac{P_3}{P_2} = \frac{335.2}{381.8} = 0.878$

STATOR HAS MUCH GREATER PRESSURE DROP THAN ROTOR SO WOULD EXPECT EASIER TO DESIGN $\Rightarrow$ LOWER LOSS.
WOULD EXPECT TO BE ABLE TO IMPROVE <u>STATOR</u> DESIGN