**2010 IIA 3D5 - Water Engineering Dr D. Liang**

## Question 1

(a) Evapotranspiration (ET) is the sum of evaporation and plant transpiration from the Earth's land surface to lower atmosphere. Evaporation is the movement of water to the air from sources such as soil, canopy interception, and water bodies. Transpiration describes the movement of water within a plant and the subsequent loss of water as vapour through stomata in its leaves.

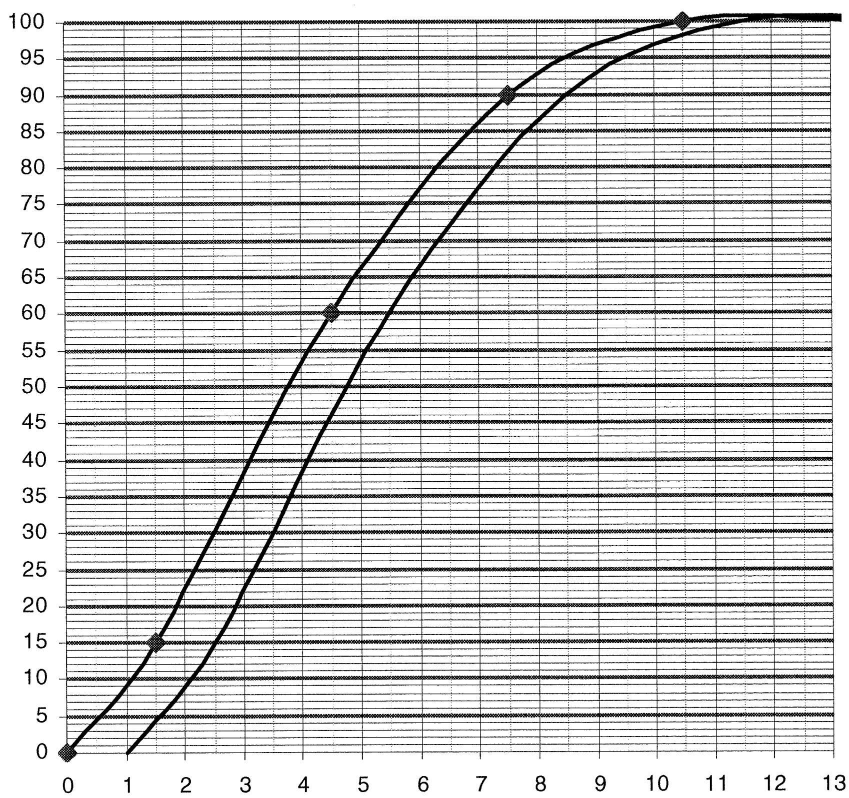
Potential evapotranspiration (PET) is the amount of water that can be evaporated and transpired if there is ample water available. The soil is fully saturated.

ET and PET depend on sunlight, temperature, humidity, wind, crop type, soil type and etc.

Evapotranspiration occurs relatively slowly. In a rainfall-runoff process that lasts for only hours, the effect of evapotranspiration is negligible.

(b) Plot S curve:

| | | | | | | |
|---|---|---|---|---|---|---|
| t: | 0 | 1.5 | 4.5 | 7.5 | 10.5 | 13.5 |
| %: | 0 | 15 | 60 | 90 | 100 | 100 |




Readings on S curve at half hour points:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| t | 0 | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 |
| % | 0 | 4.5 | 15 | 30 | 46 | 60 | 72 | 82 | 90 | 95 | 98 | 100 |

Shift by one hour and subtract to get one-hour unit hydrograph:

| t | 0 | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 | 10.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| %: | 0 | 4.5 | 10.5 | 15 | 16 | 14 | 12 | 10 | 8 | 5 | 3 | 2 |

So, the peak flow occurs during 3-4 hour after the start of the rainfall.

Infiltration rate: $f = f_c + (f_0 - f_c)e^{-K_f t}$

Total infiltration during the rainfall is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$\int_0^1 f \cdot dt = 5(1-0) - \frac{1}{0.2}(20-5)\left(e^{-0.2\times 1} - e^{-0.2\times 0}\right) = 5 + 13.6 = 18.6 \text{ mm}$$

Effective rainfall: 30-18.6 = 11.4 mm

The peak discharge is: $\dfrac{16\% \times 15 \times 10^6 \times 0.0114}{3600} = 7.6 \text{ m}^3\text{/s}$

(c) (i) Total rainfall $(30 + 80 + 50 + 20 + 10) \times 2 \times 3600 = 1{,}368{,}000 \text{ m}^3$

Catchment area $\dfrac{1368000}{0.02} = 68{,}400{,}000 \text{ m}^2 = 68.4 \text{ km}^2$

(ii)

| Contribution to the flow rate at outlet | Time Interval | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0-2 hour | 2-4 hour | 4-6 hour | 6-8 hour | 8-10 hour | 10-12 hour | 12-14 hour | 14-16 hour | 16-18 hour |
| 40 mm Rain | 60 | 160 | 100 | 40 | 20 | 0 | | | |
| 20 mm Rain | | | 30 | 80 | 50 | 20 | 10 | 0 | |
| 10 mm Rain | | | | 15 | 40 | 25 | 10 | 5 | 0 |
| Total | 60 | 160 | 130 | 135 | 110 | 45 | 20 | 5 | 0 |

## Question 2

(a) $C$ has a unit of [m$^{1/2}$ s$^{-1}$]

(b)

(i)

Manning formula $U = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2}$

$$Q = \frac{1}{n} \cdot \left(\frac{6h}{6+2h}\right)^{2/3} \cdot S_b^{1/2} \cdot 6h$$

$$10 = \frac{1}{0.013} \cdot \left(\frac{6h}{6+2h}\right)^{2/3} \cdot 6h \cdot \sqrt{0.0001}$$

$$0.656 = \frac{h^{5/3}}{(6+2h)^{2/3}}$$

$h$ = 1.94 m is the solution.

(ii)

$$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f \quad => \quad \frac{\left.\left(h+\frac{U^2}{2g}\right)\right|_{h=1.5} - \left.\left(h+\frac{U^2}{2g}\right)\right|_{h=1.6}}{\Delta x} = 0.0001 - \overline{\frac{n^2 \cdot U^2}{R_h^{4/3}}}$$

$h$ = 1.5 m:

$$U = \frac{10}{1.5 \times 6} = 1.11 \text{ m/s}$$

$$R_h = \frac{1.5 \times 6}{6 + 1.5 \times 2} = 1 \text{ m}$$

$$\left.\left(h + \frac{U^2}{2g}\right)\right|_{h=1.5} = 1.5 + \frac{1.11^2}{2 \times 9.8} = 1.563 \text{ m}$$

$$\left.\left(\frac{n^2 \cdot U^2}{R_h^{4/3}}\right)\right|_{h=1.5} = \frac{0.013^2 \cdot 1.11^2}{1^{4/3}} = 0.000208$$

$h$ = 1.6 m:

$$U = \frac{10}{1.6 \times 6} = 1.042 \text{ m/s}$$

$$R_h = \frac{1.6 \times 6}{6 + 1.6 \times 2} = 1.0435 \text{ m}$$

$$\left.\left(h + \frac{U^2}{2g}\right)\right|_{h=1.5} = 1.6 + \frac{1.042^2}{2 \times 9.8} = 1.6554 \text{ m}$$

$$\left.\left(\frac{n^2 \cdot U^2}{R_h^{4/3}}\right)\right|_{h=1.5} = \frac{0.013^2 \cdot 1.042^2}{1.0435^{4/3}} = 0.000173$$

So,

$$\frac{1.563 - 1.6554}{\Delta x} = 0.0001 - \frac{0.000208 + 0.000173}{2} = -0.0000905$$

$$\Delta x = 1021 \text{ m}$$

(c)

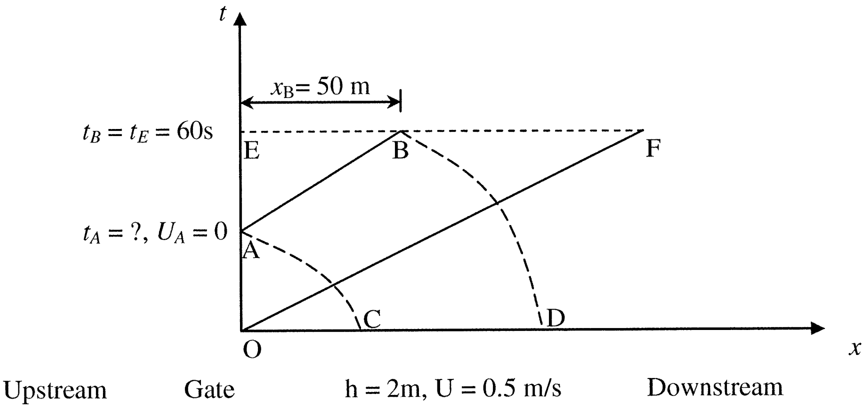
(i)

$$R_h = \frac{10 \times 2}{10 + 2 \times 2} = 1.429 \text{ m}$$

$$C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = 7.8 \ln\left(\frac{12.0 \cdot 1.429}{0.02}\right) = 52.68 \text{ m}^{1/2}/\text{s}$$

$$0.5 = 52.68 \times \sqrt{1.429 \times S_b} \Rightarrow S_b = 6.3 \times 10^{-5}$$

(ii)




The question asks to find $h_B$.

The slope of +ve line OF is $\frac{dx}{dt} = U + \sqrt{gh} \Rightarrow \frac{x_F - 0}{60 - 0} = 0.5 + \sqrt{9.8 \times 2}$

$$\Rightarrow x_F = 295.63 \text{ m}$$

Therefore Point B is not located in the unaffected region. In other words, Point B is located between E and F, and Point A is located between O and E.

Along –ve line AC: $U_A - 2\sqrt{gh_A} = U_C - 2\sqrt{gh_C}$

$$\Rightarrow \quad 0 - 2\sqrt{gh_A} = 0.5 - 2\sqrt{9.8 \times 2} \Rightarrow h_A = 1.78 \text{ m}$$

Along +ve line AB, the water level remain the same, so $h_B = 1.78$ m

In fact, the water depths downstream of the gate (along EF) remain constant (1.78 m) after the sudden closure of the gate.

**Question 3**

(a)

The Rouse profile is derived from the flux balance through a horizontal plane with a distance $z$ above the bed.

In uniform flow, the downward flux of sediment is $w_s\bar{c}$

The upward flux is due to the turbulent diffusion $-D_z\dfrac{d\bar{c}}{dz}$

**The following explanation goes into far more detail than what is expected for complete credit to be given.**

Assume the diffusion coefficient is equal to the turbulent viscosity, then

$$-D_z\frac{d\bar{c}}{dz} = -\nu_t\frac{d\bar{c}}{dz} = -\kappa u_* z\left(1-\frac{z}{h}\right)\frac{d\bar{c}}{dz}$$

In the equilibrium condition, the downward and upward fluxes are equal, so

$$\frac{d\bar{c}}{\bar{c}} = -\frac{w_s h}{\kappa u_* z(h-z)}dz$$

$$\frac{d\bar{c}}{\bar{c}} = -\frac{w_s}{\kappa u_*}\cdot\left(\frac{1}{z}+\frac{1}{h-z}\right)dz$$

$$\frac{d\bar{c}}{\bar{c}} = -\frac{w_s}{\kappa u_*}\cdot\left[\frac{dz}{z}-\frac{d(h-z)}{h-z}\right]$$

$$\ln(\bar{c}) = -\frac{w_s}{\kappa u_*}\cdot\left[\ln z - \ln(h-z) + \ln(A)\right],$$ with $A$ being a arbitrary constant

$$\ln(\bar{c}) = -\frac{w_s}{\kappa u_*}\cdot\ln\left(\frac{Az}{h-z}\right)$$

$$\bar{c} = \left(\frac{h-z}{Az}\right)^{\frac{w_s}{\kappa u_*}}$$

The constant $A$ can be determined by knowing a reference concentration at an elevation $a$.

$$\bar{c}(a) = \left(\frac{h-a}{Aa}\right)^{\frac{w_s}{\kappa u_*}} \quad => \quad A = \frac{h-a}{a\cdot[\bar{c}(a)]^{\frac{\kappa u_*}{w_s}}}$$

So

$$\bar{c} = \left(\frac{h-z}{z}\cdot\frac{a\cdot[\bar{c}(a)]^{\frac{\kappa u_*}{w_s}}}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{\bar{c}}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$, which is Rouse profile.

(b.i)

The flow is uniform, so

$$C' = 7.8\ln\left(\frac{12h}{k_s'}\right) = 7.8\ln\left(\frac{12\cdot 0.6}{3\cdot 0.0008}\right) = 62.45\ \text{m}^{1/2}/\text{s}$$

$$\tau_b' = \rho g\frac{U^2}{C'^2} = 1000\cdot 9.8\cdot\frac{1^2}{62.45^2} = 2.51\ \text{Pa}$$

$$\theta' = \frac{\tau_b'}{g(\rho_s-\rho)d} = \frac{2.51}{9.8\cdot(2650-1000)\cdot 0.0008} = 0.194$$

$$d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0008\times\left(\frac{9.8\times(2.65-1)}{10^{-12}}\right)^{1/3} = 20.23$$

$$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055[1-\exp(-0.02d_*)] = \frac{0.30}{1+1.2\cdot 20.23} + 0.055[1-\exp(-0.02\cdot 20.23)]$$

$$= 0.01236 + 0.055\cdot 0.333 = 0.0307$$

$$T = \frac{\tau_b' - \tau_{bc}}{\tau_{bc}} = \frac{\theta' - \theta_c}{\theta_c} = \frac{0.194-0.0307}{0.0307} = 5.32$$

$$q_b = 0.053\frac{T^{2.1}}{d_*^{0.3}}\times\sqrt{g(s-1)\cdot d^3} = 0.053\frac{5.32^{2.1}}{20.23^{0.3}}\times\sqrt{9.8\cdot(2.65-1)\cdot 0.0008^3}$$

$$= 0.053\times 13.57\times 9.1\times 10^{-5} = 6.5\times 10^{-5}\ \text{m}^2/\text{s} = 2650\times 6.5\times 10^{-5} = 0.17\ \text{kg/s/m}$$

(b.ii)

$$\dot{M} = 0.1\times 50 = 5\ \text{kg/s}$$

$$\tau_b = \rho g R_h S_b = 1000\times 9.8\times 0.6\times 0.0008 = 4.704\ \text{Pa}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{4.704}{1000}} = 0.0686\ \text{m/s}$$

$$D_y = D_{ty} = 0.15hu_* = 0.15\times 0.6\times 0.0686 = 0.0062\ \text{m}^2/\text{s}$$

Contribution due to the real source:

$$c(x=20, y=1.5) = \frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y x/U}\right) = \frac{5/0.6}{1\cdot\sqrt{4\pi\cdot\frac{20}{1}\cdot 0.0062}}\exp\left(-\frac{1.5^2}{4\times 0.0062\times 20/1}\right)$$

$$= 6.677\times 0.0107 = 7.15\times 10^{-2}\ \text{kg/m}^3$$

Contribution due to the image source:

$$c(x=20, y=2.5) = \frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y x/U}\right) = \frac{5/0.6}{1\times\sqrt{4\pi\cdot\frac{20}{1}\cdot 0.0062}}\exp\left(-\frac{2.5^2}{4\times 0.0062\times 20/1}\right)$$

$$= 6.677\times 3.37\times 10^{-6} = 2.25\times 10^{-5}\ \text{kg/m}^3\ \text{(negligible)}$$

So, $c(x=20, y=1.5) = 7.15\times 10^{-2}\ \text{kg/m}^3$.

## Question 4

(a)

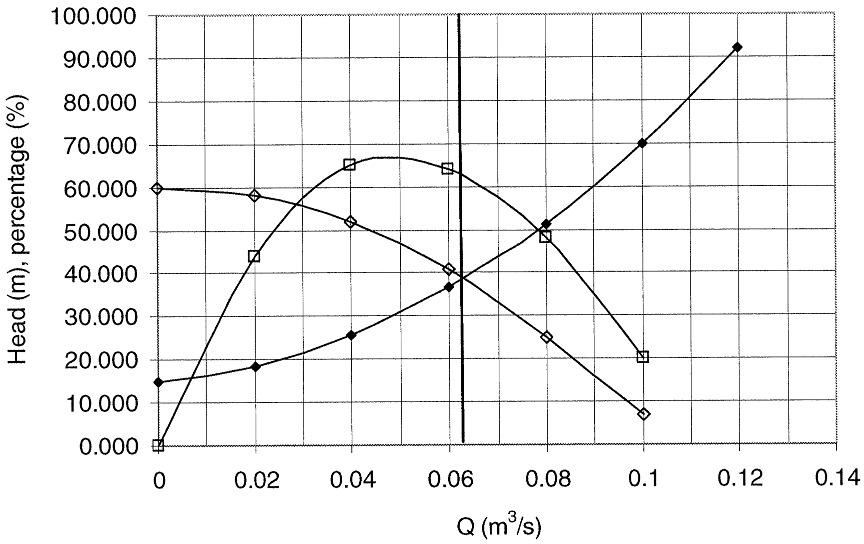
The question is basically to find the interception point between the system and pump curves. The system curve is:

$$H_S = Z_W + 10 + H_f + H_{local}$$

$$H_S = 5.0 + 15 \times Q \times \ln(45000 \cdot Q) + 10 + \lambda \frac{L}{D}\frac{U^2}{2g} + 10\frac{U^2}{2g}$$

It should be noted that λ is related to Re and $k_s/D$ (0.2/200=0.001)and should be obtained from the Moody diagram.

| Q (m³/s) | 0 | 0.02 | 0.04 | 0.06 | 0.08 | 0.1 | 0.12 |
|---|---|---|---|---|---|---|---|
| u (m/s) | 0.000 | 0.637 | 1.273 | 1.910 | 2.546 | 3.183 | 3.820 |
| Re | 0.00E+00 | 1.27E+05 | 2.55E+05 | 3.82E+05 | 5.09E+05 | 6.37E+05 | 7.64E+05 |
| λ (Moody Diagram) | | 0.022 | 0.021 | 0.0207 | 0.0203 | 0.0202 | 0.0201 |
| Zw (m) | 5.000 | 6.243 | 7.763 | 9.388 | 11.081 | 12.824 | 14.608 |
| $H_f$ (m) | 0.000 | 1.820 | 6.948 | 15.409 | 26.865 | 41.769 | 59.850 |
| $H_{local}$ (m) | 0.000 | 0.207 | 0.827 | 1.861 | 3.308 | 5.169 | 7.444 |
| $H_{System}$ (m) | 15.000 | 18.269 | 25.538 | 36.658 | 51.254 | 69.763 | 91.901 |

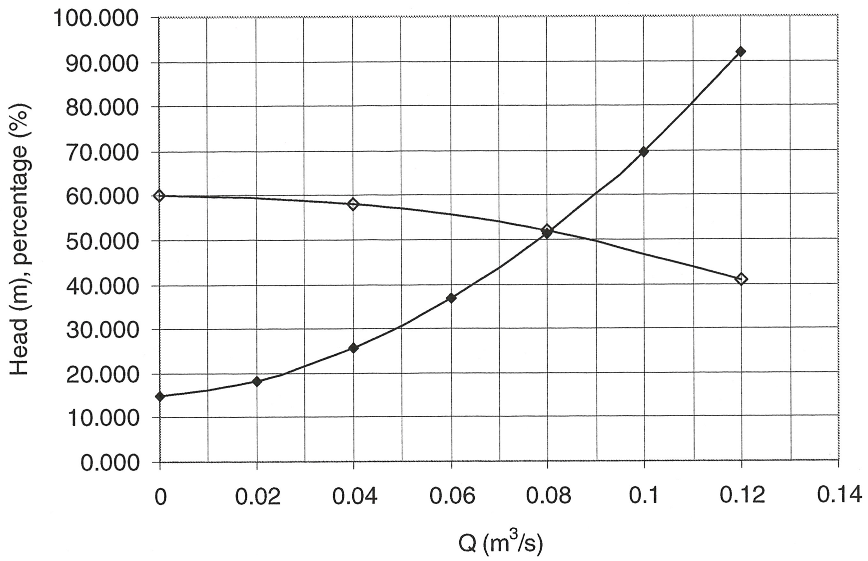



$Q = 0.063$ m³/s, $H = 39$ m, $\eta = 62.5\%$

$$P_p = \rho g Q_p H_p / \eta_p = \frac{1000 \times 9.8 \times 0.063 \times 39}{0.625} = 38526 \text{ Watts} \approx 38.5 \text{ kW}$$

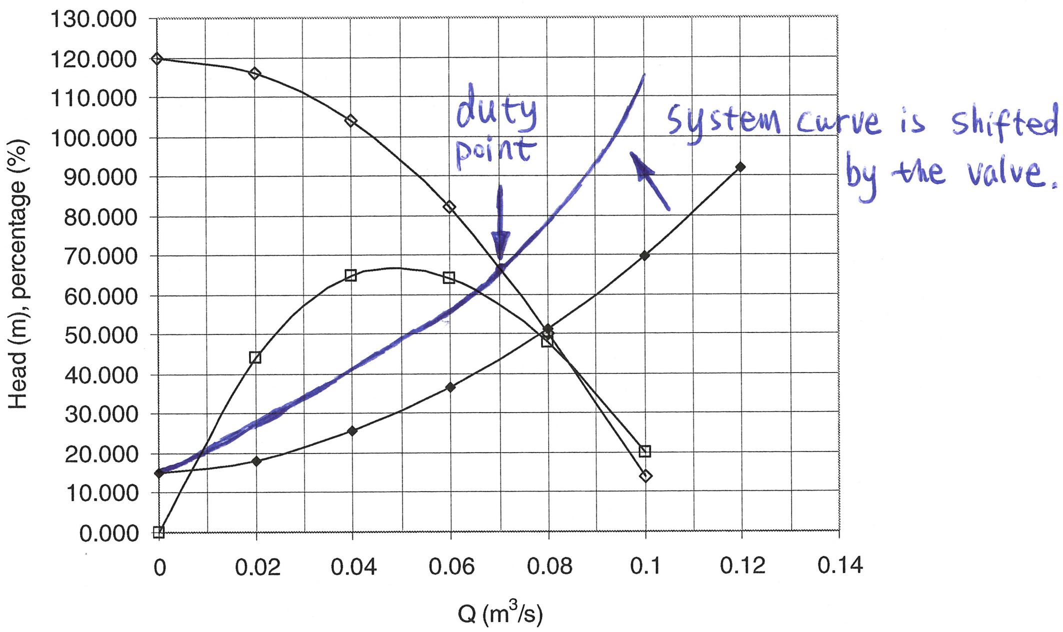
(b.i)

The predicted head vs. discharge curve for dual pump operation in parallel mode is obtained by doubling the discharge over the range of head. The system is the same. From the intersection of the characteristic and system curve shown below, Discharge $Q = 0.081$ m³/s.




(b.ii)

The predicted head vs. discharge curve for dual pump operation in series mode is obtained by adding the individual pump manometric heads at any arbitrary discharges.




The system will curve will shift upwards to intersect with the pump curve at

$$Q_p = 70 \text{ liter/s}, \quad H_p = 67 \text{ m}, \quad \eta_p = 57.5\%$$

So,

$$P_p = \rho g Q_p H_p / \eta_p = \frac{1000 \times 9.8 \times 0.07 \times 67}{0.575} = 79934 \text{ Watts} \approx 79.9 \text{ kW}$$

(c) Clarification: coagulation, flocculation, settlement.
Filtration: rapid gravity, slow sand filters and etc.
Disinfection: chlorine, ozone and etc.
Fluoridation.

It should not be confused with the sewage treatment.

**2010 IIA 3D5 WATER ENGINEERING**
List of Numerical Answers

Q1. (b) 7.6 $m^3 s^{-1}$
(c) (i) 68.4 $km^2$
(ii)

| Time Interval (hour) | 0-2 | 2-4 | 4-6 | 6-8 | 8-10 | 10-12 | 12-14 | 14-16 | 16-18 |
|---|---|---|---|---|---|---|---|---|---|
| Flow rate at outlet ($m^3 s^{-1}$) | 60 | 160 | 130 | 135 | 110 | 45 | 20 | 5 | 0 |

Q2. (a) $m^{1/2} s^{-1}$
(b) (ii) 1021 m
(c) (i) $6.3\times10^{-5}$

Q3. (b) (i) 0.17 kg $s^{-1}$ $m^{-1}$
(ii) $7.15\times10^{-2}$ kg $m^{-3}$

Q4. (a) 38.5 kW
(b) (i) 81 litre $s^{-1}$
(ii) 79.9 kW