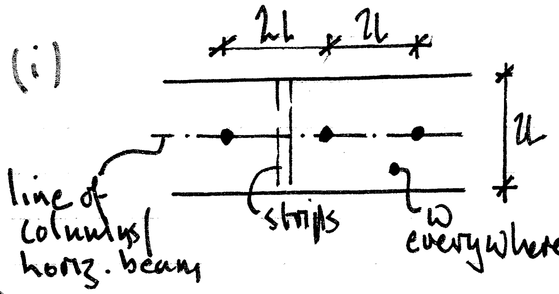
2010 PART IIA 3D3
STRUCTURAL MATERIALS AND DESIGN
DR K SEFFEN

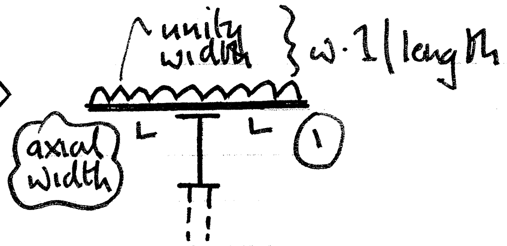
3D3/2010 Qu 1

a) A loadpath is any viable eqlm system, describing the transmission of loads to the foundation. If yield is nowhere violated, then by the lower bound theorem, the structure safely carries the loads.

b) (i)



$\Rightarrow$



(1) Maximum bending moment per unit width $= [(w \cdot 1) \cdot L] \times \frac{L}{2}$

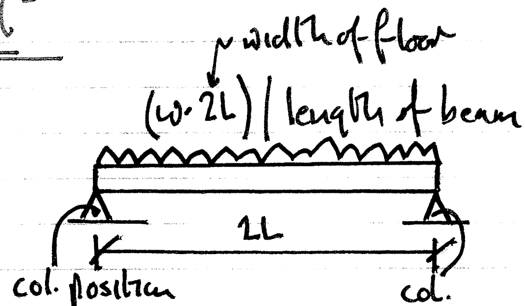
$= wL^2/2$ Nm/m (at central line) $= M_p$ (per unit width)

$\Rightarrow \underline{w_{max} = 2M_p/L^2 \text{ N/m}^2}$

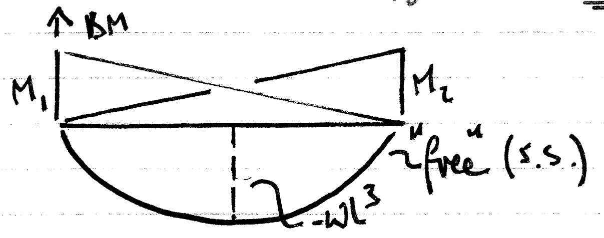
In each column, the vertical force on average, supports w over an area $2L \times 2L$

$\Rightarrow \underline{\text{column force} = 4wL^2}$

(ii) Side view of bridge/beam, where it is first assumed that the beam is simply-supported at each column.



$M_{max} = (w \cdot 2L)(2L)^2/8 \quad \Rightarrow \quad \underline{M_{max} = wL^3}$



If instead, the horiz. beam is continuously supported over columns, the b.m. profile is stat. indeterminate via M_1 and M_2 at column positions.

2 redundancies

Optimal set-up has $M_1 = M_2 = M$: max +ve b.m. $= M$

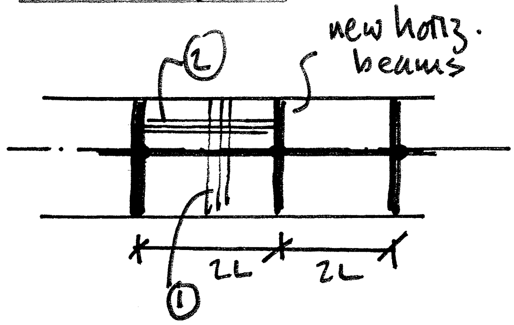
max -ve b.m. $= wL^3 - \frac{M}{2} \times 2$

Set both equal to $\pm M_{max} \Rightarrow \underline{M_{max} = \frac{wL^3}{2}}$ ($\frac{1}{2}$ s.s. case)

$\Rightarrow$ range of max. moment $\underline{\frac{wL^3}{2} \rightarrow wL^3}$

3D3/2010 Qu 1

b) (iii)



If horizontal beams are added transversely, this permits the floor slab to "develop" orthogonal strips, for carrying w.

For ① strips, let the load proportion be βw (constant)
" ② " " " " " " $(1-\beta)w$

①: result is the same as (b.i) $\Rightarrow M_{max} = M_p = \dfrac{(w\beta \cdot 1)L^2}{2}$ — Ⓐ

②: assume strips are clamped over transverse horiz. beams (recall in (b.ii), that max. moment = ½ s.s. case)

$$\Rightarrow M_{max} = \underbrace{[(1-\beta)w \cdot 1]}_{\text{load intensity/width}} \times (2L)^2 \cdot \frac{1}{8} \times \frac{1}{2}$$

$$\Rightarrow M_{max} = (1-\beta)wL^2/4 = M_p \quad — Ⓑ$$

Ⓐ: $w_{max} = 2M_p/\beta L^2$
Ⓑ: $w_{max} = 4M_p/(1-\beta)L^2$

optimal $\Rightarrow \dfrac{2}{\beta} = \dfrac{4}{(1-\beta)}$

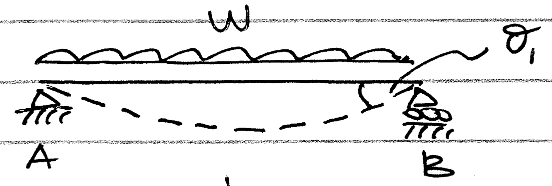
$\Rightarrow \beta = 1/3$

$$w_{max} = \overset{1/\beta}{3} \times \frac{2M_p}{L^2} = 3 \times (b.i)$$

Common mistakes:

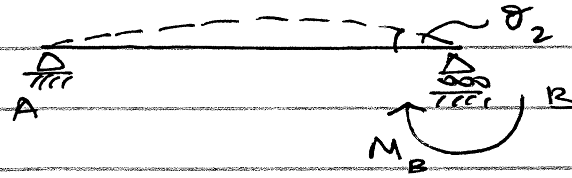
(a) i STRUCTURE IS STATICALLY INDETERMINATE

TAKE CUT AT B:



$$-\theta_1 = Wl^2/24EI$$

+



$$\theta_2 = Ml/3EI$$

BY INSPECTION OF ORIGINAL STRUCTURE ROTATION AT B = 0 $\therefore \theta_1 + \theta_2 = 0$

$$\therefore Ml/3EI = Wl^2/24EI$$

$$\therefore M_B = \underline{Wl/8}$$

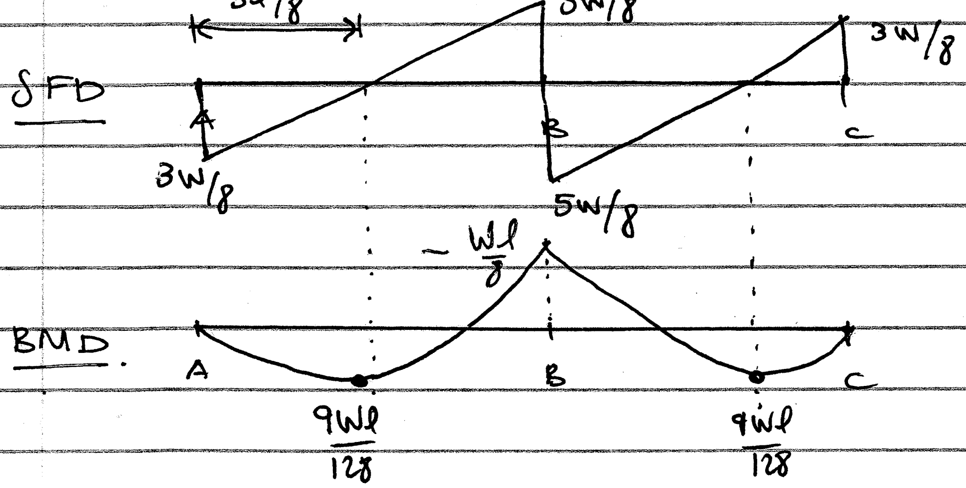
$\circlearrowleft A$: $\dfrac{Wl}{2} + \dfrac{Wl}{8} - P_1 l = 0$ (WHERE P_1 = SHEAR FORCE AT B)

$\therefore P_1 = 5W/8$ FOR SPAN AB

$= 5W/8$ FOR SPAN BC

$\therefore$ TOTAL SHEAR FORCE AT B $= \underline{5W/4}$

$\therefore$ SHEAR FORCE AT A AND C $= \dfrac{1}{2}\left(2W - \dfrac{5W}{4}\right) = \underline{3W/8}$



SAGGING MOMENT IS MAX AT $\dfrac{xW}{l} - \dfrac{3W}{8} = 0$

$\therefore x = 3l/8$

MOMENT AT $x = 3l/8$

$= \dfrac{9Wl}{64} - \left(\dfrac{W}{l} \cdot \dfrac{3l}{8} \cdot \dfrac{3l}{16}\right)$

$= 9Wl/128$

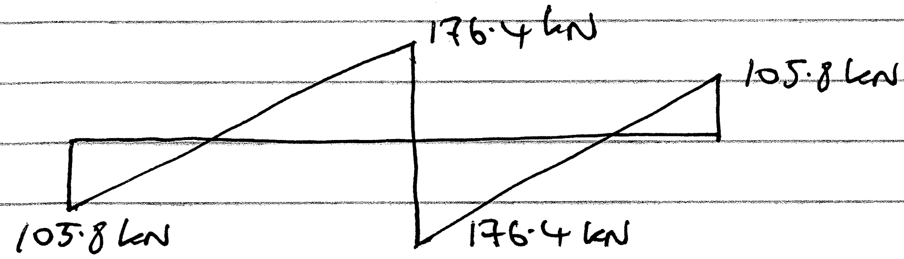
DESIGN LIVE LOAD PER SPAN $(W_L) = 14\,kN/m \times 4m \times 1.6 = \underline{89.6\,kN}$

DESIGN DEAD LOAD PER SPAN $(W_D) = \left[\left(24\,kN/m^3 \times 0.4 \times 0.25 \right) + \left(32\,kN/m \right) \right] \times 4 \times 1.4$

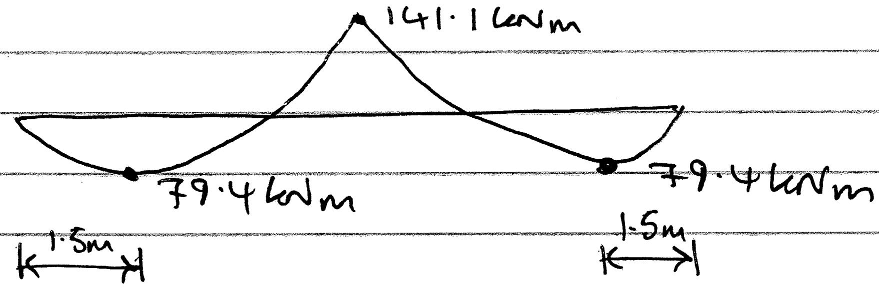
$= \underline{192.6\,kN}$

$\therefore$ TOTAL DESIGN LOAD PER SPAN $(W) = \underline{\underline{282.2\,kN}}$

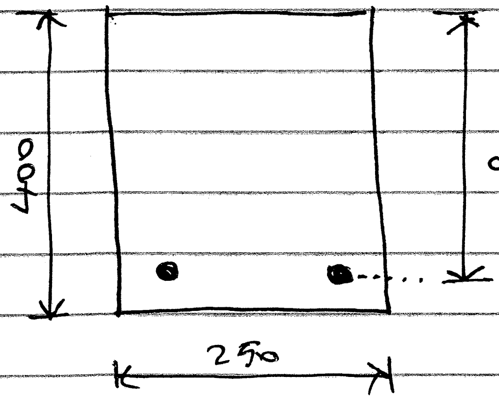
SFD



BMD



(ii)



$f_{cu} = 50\,MPa \;;\; f_y = 460\,MPa$

$\gamma_c = 1.5 \;;\; \gamma_s = 1.15$

$COVER = 40\,mm \;;\; \phi = 20\,mm$

$\therefore\; d = h - \phi/2 - COVER$

$= 400 - 10 - 40$

$= 350\,mm$

ASSUME UNDER REINFORCED AND SINGLY REINFORCED:

$M_u = 0.225 f_{cu} b d^2 / \gamma_c$

$= 0.225 \times (50 \times 10^6) \times 0.25 \times 0.35^2 / 1.5$

$= 229.7\,kNm > 141.1\,kNm$

$\therefore$ NO COMPRESSION REINFORCEMENT REQUIRED.

$$M_u = A_s f_y \frac{d}{\gamma_s}\left(1 - \frac{x}{2d}\right) \quad -(1)$$

WHERE $\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s \, 0.6 f_{cu} b d}$

$$= \frac{1.5 \times 460 \times 10^6 \; A_s}{1.15 \times 0.6 \times 50 \times 10^6 \times 0.25 \times 0.35}$$

$$= 228.6 \, A_s$$

SUBSTITUTE INTO (1):

$$M_u = A_s \times 460 \times 10^6 \times \frac{0.35}{1.15}\left(1 - 114.3 A_s\right)$$

$$= 140 \times 10^6 A_s \left(1 - 114.3 A_s\right) \quad -(2)$$

<u>FOR SAGGING MOMENT</u> ($M_{app} = 79.4$ kNm) EQUATION 2 BECOMES:

$$114.3 A_s^2 - A_s + 5.67 \times 10^{-4} = 0$$

SOLVE FOR LOWER ROOT:

$$A_s = \frac{+1 \pm 0.86}{228.6} = 6.12 \times 10^{-4} \text{ m}^2$$

$$\therefore A_s = 612 \text{ mm}^2$$

1 IN No. 20 mm ϕ BAR $= \frac{\pi 20^2}{4} = 314$ mm²

$\therefore$ <u>PROVIDE 2T20 (628 mm²)</u>

<u>FOR HOGGING MOMENT</u> ($M_{app} = 141.1$ kNm) EQUATION 2 BECOMES:

$$114.3 A_s^2 - A_s + 1.01 \times 10^{-3} = 0$$

SOLVE FOR LOWER ROOT:

$$A_s = \frac{+1 \pm 0.73}{228.6} = 1.18 \times 10^{-3} \text{ m}^2$$

$\therefore$ PROVIDE 4T20 (1256 mm²)

(iii) $V_{MAX} = 176.4$ kN
CHECK CAPACITY WITHOUT INTERNAL STIRRUPS $V_{Rd,c}$
BUT THIS MAY BE IGNORED FOR BREVITY.

ASSUME $\cot\theta = 2.5$; $f_{cu} = 0.8 f_{cu} = 40$

$$\frac{V_{Rd,MAX}}{b_w d} = f_{c,MAX}(0.9) / (\cot\theta + \tan\theta)$$

WHERE $f_{c,MAX} = 0.6\left(1 - 40/_{250}\right)\left(0.85 \times 40/1.5\right)$

$= 11.42$ MPa

$$\therefore \frac{V_{Rd,MAX}}{b_w d} = 11.42(0.9)/(2.5 + 1/2.5)$$

$= 4.24$ MPa (CONCRETE RESISTANCE WITH STIRRUPS)

$$v_{APP} = \frac{V_{APP}}{b_w d} = \frac{176.4 \times 10^3}{250 \times 350} = 2.02 \text{ MPa}$$ < CONCRETE RESISTANCE $\therefore$ OK.

DESIGN STIRRUPS.

$$\frac{A_{SW}}{S} = \frac{V_{APP} \cdot \gamma_s}{f_y(0.9d)\cot\theta} = \frac{176.4 \times 10^3 \times 1.15}{250 \times 0.9 \times 350 \times 2.5}$$

$= 1.03$

$$A_{SW} = 2 \text{ LEGS} \times \pi \times 8^2/4 = 100 \text{ mm}^2$$

$\therefore S = 97$ mm (100 mm WILL PROBABLY BE ACCEPTABLE!)

$\therefore$ PROVIDE R8 STIRRUPS AT 100mm CENTRES.

(iv) AXIAL LOAD IN COLUMN $= 176.4 \text{ kN} \times 2 = 352.8 \text{ kN}$

$$\sigma_u = 352.8 \times 10^3 / 250^2$$
$$= 5.64 \text{ N/mm}^2$$

$$\rho_c = \left(\sigma_u - 0.6\, f_{cu}/\gamma_c\right) \cdot \frac{\gamma_s}{f_y}$$
$$= \left(5.64 - 0.6 \times 50/1.5\right) \cdot \frac{1.15}{460}$$

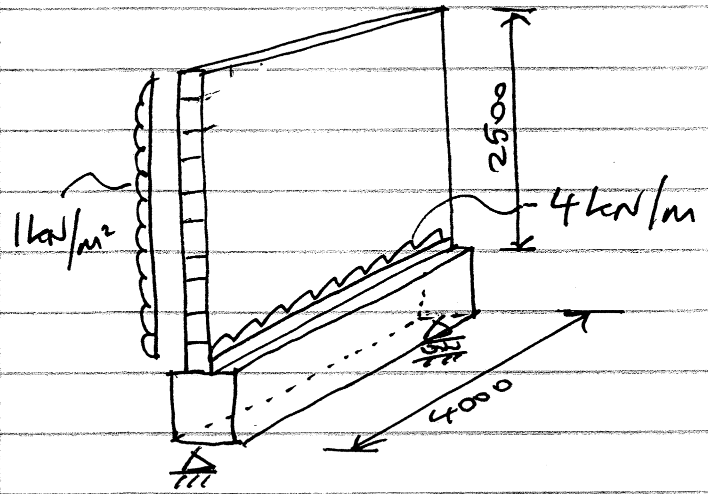
= −VE VALUE ∴ CONCRETE ALONE IS SUFFICIENT

∴ PROVIDE MIN. REINFORCEMENT OF 0.2% OF A_c
$= 125 \text{ mm}^2$

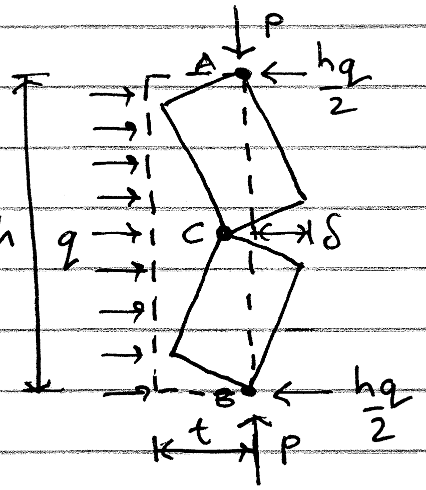
SAY 4T12 (452 mm^2)

(b) REMOVAL OF LIVE LOAD FROM ONE SPAN WOULD:

- INCREASE THE MAXIMUM SAGGING MOMENT OF THE FULLY LOADED SPAN AND REDUCE THE SAGGING MOMENT IN THE OTHER SPAN.
- REDUCE THE HOGGING MOMENT AT B
- SHIFT THE POSITION OF THE MAXIMUM SAGGING MOMENT CLOSER TO MID-SPAN OF AB.
- REDUCE THE SHEAR FORCES ON THE 'UNLOADED' SPAN
- INCREASE THE BOTTOM STEEL AND REDUCE THE TOP STEEL IN THE CRITICAL SECTIONS.
- REDUCE THE SHEAR REINFORCEMENT AT B.
- INDUCE A BENDING MOMENT IN THE COLUMN THEREBY INCREASING THE REINFORCEMENT.



(a)



CONSIDER CRACK ALONG MID-HEIGHT OF WALL ⇒ HINGES DEVELOP AT A, B AND C.

$\circlearrowright C$ FOR TOP HALF OF WALL:

$$P(t-\delta) + q\left(\frac{h}{2}\cdot\frac{h}{4}\right) - \frac{hq}{2}\left(\frac{h}{2}\right) = 0$$

ALSO $\delta \ll t$ ∴ SUBSTITUTE t FOR $(t-\delta)$:

$$P = \frac{qh^2}{8t}$$

$$= \frac{0.25\times10^3\ \text{N/m}^2 \times 2.5^2}{8 \times 0.1025}$$

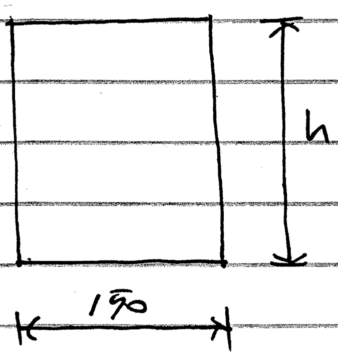
$$= 1905\ \text{N/m}$$

$$= \underline{\underline{1.9\ \text{kN/m}}}\ \text{(UNFACTORED)}$$

OR

$$\underline{\underline{7.6\ \text{kN}}}$$

(b)



$k_{es} = 1.1$; $k_h = 1.0$; $k_{mod} = 1.0$

$\gamma_m = 1.3$; $f_{m,k} = 24\,\text{MPa}$; $f_{v,k} = 2.5\,\text{MPa}$

$k_{crit} = 1.0$ (RESTRAINED BY SLAB & WALL).

BENDING MOMENT

DESIGN B.M. @ MID-SPAN $M_d = \dfrac{(7.6\text{ kN} + 16\text{ kN}) \times 1.5 \times 4\text{ m}}{8}$

$= 17.7\text{ kNm}$

DESIGN BENDING STRENGTH $f_{m,d} \geq$ ELASTIC FLEXURAL STRESS σ_{MAX}

WHERE $f_{m,d} = \dfrac{k_{mod} \times k_h \times k_{crit} \times k_{ls} \times f_{m,k}}{\gamma_m}$

$= \dfrac{1 \times 1 \times 1 \times 1.1 \times 24\text{ N/mm}^2}{1.3} = 20.3\text{ N/mm}^2$

AND $\sigma_{MAX} = M_d/Z = M/(bh^2/6)$

$\therefore h = \sqrt{6M/\sigma_{MAX}b}$

MIN. $h = \sqrt{\dfrac{6 \times 17.7 \times 10^6}{20.3 \times 150}} = \underline{187\text{ mm}}$

SHEAR

DESIGN SHEAR FORCE @ SUPPORT $V_d = \dfrac{(7.6\text{ kN} + 16\text{ kN}) \times 1.5 \times 4\text{m}}{2}$

$= 70.8\text{ kN}$

DESIGN SHEAR STRENGTH $f_{v,d} \geq$ ELASTIC SHEAR STRESS v_d

WHERE $f_{v,d} = \dfrac{k_{mod} \times k_{ls} \times f_{v,k}}{\gamma_m}$

$= \dfrac{1 \times 1.1 \times 2.5\text{ N/mm}^2}{1.3} = 2.12\text{ N/mm}^2$

AND $v_d = V_d/bh$

$\therefore h = V_d/v_d b$

MIN $h = 70.8 \times 10^3 / 2.12 \times 150 = \underline{223\text{ mm}}$

DEFLECTION

(2) TOTAL DEFLECTION (V_{TOT}) = FLEXURAL DEFLECTION (V_f) + SHEAR DEFLECTION (V_s)

$$V_f = \frac{5Wl^3}{384EI} = \frac{5Wl^3}{32Ebh^3}$$

V_s : LONGITUDINAL SHEAR STRESS $\tau = SA\bar{y}/It$

WHERE $S = \frac{W}{2} - \frac{Wx}{l}$

$$\therefore \quad \tau = \frac{3}{2bh}\left(\frac{W}{2} - \frac{Wx}{l}\right)$$

BY COMPLIMENTARY SHEAR $\gamma_{NA} G = \frac{3}{2bh}\left(\frac{W}{2} - \frac{Wx}{l}\right)$

ALSO $dv_s = \gamma_{NA}\, dx$

$$\therefore \quad dv_s = \frac{3}{2bhG}\left(\frac{W}{2} - \frac{Wx}{l}\right)dx$$

$$v_s = \frac{3}{2bhG}\left(\frac{Wx}{2} - \frac{Wx^2}{2l}\right) + A$$

BOUNDARY CONDITIONS : AT $x=0$, $v_s=0$ $\therefore$ $A=0$

SHEAR DEFLECTION AT $x = l/2$:

$$v_s = \frac{3Wl}{2bhG}\left(\frac{l}{4} - \frac{l}{8}\right) = \frac{3Wl^2}{16bhG}$$

$$\therefore \quad V_{TOT} = \frac{5Wl^3}{32Ebh^3}\left(1 + \frac{6Eh^2}{5Gl}\right)$$

TRY $h = 246$ mm

$$V_{TOT} = \frac{5 \times 23.6 \times 10^3 \times 4^3}{32 \times 11 \times 10^9 \times 0.15 \times 0.246^3}\left(1 + \frac{6 \times 11 \times 10^9 \times 0.246^2}{5 \times 0.69 \times 10^9 \times 4}\right)$$

$= 0.0124$ m $= 12.4$ mm $<$ SPAN/300

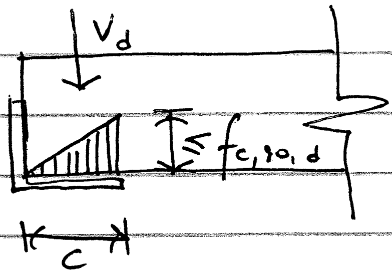
$\therefore$ $\underline{\underline{h = 246 \text{ mm}}}$ (ROUND UP TO 250 mm)

(c) DESIGN BEARING STRENGTH $f_{c,90,d} = k_{ls}\, k_{c,90}\, k_{mod}\, f_{c,90,k} / \gamma_m$

$= 1.1 \times 1.1 \times 1 \times 5.3 / 1.3$

$= 4.93\ N/mm^2$

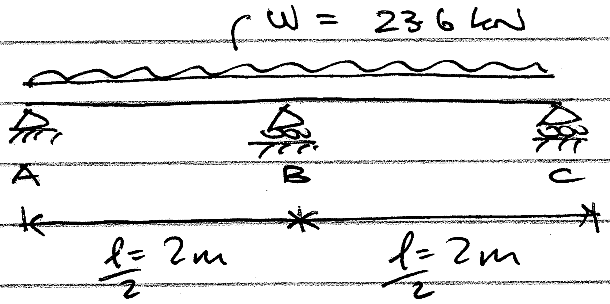
BEARING STRESS DISTRIBUTION :



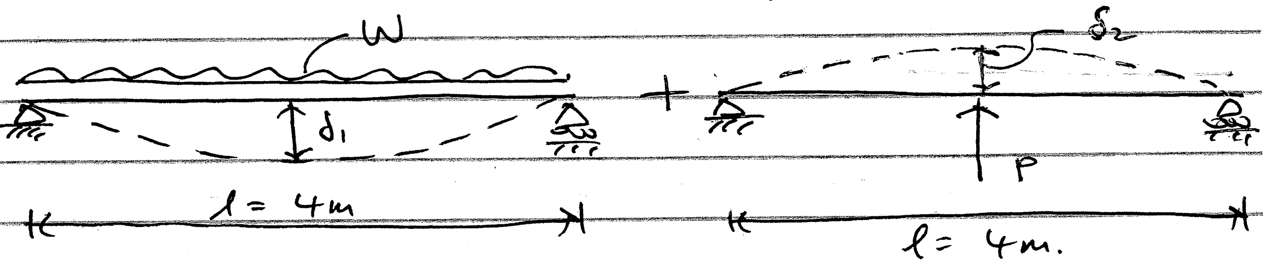
$\therefore \left(\frac{1}{2} c\, f_{c,90,d}\right) \times b \geq V_d$

$\therefore c \geq \frac{2 \times 70.8 \times 10^3}{4.93 \times 150} \geq \underline{\underline{191\ mm}}$

(d) VARIATION TO ORIGINAL DESIGN :



LOAD ON BOLTS = SHEAR FORCE (REACTION) AT B.

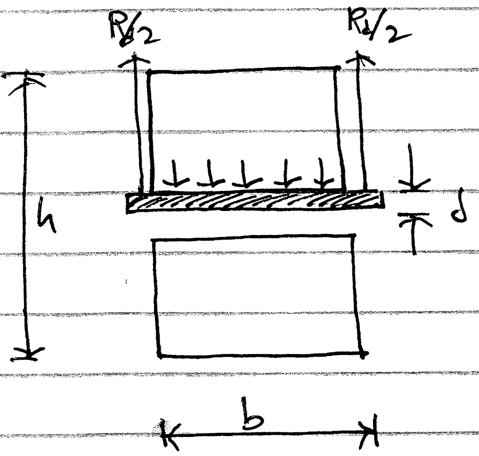


$\delta_1 = \frac{5 W l^3}{384 EI}$ $\qquad$ $\delta_2 = \frac{P l^3}{48 EI}$

BUT $\delta_1 = \delta_2$ $\quad \therefore \quad P = \frac{240 W}{384} = \frac{5 W}{8}$

$\therefore$ LOAD ON BOLTS $= \frac{5}{8} \times 23.6\ kN \times 1.5 = 22.13\ kN$

CAPACITY OF ONE BOLT:



WORK DONE BY LOAD:

$$W_E = R_d \delta$$

ENERGY DISSIPATED IN TIMBER:

$$W_I = f_{h,1,d}\, b\, d\, \delta$$

$$\therefore \quad R_d = f_{h,1,d}\, b\, d$$

BOLTS LOADED ⊥ TO GRAIN

$$\therefore f_{h,90,k} = \frac{\overbrace{0.082(1-0.01\times 30)\times 350}^{f_{h,0,k}}}{\underbrace{[1.35+(0.015\times 30)]}_{k_{90}}\times \sin^2 90 + \cos^2 90}$$

$$= 15.9\ \mathrm{N/mm^2}$$

$$f_{h,1,d} = f_{h,90,k}\times k_{mod}/\gamma_m$$

$$= 15.9\times 1/1.3$$

$$= 12.2\ \mathrm{N/mm^2}$$

$$\therefore R_d = 12.2\times 150\times 30 = 54.9\ \mathrm{kN} > 22.13\ \mathrm{kN}$$

$\therefore$ A SINGLE 30 mm ϕ BOLT IS SUFFICIENT.

3D3/2010 Qu 4

a) For bending up to first yield $\phi_f = Z_e / Z_o$ (any section / sq. section).

$Z_o = b^3/6 \Rightarrow \phi_f = Z_e/(b^3/6)$: A = crossectional area $= b^2$

$\Rightarrow \underline{\phi_f = 6Z_e/A^{3/2}}$: Z_e = elastic section modulus

When fully plastic ; $Z = Z_p \Rightarrow \phi_f = Z_p/Z_o$ $Z_o = b^3/4$

$\Rightarrow \underline{\phi_f = 4Z_p/A^{3/2}}$

Ratio $= \frac{4Z_p}{A^{3/2}} \cdot \frac{A^{3/2}}{6Z_e} = \underline{2/3 \cdot Z_p/Z_e}$ not dependent on area A. (A)

b) (i) For a 914×419×388 UB (steel) $Z_p = 17670\ cm^3$, $Z_e = 15630\ cm^3$ } major axis bending

$\rho = \frac{2}{3} \cdot \frac{17670}{15630} \simeq \underline{0.75}$ — (1)

(ii) For a thin-walled square section: $b \times b$ (thickness t)

$Z_e = I/y_{max}$: $I = \frac{b^4}{12} - \frac{(b-t)^4}{12} = b^4/12 \cdot [1-(1-4t/b)] = \frac{b^3t}{3}$

$\Rightarrow Z_e = \frac{b^3t}{3} \cdot \frac{2}{b} = \underline{\frac{2}{3} b^2 t}$.

$Z_p = b^3/4 - \frac{(b-t)^3}{4} = b^3/4 [1-(1-3t/b)] = \underline{\frac{3}{4} b^2 t}$

Ratio (A) : $\frac{2}{3} \cdot \frac{Z_p}{Z_e} = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}\cancel{b^2t}}{\cancel{4}2} \cdot \frac{3}{2\cancel{b^2t}} = \underline{\frac{3}{4}}$ — (2)

(1) & (2) very similar : theoretical limit has $Z_p = Z_e$ if all of cross-sectional area concentrated in flange $\Rightarrow$ (A) $= 2/3$.

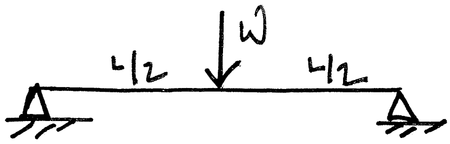
(1) & (2) are close to this limit.

c) Span-to-depth ratio controls the performance of beam either in terms of stiffness or strength : if deep $\Rightarrow$ high stiffness, but proximal to yield since outer fibre is more strained, and vice versa.

P.T.O.

3D3/2010 Qu 4

c) contd



$$B.M._{max} = \frac{WL}{4} = \underbrace{M_{max} = M_p}_{\text{using full strength}}$$

Using load factor $(\gamma_f \cdot W)\frac{L}{4} \leq Z_p \cdot \sigma_y$ — (1)

For deflection: (central, max) $\delta = \underset{\text{given}}{c}\underbrace{\left(\frac{WL^3}{48EI}\right)}_{\text{struct. databk}} \leq L/F$ given — (2)

From (1): $W \leq \left[4Z_p / \gamma_f \cdot L\right] \cdot \sigma_y$ substitute into (2):

$$\frac{L}{F} \geq \frac{c \cdot L^{\cancel{3}\,2}}{{}_{12}\cancel{48}EI} \cdot \frac{\cancel{4} Z_p \sigma_y}{\cancel{L}\gamma_f} \times F/d$$

$$\frac{12EI \cdot \cancel{L}\,\gamma_f}{cL^2 \cdot Z_p \sigma_y} \cdot \frac{1}{F} \geq L^2 \quad \div \text{ both sides by } d$$

$$\Rightarrow \quad L/d \leq \frac{12\gamma_f}{F \cdot c} \cdot E/\sigma_y \cdot \frac{I}{Z_p \cdot d}$$

This formula cannot be used for brittle materials, since (1) relies on plastic ductile behaviour. It may be possible to replace (1) by an equivalent check for fracture, e.g. $\sigma_{max} \leq My/I$ but there are flaws thro' out, so the outer fibre may not fail first.