ENGINEERING TRIPOS PART IIA **2009/2010**

Crib for Paper 3C9: Fracture Mechanics of Materials and Structures

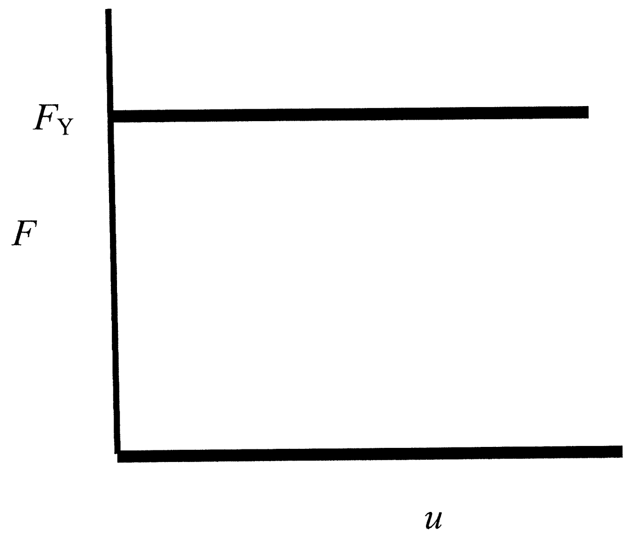
Q1. (a) (i) The J-integral is defined as the loss in potential energy per unit crack extension. It is possible to define the potential energy for a non-linear elastic solid but nor an elastic-plastic solid. However, for a stationary crack without crack extension, the stress-strain field surrounding the crack tip in an elastic-plastic solid is very close to that in a non-linear elastic solid provided the stress-strain curves match.

(ii) When a crack extends in a non-linear elastic solid, the drop in potential energy is released at the crack tip. In contrast, hysteresis occurs in the elastic-plastic solid and this lost energy does not appear at the crack tip in order to drive the fracture processes. The stress-strain fields associated with UNLOADING are very different for a non linear elastic solid and for an elastic-plastic solid. Since the J-integral is defined only for the non-linear elastic solid, it can only capture in an approximate way the growth of a crack. Thus, it is an approximate characterising parameter for a growing crack.

(b) (i)

$$F_Y = 2k(W-a)h$$

(ii)



The potential energy for a prescribed displacement is $\Psi = F_Y u = 2k(W-a)u$
(The potential energy for a prescribed force = 0)

(iii) $J = -\dfrac{\partial \Psi}{\partial a} = 2ku$ Now put $J = J_{IC}$ to get $u_C = \dfrac{J_{IC}}{2k}$

Q2. (a) The energy release rate G is the rate of energy release per unit crack extension for a linear elastic solid. Energy is released to the crack tip and drives any fracture processes there such as cleavage. Irwin demonstrated that $EG = K^2$ and thereby showed that the energy-approach and the mechanics-approach to fracture are actually the same. In order for fracture to occur, the energy criterion is necessary but not sufficient: a fracture mechanism must also be activated, such as cleavage of void growth.

G is an appropriate parameter under *small scale yielding* and is identical to the J-integral under these conditions.

(b) (i) Consider a beam of section h x h. The second moment of area for bending is $I_1 = h^4/12$. The compliance of this beam is $C_1 = \dfrac{a^3}{3EI_1}$.

Consider a beam of section $2h$ x h. The second moment of area for bending is $I_2 = 8h^4/12 = 2h^4/3$. The compliance of this beam is $C_2 = \dfrac{a^3}{3EI_2}$. The total compliance of the pair of beams is $C = C_1 + C_2 = \dfrac{9a^3}{2Eh^4}$.

(ii) $G = \dfrac{1}{2}\dfrac{P^2}{h}\dfrac{dC}{da}$ where $P = \delta / C$. Thus, $G = \dfrac{1}{2}\dfrac{\delta^2}{C^2 h}\dfrac{dC}{da}$

Now $\dfrac{dC}{da} = \dfrac{27a^2}{2Eh^4}$, giving $G = \dfrac{27}{4}\dfrac{\delta^2}{C^2}\dfrac{a^2}{Eh^5} = \dfrac{3}{2}\dfrac{\delta^2}{haC} = \dfrac{Eh^3\delta^2}{3a^4}$.

The asymmetry in geometry leads to mixed mode loading at the crack tip and so the crack tip may kink. It will kink downwards towards the mid-plane, and grow as a local mode I crack.

(iii) As the crack gets longer, G drops and so crack growth will arrest.

3. (a) The ductility ε_f scales as $\varepsilon_f = f^{-1/2}\exp\left(\frac{-3\sigma_h}{2\sigma_Y}\right)$ for a steel containing a volume fraction f of inclusions, where σ_h is the hydrostatic stress and σ_Y is the yield strength. Ahead of a crack tip we have $\sigma_h \approx 3\sigma_Y$. Also, $J_{IC} \approx 3\sigma_Y \ell \varepsilon_f$, where $\ell \approx R/\sqrt{f}$ is the inclusion semi-spacing in terms of inclusion radius R. Thus, a high f lead to a small ε_f and a small ℓ and thereby to as low toughness.

(b) Under small scale yielding conditions, an outer K field exists beyond the plastic zone and can thereby be used as a valid failure criterion. In contrast, when plastic collapse occurs, no K field exists.

(c) After an overload cycle, residual compression exists ahead of the fatigue crack. The fatigue crack nibbles through this zone of compression and this leads to an elevation in the crack-opening-stress-intensity factor. Consequently, the effective stress intensity range ΔK_{eff} drops and the crack growth rate retards, and may even arrest.

(d) As the crack grows, plastic dissipation occurs within the plastic zone and this elevates the observed toughness by a large multiplicative factor above the surface energy.

4. (i) When $a < W$, we have a net stress σ_{NET} on the crack of uniform value

$$\sigma_{NET}(t) = \sigma_Y - \sigma(t)$$

with $\sigma(t)$ varying from $\sigma(t) = 0$ to $\sigma(t) = \sigma_Y / 2$.

Thus, $K_{min} = \dfrac{\sigma_Y \sqrt{\pi a}}{2}$ $\quad K_{max} = \sigma_Y \sqrt{\pi a}$ $\quad$ and $\Delta K = \dfrac{\sigma_Y \sqrt{\pi a}}{2}$

So at $a=W$, we have $\Delta K = \dfrac{\sigma_Y \sqrt{\pi W}}{2}$.

When $a > W$, the loading on the crack faces comprises the sum of two contributions:

Problem A: uniform compression of $-\sigma(t)$ over crack giving $K_A = -\sigma\sqrt{\pi a}$

Problem B: tensile loading of a crack of length $2a$ over the central portion of length $2W$ by a value of $\sigma_Y - \sigma(t)$. This generates $K_B = \dfrac{2\sigma_Y a}{\sqrt{\pi a}} \arcsin\left(\dfrac{W}{a}\right)$ according to the datasheet. Hence:

$$K(t) = K_A + K_B = -\sigma\sqrt{\pi a} + \frac{2\sigma_Y a}{\sqrt{\pi a}} \arcsin\left(\frac{W}{a}\right)$$

giving $$K_{min} = -\frac{\sigma_Y}{2}\sqrt{\pi a} + \frac{2\sigma_Y a}{\sqrt{\pi a}} \arcsin\left(\frac{W}{a}\right)$$

and $$K_{max} = \frac{2\sigma_Y a}{\sqrt{\pi a}} \arcsin\left(\frac{W}{a}\right)$$

Also, $$\Delta K = K_{max} - K_{min}$$

Now, for a slightly bigger than W, K_{min} is negative.

(ii) Consider $W/4 < a < W/2$

Then, from (i) we have $\Delta K = \dfrac{\sigma_Y \sqrt{\pi a}}{2}$.

$$\frac{da}{dN} = C\Delta K^n = C\left(\frac{\sigma_Y \sqrt{\pi a}}{2}\right)^n$$

Integrate this to obtain

$$\left(\frac{n-2}{2}\right)\left[\left(\frac{4}{W}\right)^{(n-2)/2} - \left(\frac{2}{W}\right)^{(n-2)/2}\right]\frac{da}{dN} = C\left(\frac{\sigma_Y \sqrt{\pi}}{2}\right)^n N$$

(iii) The overall maximum value of $K_{\max}$ is achieved at a=W, giving

$$K_{overall-\max} = \sigma_Y \sqrt{\pi W}$$

Equate this to K_{IC}.

(iv) A stress relief will remove the tensile residual stress of value σ_Y, and consequently the crack is subjected to compressive cyclic loading. The crack will not advance in fatigue.

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