3C8 MACHINE DESIGN
CRIB FOR 2010 EXAM
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1. a)

$$T = aV - b\omega$$

$$P = T\omega = aV\omega - b\omega^2$$

$$\frac{dP}{d\omega} = aV - 2b\omega = 0$$

$$aV = 2b\omega$$

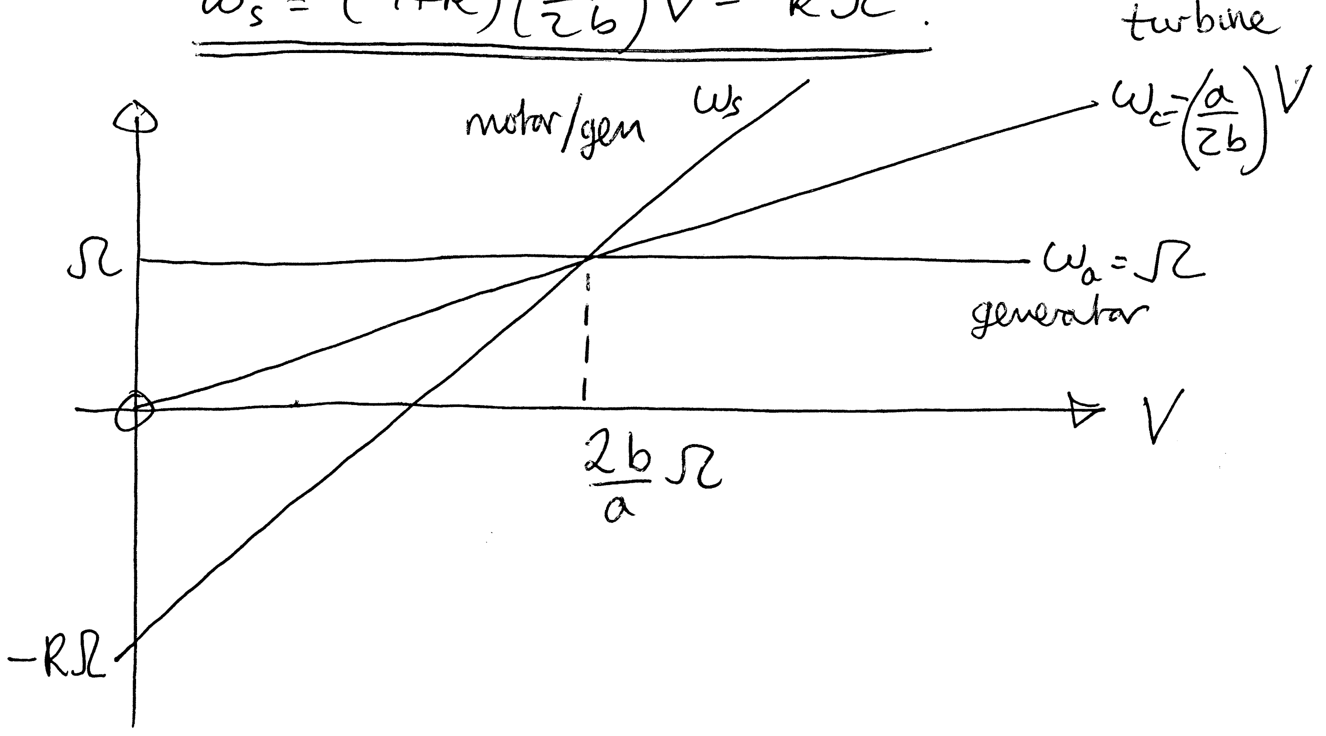
$$\omega = \left(\frac{a}{2b}\right)V$$

corresponding torque $T = \frac{aV}{2}$

b) data sheet: $\omega_s = (1+R)\,\omega_c - R\,\omega_a$

generator $\omega_a \rightarrow$ constant Ω
turbine $\omega_c \rightarrow \left(\frac{a}{2b}\right)V$.

$$\omega_s = (1+R)\left(\frac{a}{2b}\right)V - R\Omega.$$



c) To get power, need torques and speeds. Speeds from (b).

Torques: virtual power $T_s \omega_s + T_c \omega_c + T_a \omega_a = 0$

virtual speeds $\omega_s' \quad \omega_c' \quad \omega_a'$

set $\omega_s' = 0$

$$T_c \omega_c' + T_a \omega_a' = 0$$

$$\frac{T_c}{T_a} = -\frac{\omega_a'}{\omega_c'}$$

from data sheet speed rule $(1+R)\omega_c' = R\omega_a'$

$$\frac{\omega_a'}{\omega_c'} = \frac{1+R}{R}$$

$$\therefore \frac{T_c}{T_a} = -\frac{(1+R)}{R}$$

$$\therefore T_a = -\left(\frac{R}{1+R}\right) T_c$$

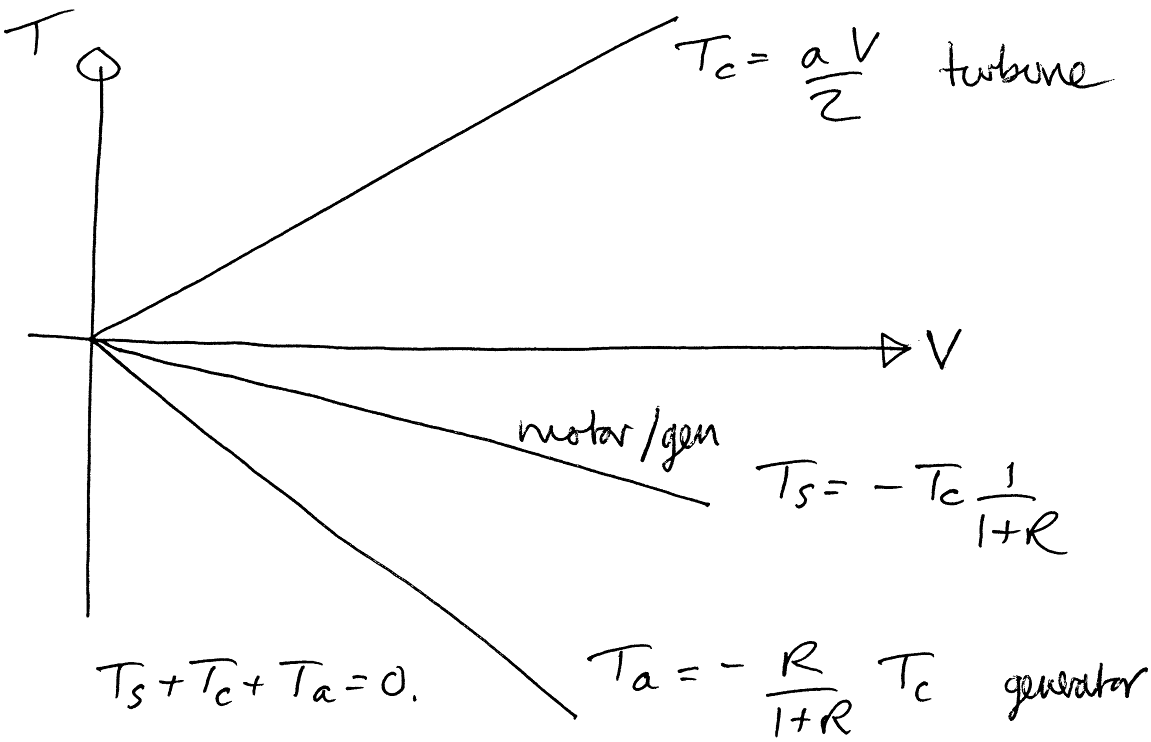
equilibrium of planetary gear $T_s + T_c + T_a = 0$

subst for T_a

$$T_s + T_c - \frac{R T_c}{1+R} = 0$$

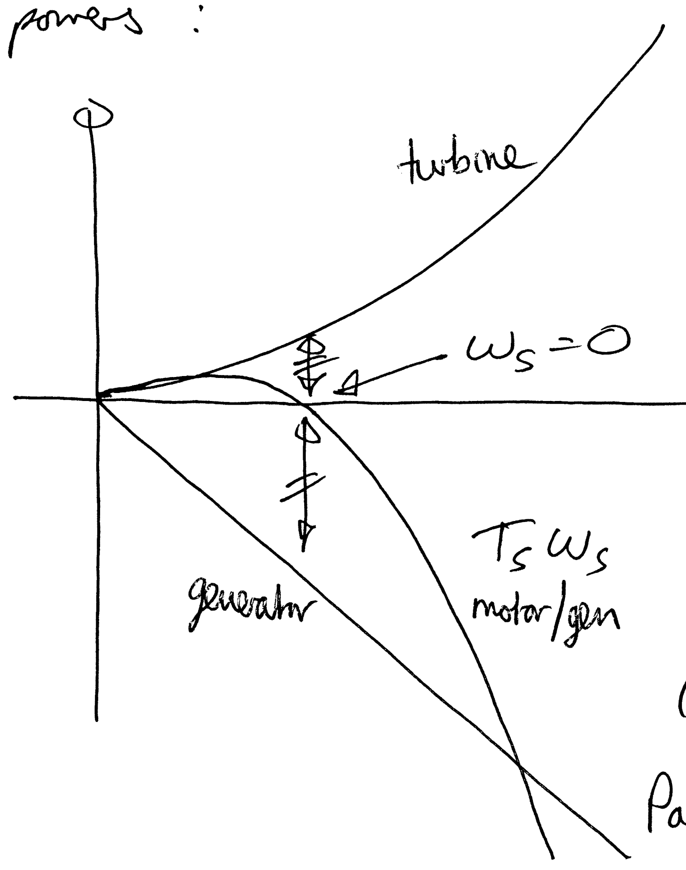
$$T_c\left(1 - \frac{R}{1+R}\right) + T_s = 0.$$

$$T_s = -\frac{T_c}{1+R}$$



$T_s + T_c + T_a = 0.$

d)
so powers :



$P_c = T_c \omega_c$ +ve, so turbine power into gearbox

+ve power when $V < \frac{R}{1+R} \frac{2b}{a} \Omega$

(from (b), $\omega_s = 0$)

$P_a = T_a \omega_a$ −ve, so power out of gearbox and into generator.

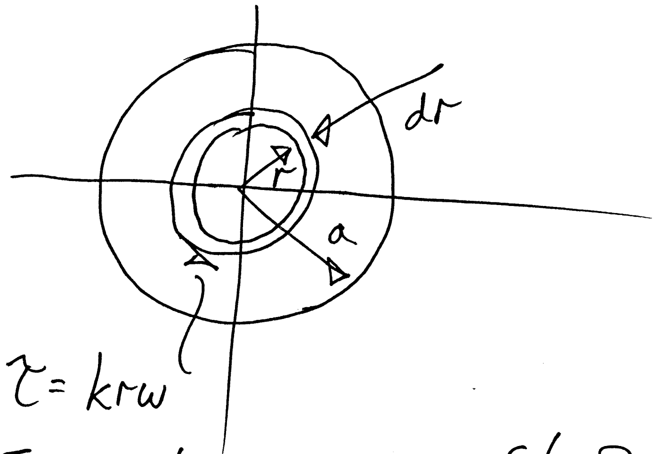
Power goes into sun when $V < V|_{\omega_s = 0}$.

from (b) this is when $\underline{\underline{V < \frac{R}{1+R} \frac{2b}{a} \Omega}}$

e) Could use a toroidal CVT, but cannot reverse direction (unlike an electric motor), so turbine operation would be limited to the speed range identified in (d).

2. a) smooth surfaces
small strains
linear elasticity
negligible surface shear stress.

b)



Find torque $T_{wet} = f(\omega, P, R, E^*, k)$

torque on ring $dT = \tau \, 2\pi r^2 \, dr$

$= k \, 2\pi \, r^3 \omega \, dr$

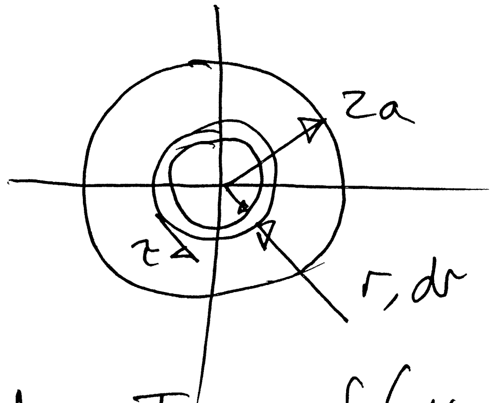
total torque $T_{wet} = \int_0^a k \, 2\pi r^3 \omega \, dr$

where $a = \left(\frac{3PR}{4E^*} \right)^{\frac{1}{3}}$ (data sheet)

$$T_{wet} = \left[\frac{k \, 2\pi}{4} r^4 \omega \right]_0^a$$

$$T_{wet} = \frac{k\pi}{2} \omega \left(\frac{3PR}{4E^*} \right)^{\frac{4}{3}} = \frac{k\pi}{2} \omega a^4$$

c)



$p(r) = \frac{A}{r}$

find $T_{dry} = f(\mu, a, A)$

ring $dT = 2\pi r\, dr\, \tau\, r$

$= 2\pi r\, dr\, \frac{\mu A}{\cancel{r}} \cancel{r}$

$\therefore T_{dry} = \int_0^{2a} 2\pi r \mu A\, dr$

$= \left[\pi r^2 \mu A\right]_0^{2a}$

$$\underline{\underline{T_{dry} = 4\pi a^2 \mu A}}$$

d) but what is A? $P = \int_0^{2a} p(r)\, 2\pi r\, dr$

$= \int_0^{2a} \frac{A}{r}\, 2\pi r\, dr$

$= \left[2\pi A r\right]_0^{2a}$

$P = 4\pi A a \quad \therefore A = \frac{P}{4\pi a}$

$T_{dry} = 4\pi a^2 \mu \frac{P}{4\pi a}$

$$\underline{\underline{T_{dry} = a\mu P}}$$

d) cont.

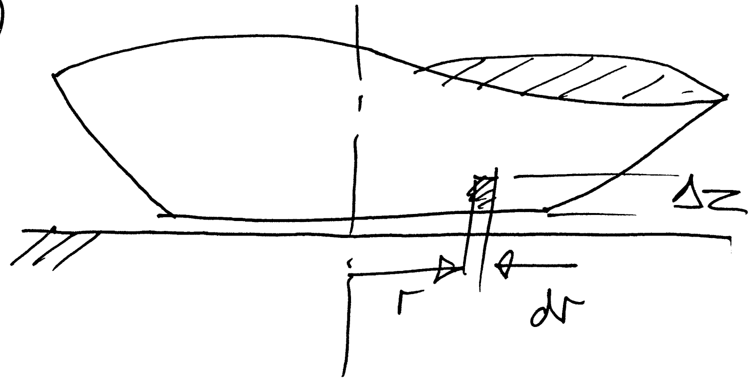
$$\frac{T_{dry}}{T_{wet}} = \frac{a\mu P}{\frac{k}{2}\pi\omega a^4} \leftarrow \text{from (b).}$$

$$\frac{T_{dry}}{T_{wet}} = \frac{2\mu P}{k\omega\pi a^3}$$

$$= \frac{2\mu \cancel{P}\, 4E^*}{k\omega\pi\, 3\cancel{P}R}$$

$$\frac{T_{dry}}{T_{wet}} = \frac{8}{3}\frac{\mu E^*}{k\omega\pi R}$$

e)



"Archard Law": wear volume $\propto$ load × sliding distance

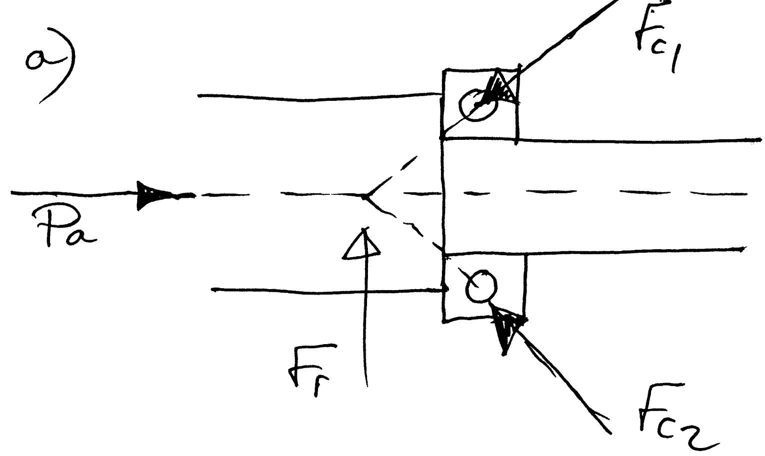
to maintain flat profile, wear rate at all radii must be the same.

So in time Δt, $\Delta V = 2\pi r\, dr . \Delta z$

$$= \underbrace{2\pi r\, dr\, p(r)}_{\text{load}} \times \underbrace{r\omega\Delta t}_{\text{distance}}$$

$$\text{i.e. } p(r) = \frac{\Delta z/\Delta t}{r\omega} = \frac{\text{const}}{r}$$

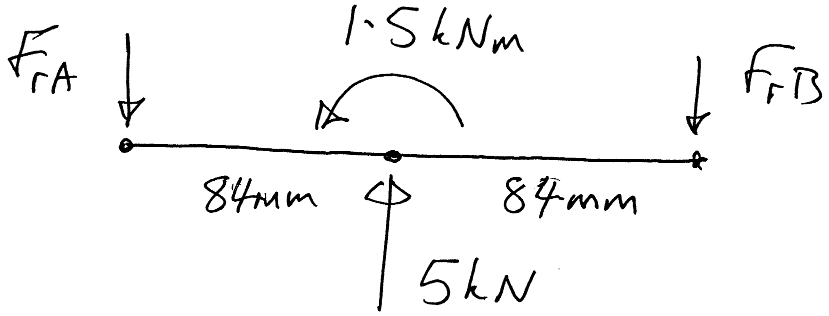
3 a)



The bearing contact forces F_{c1} and F_{c2} can only act along the diagonal lines.

For equilibrium of the shaft, the applied radial force F_r must act through the point where the axial force P_a and contact forces F_{c1} and F_{c2} intercept.

b) i) Find radial forces F_{rA} and F_{rB}



$$F_{rA} + F_{rB} = 5\,kN.$$

moments: $(F_{rB} - F_{rA}).\,0.084 = 1500$

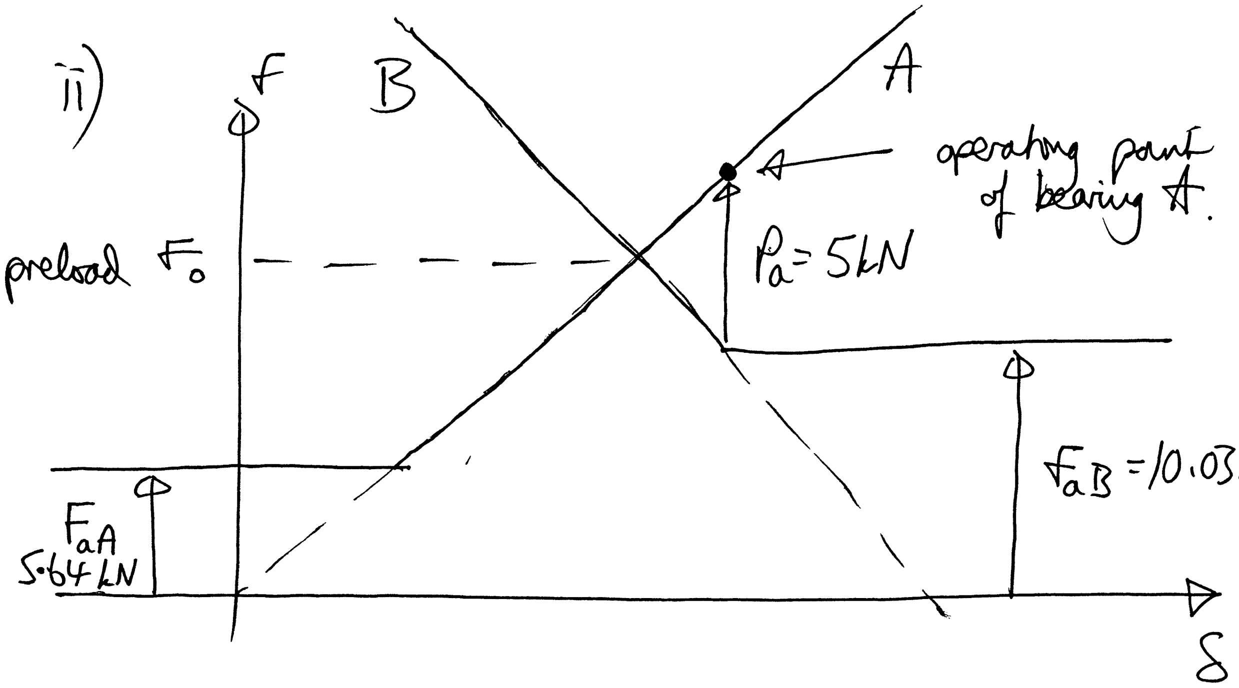
hence $F_{rB} = 11.43\,kN$

$F_{rA} = -6.43\,kN.$

$$F_{aA} = \left| F_{rA} \cdot \frac{1}{2.0.57} \right| = \underline{5.64\,kN}$$

$$F_{aB} = \left| F_{rB} \frac{1}{2.0.57} \right| = \underline{\underline{10.03\,kN}}$$

ii)



required pre-load force is $10.03 + \frac{5}{2} = \underline{\underline{12.53 kN}}$

c) life of bearing A.

axial force on bearing A is $10.03 + 5 = 15.03 kN$

radial force on bearing A is $6.43 kN$ (part (i))

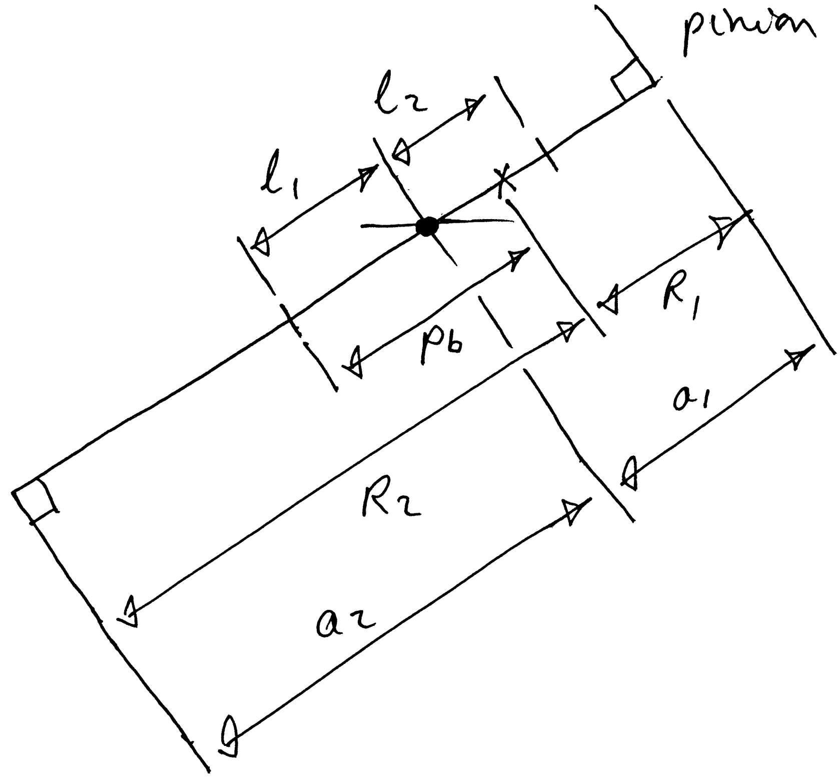
equivalent radial force is $15.03 + 6.43 = 21.46 kN$

from data sheet $C = 36400 N$

$$\therefore L = \left(\frac{C}{P}\right)^3 = \left(\frac{36400}{21460}\right)^3 = \underline{\underline{4.88 . 10^6 \text{revs}}}$$

d) If the bearings are reversed, the distance between F_{rA} and F_{rB} reduces, thus increasing the magnitudes of F_{rA}, F_{rB} and F_{aA}, F_{aB}. Consequently the preload force must increase, and the life of the bearings reduces.

4 a)



single pair contact $\quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where $\quad R_1 = a_1 + l_1 - p_b \qquad p_b = m\pi\cos 20°$

$R_2 = a_1 + a_2 - R_1$

$$a_1 = r_{b_1}\tan 20° = \frac{mN_1}{2}\cos 20 \tan 20$$

$$= \frac{mN_1}{2}\sin 20$$

$$a_2 = \frac{mN_2}{2}\sin 20.$$

data sheet. $\quad l = m\left(0.02924N^2 + N + 1\right)^{\frac{1}{2}} - 0.171N$

thus $l_1 = m\,2.298$

so. $\quad R_1 = \frac{mN_1}{2}\sin 20 + m\,2.298 - m\pi\cos 20$

$= m\,2.766$

$R_2 = \frac{mN_1}{2}\sin 20 + \frac{mN_2}{2}\sin 20 - m\,2.766$

$= m\,14.335$

thus $$R = \frac{R_1 R_2}{R_1 + R_2} = m \, \frac{2.766 \, . \, 14.335}{2.766 + 14.335}$$

$$= m \, . \, 2.3186$$

$$R \sim \underline{\underline{m \, . \, 2.32}}$$

b) i) four pinions, so four times the torque

$4 \times 500 \, Nm = \underline{\underline{2000 \, Nm}}$

ii) all dimensions doubled.

contact stress for line contact $p_0 = \left(\frac{P' E^*}{\pi R} \right)^{\frac{1}{2}}$

R doubles, so P' can also double for the same stress p_0

face width doubles. and wheel radius doubles

So wheel torque can increase $\underset{\text{wheel radius}}{\times 2} \; \underset{p'}{\times 2} \; \underset{\text{width}}{\times 2} = 8$

new torque $8 \times 500 = \underline{\underline{4000 \, Nm}}$

iii) Need to find new equivalent radius, so redo calculation of (a) with $N_2 = 60$ not 80.

$$R_1 = m\,2.766 \text{ as before}$$

$$R_2 = \frac{m}{2}.20 \sin 20 + \frac{m}{2}.60 . \sin 20 - m\,2.766$$

$$R_2 = 10.92\,m$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{m.\,2.766 . 10.92}{2.766 + 10.92}$$

$$R = m\,2.207$$

new torque is smaller because of smaller R, according to $\frac{2.207}{2.32}$ (keeping $\frac{R'}{R}$ const)

and new torque is also smaller because of smaller wheel radius, according to $\frac{60}{80}$.

so new torque is $500\,Nm \times \frac{2.207}{2.32} \times \frac{60}{80}$

$$= 356.7\ Nm$$

c) Arranging for the torque to be shared equally between the pinions is difficult. However, the volume and mass are potentially smaller than a single pinion design of the same torque capacity.

Answers 3C8 2010

1 (b) $(1+R)\left(\dfrac{a}{2b}\right)V - R\Omega$

(c) $T_{turbine} = \dfrac{aV}{2}$, $T_{motor/gen} = -\dfrac{1}{(1+R)}\dfrac{aV}{2}$, $T_{gen} = -\dfrac{R}{(1+R)}\dfrac{aV}{2}$

(d) $0 < V < \dfrac{R}{(1+R)}\dfrac{2b}{a}\Omega$

2 (b) $a = \left(\dfrac{3PR}{4E^*}\right)^{\frac{1}{3}}$, $T = \dfrac{k\pi w}{2}\left(\dfrac{3PR}{4E^*}\right)^{\frac{4}{3}}$

(c) $T' = 4\pi a^2 \mu A$

(d) $\dfrac{T'}{T} = \dfrac{8}{3}\dfrac{\mu E^*}{kw\pi R}$

3 (b) (i) F_{aA} =5.6 kN, F_{aB} =10.0 kN

(ii) 12.5 kN

(c) 4.9 million revolutions

4 (b) (i) 2 kNm

(ii) 4 kNm

(iii) 357 Nm