**Engineering Tripos Part IIA**

**3E3 Modelling Risk / 2010 Exam/ Cribs/ Version 2**

**Problem 1**

**a)**

**i)**

| Alternatives | Successful | Unsuccessful |
|---|---|---|
| Introduce new product | £ 40 million | -£15 million |
| Do not introduce | 0 | 0 |
| Prior probabilities | 0.5 | 0.5 |

Exp.Payoff(Introduce)=12.5 , so introduce

ii) Regret Table

| Alternatives | Successful | Unsuccessful | Max Regret |
|---|---|---|---|
| Introduce new product | 0 | 15 | 15 |
| Do not introduce | 40 | 0 | 40 |

MinMaxRegret= Introduce new product

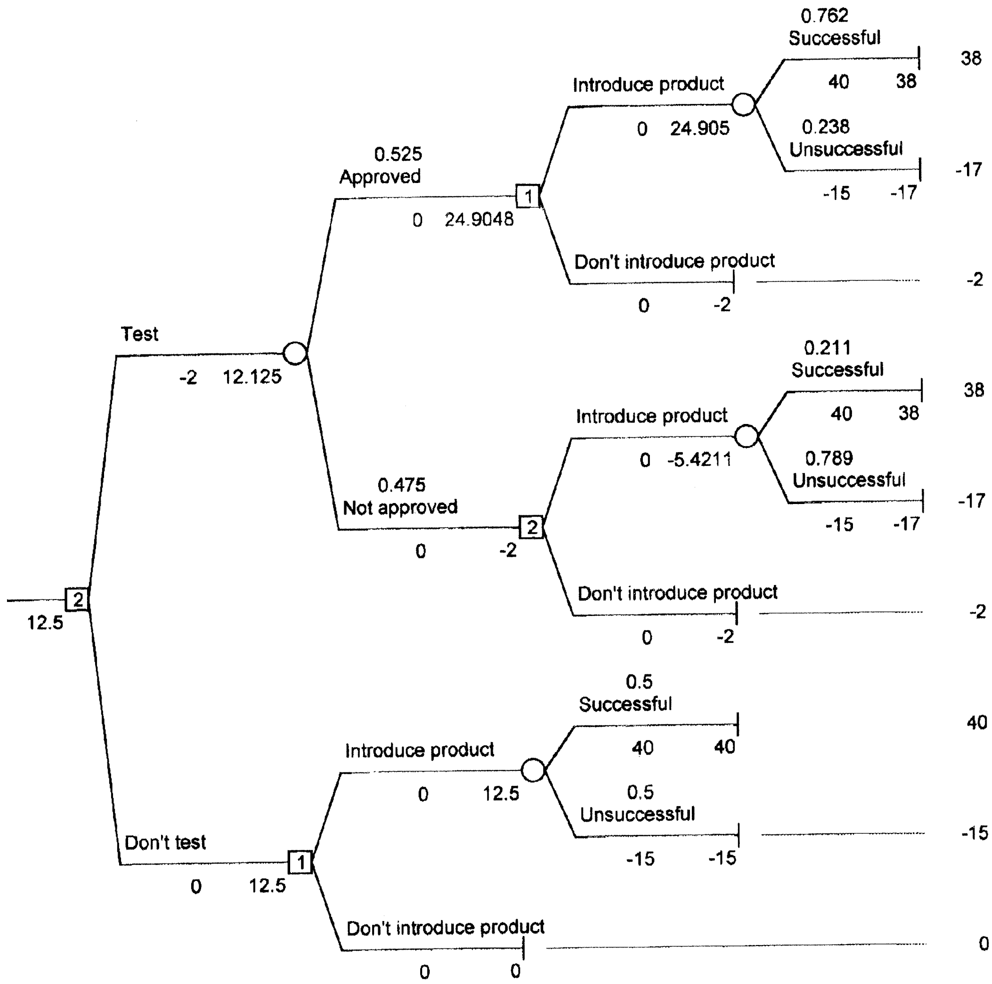
iii) with Perfect information: Expected value=0.5x40=£20 million

so EVPI= 20-12.5=7.5

iv)

| Posterior | | P(state\|finding) | |
|---|---|---|---|
| Finding | P(Finding) | Successful | Unsuccessful |
| Approved | 0.525 | 0.762 | 0.238 |
| Not Approved | 0.475 | 0.211 | 0.789 |

v) The optimal decision is to not try it out in the test market and introduce it as it is. That course of action has expected payoff of 12.5. Therefore the EVSI=0. See decision tree in the next page.




b)

i) Being willing to accept an amount X with certainty versus a risky situation with an expected value of Y even if Y>X.

ii) Certainty Equivalent - the amount that is equivalent in the decision maker's mind to a situation involving risk.

iii) The expected monetary value the decision maker is willing to give up to avoid a risky decision. Or equivalently, the minimum difference a person requires to be willing to take an uncertain bet, between the expected value of the bet and the certain value that he is indifferent to.

**Problem 2**

i) $P(X_1 = 0) = P(X_1 = 0|X_0 = 0) \cdot P(X_0 = 0) + P(X_1 = 0|X_0 = 1) \cdot P(X_0 = 1) + P(X_1 = 0|X_0 = 2) \cdot P(X_0 = 2)$
$= (1/3)(1/3) + (1/3)(1/2) + (1/3)(1/4) = 0.36$

ii) We need to find $P^{(2)}$ which is

| | | |
|---|---|---|
| 0.36 | 0.36 | 0.28 |
| 0.375 | 0.375` | 0.25 |
| 0.25 | 0.38 | 0.38 |

Therefore $P(X_2 = 1|X_0 = 0) = 0.36$

iii) the answer is $\pi_1$ which is obtained by solving the following system:

$\pi_0 = 1/3\ \pi_0 + 3/4\ \pi_2$

$\pi_1 = 1/3\ \pi_0 + 1/2\ \pi_1 + 1/4\ \pi_2$

$\pi_2 = 1/3\ \pi_0 + 1/2\ \pi_1$

$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$

the solution gives $\pi_1 = 10/27$

iv) We need to solve the following system:
$u_0 = 1 + 1/3\ u_0 + 1/3\ 0 + 1/3\ u_2$
$u_1 = 0$
$u_2 = 1 + 3/4\ u_0 + 1/4\ (0)$
Which gives $u_0 = 16/5$

b)

i) Using "R" to denote "rain" and "N" to denote "no rain", the four states are RR,RN,NR, and NN. Since the weather today depends only on the weather on the last two days and since two days has been lumped into one state, the system is a Markov chain. The transition probability matrix is

$$P = \begin{array}{c} \\ RR \\ RN \\ NR \\ NN \end{array} \begin{array}{c} \begin{array}{cccc} RR & RN & NR & NN \end{array} \\ \left\| \begin{array}{cccc} 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 \\ 0.6 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0.7 \end{array} \right\| \end{array}$$

ii) The pattern is N,N, ,R. ie: it is N,N,R,R or N,N,N,R which can be thought of as

NN → NR → RR or NN → NN → NR, which also can be thought of as going from NN to RR or from NN to NR in two steps. After squaring the matrix P, the NN to RR entry is 0.18 and the NN to NR entry is 0.21. So, the final answer is 0.18 + 0.21 = 0.39

iii) Let the states RR,RN,NR, and NN be denoted by 0, 1, 2, and 3, respectively. Then the stationary distribution is given by the system

$$\pi_0 = 0.6\,\pi_0 + 0.6\,\pi_2$$
$$\pi_1 = 0.4\,\pi_0 + 0.4\,\pi_2$$
$$\pi_2 = 0.6\,\pi_1 + 0.3\,\pi_3$$
$$\pi_3 = 0.4\,\pi_1 + 0.7\,\pi_3$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

which gives $\pi_0 = 9/29$, $\pi_1 = \pi_2 = 6/29$, $\pi_3 = 8/29$

**Problem 3**

a)

i) High R Square, significant t-stat for SPEED and TYPE (or to say that 95% confidence intervals does not contain 0). The t-stat for WEIGHT is also reasonably good.

ii) The t-stat for PROC is quite weak and the 95% confidence interval for PROC obviously contains 0. You could also point out that the t-stat for b0 is very bad and the confidence interval for the intercept is very wide. There is most likely a problem of multicollinearity. The least you can do to improve the model is to remove BDRM as an independent variable and run the regression again. Better yet, you should compute the correlation between SPEED and PROC and confirm that they are indeed very highly correlated.

iii) On average, a unit increase (MHz) in SPEED and keeping all the other characteristics the same will increase the estimated price by £0.057

iv) Based on the regression model, the price would be £ 345 so £ 400 seems to be too high.

(b)

i) Standard deviation is a good proxy for the risk associated with a stock/investment/project when you are considering this asset in isolation. For example, when you only have this specific stock in your portfolio or when all the other assets are perfectly correlated with that stock.
Standard deviation is not appropriate for the risk contribution when you have other assets which are not perfectly correlated with that asset. For example, an asset with very high standard deviation might reduce the overall risk of your portfolio when it is negatively correlated with the rest of your assets.

ii) For A: the slope is $[E(A)-R_f]/\beta_A=(22\%-7\%)/1.8=8.33\%$
For B: the slope is $[E(B)-R_f]/\beta_B=(20.44\%-7\%)/1.6=8.4\%$
Since this slope is the reward-to-risk, we see that stock B offers a higher reward-to-risk ratio, therefore, higher reward per unit of risk.
In practice, such opportunities would not last for long and the market would react in such a way so that both stocks would have the same reward-to-risk ratio. For example, by lowering the E[B].

**Problem 4**

a)

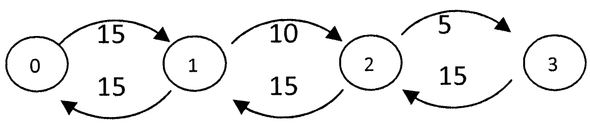
i) $\pi_0 = 0.05$

ii) $\pi_0 + \pi_1 + \pi_2 + \pi_3 = 0.7$

iii) $L_q=1\,\pi_5 + 2\,\pi_6=0.2$

b)

i)




ii) $15p_0= 15p_1$
$15p_0+ 15p_2=25\, p_1$
$10p_0+ 15p_3=20\, p_2$
$5p_2= 15p_3$
iii) $p_0= 9/26$, $p_1= 9/26$, $p_2= 3/13$, $p_3= 1/13$
iv) ) $L= 0(P_0) + 1(P_1) +2(P_2) +3(P_3) =27/26$ cars