2010 PART IIA 3B5 SEMICONDUCTOR ENGINEERING
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ENGINEERING TRIPOS PART IIA

Tuesday 4 May 2010 9 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING - SOLUTIONS

Version: 4

1 (a) The first term in the Time-Independent Schrödinger Equation (TISE) represents the kinetic energy of a particle multiplied by the wavefunction, ψ, where $\hbar$ is the Plank constant divided by 2π, m is the mass of the particle and x is position. The second term is the potential energy of the particle, V, multiplied by the wavefunction and the third term is the total energy of the particle, E, multiplied by the wavefunction. Therefore, the TISE is really a statement of the conservation of energy: total energy of a particle is the sum of its kinetic and potential energy. [20%]

(b) (i) We need to find a solution to the TISE for the potential given by

$$V = \begin{cases} \infty & \text{for } x < 0 \text{ and } x > L \\ 0 & \text{for } 0 < x < L \end{cases} .$$

Outside the well ($x < 0$ and $x > L$), the TISE becomes

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \infty\psi = E\psi ,$$

which only has two possible solutions: $\psi(x) = 0$ or $\psi(x) = \infty$. However, as the integral of $|\psi|^2$ over all space is unity, then ψ must be finite, and so the only possible solution is that $\psi(x) = 0$.

Inside the well, the TISE becomes

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + 0\psi = E\psi ,$$

which has the general solution

$$\psi = A\cos(kx) + B\sin(kx) ,$$

where A and B are unknown constants which much be determined by applying boundary conditions. As the wavefunction must be single-valued and continuous at all points, the wavefunctions for inside and outside the boundary must by the same at the boundary itself ($x = 0$ and $x = L$). Therefore, $\psi(0) = \psi(L) = 0$. Substituting into the general solution at $x = 0$ gives $A = 0$, and so

$$\psi = B\sin(kx)\ .$$

Repeating for $x = L$ gives

$$0 = B\sin(kx)\ ,$$

which means that k can only take certain values given by

$$k = \frac{n\pi}{L}\ ,$$

where n is an integer greater than 0. Finally, we must determine the unknown constant B, which can be done by applying the fact that the integral of $|\psi|^2$ over all space is unity. Hence,

$$\begin{aligned}
\int_0^L |\psi|^2 dx &= \int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right)dx \\
1 &= \int_0^L \frac{B^2}{2}\left\{1 - \cos\left(\frac{2n\pi x}{L}\right)\right\}dx \\
1 &= \frac{B^2}{2}\left[x - \frac{L}{2n\pi}\sin\left(\frac{2n\pi x}{L}\right)\right]_0^L \\
\frac{2}{B^2} &= \left[L - \frac{L}{2n\pi}\sin(n\pi)\right] - [0] \\
B &= \left(\frac{2}{L}\right)^{1/2}
\end{aligned}$$

Therefore,

$$\psi = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right).$$ [40%]

(ii) From the TISE in the potential well, we know that

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} .$$

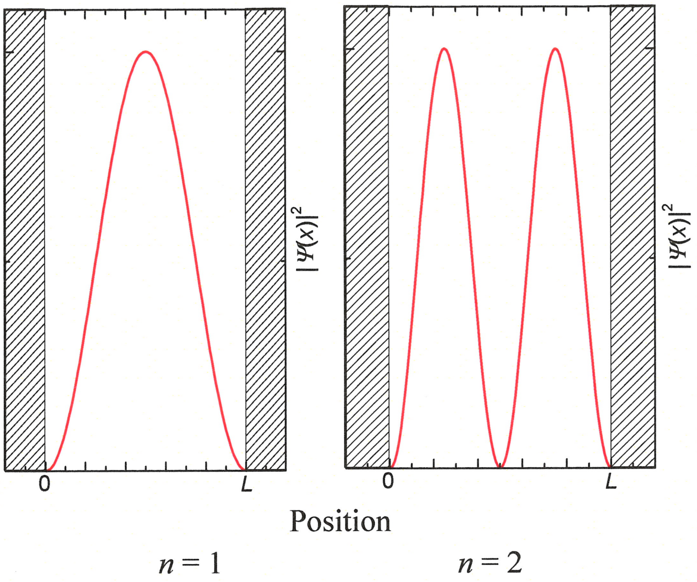
Therefore, for the ground state, where $n = 1$, we have that

$$\begin{aligned}\frac{d^2\psi}{dx^2} &= \frac{d^2}{dx^2}\left\{\left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{\pi x}{L}\right)\right\} \\ &= -\left(\frac{2}{L}\right)^{1/2}\left(\frac{\pi}{L}\right)^2 \sin\left(\frac{\pi x}{L}\right) \\ \frac{d^2\psi}{dx^2} &= -\left(\frac{\pi}{L}\right)^2 \psi\end{aligned}$$

and so

$$E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$$ [20%]

(iii) The probability of finding the particle in a small element of space *dx* as a function of position is given by $|\psi|^2$, which is platted for $n = 1$ and $n = 2$ below:



This is radically different to the classical equivalent in several ways. Firstly, the minimum energy that the particle could possess classically is zero. At this energy, the particle would not be moving. The particle could also possess any energy above this minimum. In both cases, there would be an equal probability of finding the particle at any position inside the potential well. It would certainly not have 'favoured locations' as suggested by the quantum model. [20%]

2 (a) The Hall Effect can be used to determine both the sign and number density of majority carriers in a semiconductor by applying a magnetic field, B, to a semiconductor sample in a perpendicular direction to the flow of a current that is driven through the material. The movement of carriers due to the current will result in the carriers experiencing a force due to the magnetic field. This force will be perpendicular to each of the current flow direction and the magnetic field. As a result, majority carriers will be deflected towards one side of the semiconductor, and an electric field will result. In a short space of time, an equilibrium will be reached when the force on the carriers due to the magnetic field is balanced by the force due to the electric field. This will result in a constant Hall voltage, *V*, being developed. The sign of the Hall voltage indicates the sign of the charge of the majority carriers, and hence whether they are electrons or holes. Its magnitude allows the carrier concentration to be determined. [15%]

(b) The force, F_B, acting on a carrier moving with a velocity, *v*, due to the magnetic field will be

$$F_B = evB \ .$$

The force, F_E, due to the electric field is

$$F_E = eE = \frac{eV_H}{w} .$$

At equilibrium, these two forces are equal, and so

$$evB = \frac{eV_H}{w}$$
$$V_H = vBw$$

However, the current, *I*, is related to the carrier velocity, *v*, by

$$I = qvwt$$

and so

$$V_H = \frac{IB}{qt} \ .$$ [35%]

(c) We will assume that all acceptors are ionised. Boron is a p-type dopant in silicon, and so we have holes as the majority carriers. As a result, p is equal to the doping density of boron atoms ($10^{22}\ \mathrm{m}^{-3}$). Therefore,

$$q = pe = 10^{22} \times 1.602 \times 10^{-19} = 1.602 \times 10^{3}\ \mathrm{m}^{-3} .$$

Hence,

$$\begin{aligned} V_H &= \frac{IB}{qt} \\ &= \frac{100 \times 10^{-9} \times 0.1}{1.602 \times 10^{3} \times 1 \times 10^{-6}} \\ V_H &= 6.24\ \mu\mathrm{V} \end{aligned}$$ [20%]

(d) The number density of electrons can be determined as the number density of holes is known from the Law of Mass Action,

$$\begin{aligned} n &= \frac{n_i^2}{p} \\ &= \frac{\left(10^{16}\right)^2}{10^{22}} \\ n &= 10^{10}\ \mathrm{m}^{-3} \end{aligned}$$

Therefore, the conductivity due to electrons is

$$\sigma_e = ne\mu_e = 10^{10} \times 1.602 \times 10^{-19} \times 0.14 = 2.24 \times 10^{-10}\ \Omega^{-1}\mathrm{m}^{-1} ,$$

While that due to holes is

$$\sigma_h = pe\mu_h = 10^{22} \times 1.602 \times 10^{-19} \times 0.048 = 76.9\ \Omega^{-1}\mathrm{m}^{-1}$$

Therefore, the total conductivity is essentially the same as that due to holes alone (76.9 Ω^{-1} m^{-1}) and the ratio of the electron conductivity to that of the holes is 2.9×10^{-12} .

If the number density of boron atoms was increased by a factor of 10^2 , then the ratio of electrons to holes would change by 10^4 , to become 2.9×10^{-16} . One might expect the conductivity to also simply increase by a factor of 10^2 . However, we know that doping densities over 10^{23} m^{-3} are very high, and that impurity scattering will have the effect of reducing the mobility. Therefore the increase in conductivity will be less than a factor of 10^2, and may be as little as 10^1, resulting in a conductivity between 700 and 7000 Ω^{-1} m^{-1} . [30%]

3 (a) The position of the Fermi level in the n-Si can be calculated via:

$$n = N_C \exp\left(\frac{E_F - E_C}{kT}\right)$$

Assuming that all donors are ionised, so that n=N$_D$, then

$$E_F - E_C = kT\ln\left(\frac{N_D}{N_C}\right) = 0\cdot 862\times 10^{-4}.298\ln\left(\frac{8\times 10^{22}}{2\cdot 8\times 10^{25}}\right)$$

$$E_F - E_C = -0.15\ \text{eV}$$

Hence the work function of the n-Si is given by

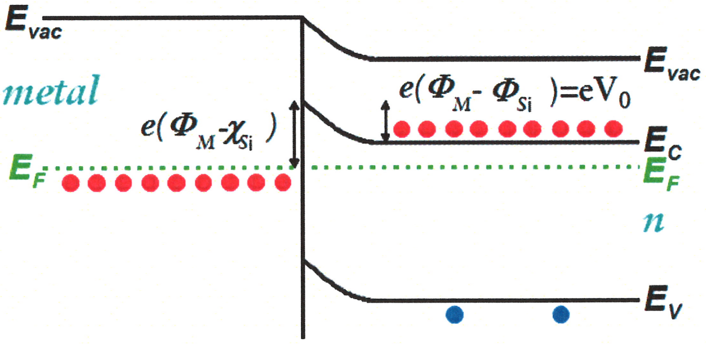
$$\begin{aligned} e\Phi_{Si} &= e\chi_{Si} + \left(E_C - E_F\right) \\ &= 4\cdot 05 + 0\cdot 15 \\ e\Phi_{Si} &= 4\cdot 2\ \text{eV} \end{aligned}$$

and the built-in potential is

$$V_0 = \frac{\Phi_M - \Phi_{Si}}{e}$$

$$V_0 = 0.6\ \text{V}$$

Equilibrium band diagram for the junction:



[30%]

(b) The barrier Φ_b to electron flow from the metal to the n-type silicon is given by

$$e\phi_B = e(\phi_M - \chi_{Si}) = 0.75eV$$

The reverse saturation current of the diode at room temperature can be estimated by

$$I = \pi \times (0.2 \times 10^{-3})^2 \times 1.2 \times 10^6 \times 298^2 \exp\left(\frac{-0.75}{0.025}\right)$$

$$I = 1.25nA$$

[20%]

(c) Due to the very large carrier density in the metal, the depletion width is entirely dominated by the n-Si. Starting with the Poisson equation and given that V only varies in the x direction across the junction

$$\frac{d^2V}{dx^2} = \frac{-\rho}{\varepsilon_0\varepsilon_r} = \frac{-eN_D}{\varepsilon_0\varepsilon_r}$$

Integrating this with respect to *x* and assuming that there are no electric fields outside the depletion region, i.e. ε= 0 at *x* = *w*, gives

$$-\varepsilon = \frac{dV}{dx} = \frac{eN_D(w-x)}{\varepsilon_0\varepsilon_r}$$

Integrating again using the condition that *V* = 0 at *x* = 0 gives

$$V = \frac{eN_D}{2\varepsilon_0\varepsilon_r}\left(2wx - x^2\right)$$

Hence the built-in potential (potential at x=w) can be written as

$$V_0 = \frac{eN_D w^2}{2\varepsilon_0\varepsilon_r}$$

Rearranging with respect to w, and also considering the voltage V applied to the gate with respect to the channel, gives

$$w = \left(\frac{2\varepsilon_0\varepsilon_r(V_0 - V)}{eN_D}\right)^{1/2}$$

4 (a) The flat band condition refers to flat conduction and valance bands in the semiconductor, i.e. no band bending is present. The gate voltage required to achieve the flat band condition is

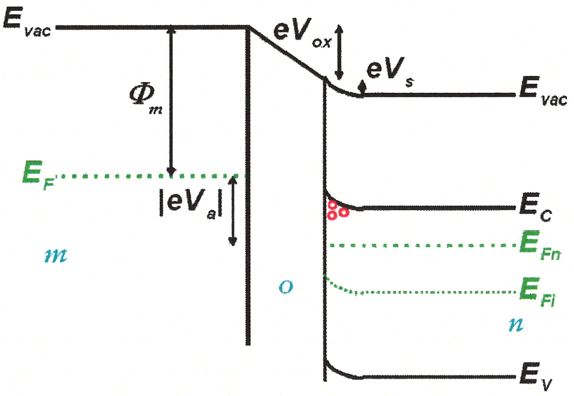
$$V_{FB} = \frac{\phi_m - \phi_{sc}}{e}$$

where Φ_m and Φ_{sc} are the work functions of the metal and semiconductor, respectively. For real MOS capacitors, interface trapped charges, fixed oxide charges, oxide trapped charges and mobile ionic charges will affect V_{FB}. Typically these charges are rolled into an effective charge Q_i and V_{FB} is written as

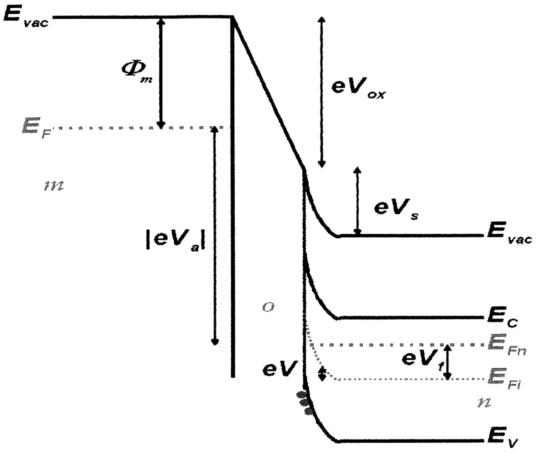
$$V_{FB} = \frac{\phi_m - \phi_{sc}}{e} - \frac{Q_i}{C_i}$$

where C_i is the capacitance (per unit area) of the gate insulator.

Depletion occurs when a gate voltage is applied such that majority carriers are repelled away from the channel layer in the semiconductor. For an p-channel enhancement MOSFET, an n-type semiconductor connects source and drain and depletion occurs for a negative gate voltage.



If a sufficiently large negative gate voltage is applied, then the bend bending may be sufficient that the Fermi level is as close to the valence band in the channel as it is to the conduction band in the bulk of the semiconductor. This is defined as the point where strong inversion has occurred.



[15%]

(b) The resistance of a small length of the channel δx is

$$\delta R = \frac{\rho \delta x}{t_{inv} W}$$

where t_{inv} is the thickness of the inversion layer and if the density of holes in the inversion layer is p_{inv}, then

$$\delta R = \frac{\delta x}{p_{inv} e \mu_{hFE} t_{inv} W}$$

The free carrier charge density per unit area is

$$Q_f = p_{inv} e t_{inv}$$

Therefore

$$\delta R = \frac{\delta x}{Q_f \mu_{hFE} W}$$

We know that there is the same current I_{DS} along the whole length of the channel, assuming that there is no charge leakage through the oxide to the gate.

Therefore, the voltage drop across the small element of the channel δx is

$$\delta V(x) = I_{DS} \delta R$$

Substituting for δR, this becomes

$$I_{DS}\delta x = Q_f \mu_{hFE} W \delta V(x)$$

Substituting for Q_n from the given equation and integrating from the source to the drain

$$\int_0^L I_{DS}\delta x = -\int_0^{V_{DS}} C_{ox}[V_{GS} - V_T - V(x)]\mu_{hFE} W \delta V(x)$$

$$I_{DS} = -\frac{C_{ox}\mu_{hFE}W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

[35%]

(c) The expression for I_{DS} is valid up to $V_{DS} = V_{GS} - V_T$, where the parabola is at its maximum.
At this point, the channel has been pinched off. [10%]

(d) The saturation current may be found from the equation for I_{DS} when $V_{DS} = V_{GS} - V_T$

$$I_{DS}(\text{sat}) = -\frac{C_{ox}\mu_{hFE}W(V_{GS} - V_T)^2}{2L}$$

The mutual transconductance in the saturated region is given by

$$g_m = \frac{\partial I_{DS}(sat)}{\partial V_{GS}} = -\frac{C_{ox}\mu_{hFE}W}{L}(V_{GS} - V_T)$$

[25%]

(e) For short channel devices, high fields can lead to a carrier velocity saturation. For this case the saturated current follows the velocity saturation, i.e. no pinch off is required, and hence a different expression for I_{DS}(sat) is required. [15%]

ENGINEERING TRIPOS PART IIA

Tuesday 4 May 2010 9 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING – NUMERICAL SOLUTIONS

1 (b) (ii) $E = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2$

2 (c) 2.24 μV

(d) Total conductivity = 76.9 Ω^{-1} m^{-1} ; ratio of the electron conductivity to that of the holes is 2.9×10^{-12}.

3 (a) $V = 0.6$ V

(b) $e\phi_b = 0.75$ eV ; $I = 1.25$ nA

4 (c) I_{DS} is valid up to $V_{DS} = V_{GS} - V_T$.

(d) $g_m = \frac{\partial I_{DS}(sat)}{\partial V_{GS}} = -\frac{C_{ox}\mu_{hFE}W}{L}(V_{GS} - V_T)$