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2010 PART IIA 3F4 DATA TRANSMISSION
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3F4 Data Transmission 2009/10

1.) (a) To match the power spectrum of the transmitted signal to suit the frequency response of the channel. For example, many channels have a poor response at d.c. and at low frequencies owing to the use of a.c. coupling. Also a low pass channel response limits the ability to carry high signal frequency components.

To permit symbol timing self-synchronisation. That is, there should be sufficient information in the transitions of the transmitted signal to allow timing regeneration to be performed at the receiver. 4

(b) Now,

$$S_x(\omega) = \frac{1}{T} \sum_{m=-\infty}^{\infty} R(m)\, e^{j m\omega T}$$

where,

$$R(m) = E\left[a_k \; a_{k+m}\right]$$

We need to calculate $R(m)$ for the polar line coding scheme.

m	b_k .. b_{k+m}	a_k .. a_{k+m}	$R_i = a_k a_{k+m}$	p_i	$R(m) = \sum_{i=1}^{M} R_i p_i$
0	0 .. 0	−1 .. −1	$R_1 = 1$	0.5	$R(0) = (1 \times 0.5) + (1 \times 0.5) = 1$
	1 .. 1	1 .. 1	$R_2 = 1$	0.5	
≥1	0 .. 0	−1 .. −1	$R_1 = 1$	0.25	
	0 .. 1	−1 .. 1	$R_2 = -1$	0.25	$R(\geq 1) = 0$
	1 .. 0	1 .. −1	$R_3 = -1$	0.25	
	1 .. 1	1 .. 1	$R_4 = 1$	0.25	

So, $S_x(\omega) = \frac{1}{T}$.

ie, a flat spectrum. 5

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(c)(i) Again we need to calculate the discrete ACF values, $R(m)$.

Consider $m=0$.
Since the inverted symbols are equiprobable, i.e, same as those owing to the data symbols, then the ACF is the same as for the polar case,

$R(0) = \underline{1}$.

Remember the transmitted data has the form DDD1DDD-1DDD1...
Consider ~~[illegible]~~ $1 \leq m \leq 3$
Again we see that this is effectively the same as for polar binary, i.e,
$R(m) = 0$,

Now consider $m = 4$. In this case the situation is more complex ~~[illegible]~~ since there is a probability of 1/4 that a_k and a_{k+m} are both inverted symbols.
i.e,

~~[illegible]~~DDD1DDD-1 DDD1DDD-1

1 inverted 4 total.

In which case they have the opposite sign, so,

$a_k \cdot a_{k+m} = \underline{-1}$

So $R(4) = (0.25 \times -1) + (0.75 \times 0) = -0.25$
Similar argument for ~~R~~$R(m)$, for $m =$ ~~8~~ 12, 20, etc.

Now consider $m = 8$

Again probability of $1/4$ that a_k and a_{k+m} are both inverted symbols.

In this case they will have identical signs, so

$$a_k \, a_{k+m} = 1 .$$

So,

$$R(8) = (0.25 \times 1) + (0.75 \times 0) = 0.25 .$$

Similar argument applies for $R(m)$, $m = 16, 24 \ldots$

Thus the ACF sequence is

$m =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	1	0	0	0	−0.25	0	0	0	+0.25	0	0	0	−0.25	0	0	0	+0.25

Therefore

$$S_x(\omega) = \frac{1}{T} \sum_{m=-\infty}^{\infty} R(m)\, e^{j\omega m T}$$

Splitting the infinite summation into a term at $m = 0$ plus terms at $m = \ldots -16, -8, 0, 8, 16$ and $m = \ldots -20, -12, -4, 4, 12, 20 \ldots$, i.e.,

$$S_x(\omega) = \frac{1}{T}\left(0.75 + \sum_{m=-\infty}^{\infty} 0.25\, e^{j\omega 8mT} - \sum_{m=-\infty}^{\infty} 0.25\, e^{j\omega(8m-4)T}\right)$$

$$S_x(\omega) = \frac{1}{T}\left(0.75 + \sum_{m=-\infty}^{\infty} 0.25\, e^{j\omega 8mT} - e^{-j4\omega T} \sum_{m=-\infty}^{\infty} 0.25\, e^{j\omega 8mT}\right)$$

$$S_x(\omega) = \frac{1}{T}\left(0.75 + \left(1 - e^{-j4\omega T}\right) \sum_{m=-\infty}^{\infty} 0.25\, e^{j\omega 8mT}\right)$$

Now using the identity from the question gives:

(4)

$$S_{xx}(w) = \frac{1}{T}\left(0.75 + \left(1 - e^{-j4wT}\right)\frac{2\pi}{T}\sum_{m=-\infty}^{\infty} 0.25\,\delta\left(w - m\frac{2\pi}{8T}\right)\right)$$

Considering first the delta function.
This expression is only non-zero when

$$w = m\frac{2\pi}{8T}$$

$$w = \frac{m\pi}{4T}.$$

substitute this into the term e^{-j4wT} :

$$e^{-j4T \cdot \frac{m\pi}{4T}} = e^{-jm\pi}$$

Thus the term $\left(1 - e^{-j4wT}\right)$ is equivalent to
$\left(1 - e^{-jm\pi}\right)$ for ~~discrete~~ integer values of m.

In particular for odd m,
$\left(1 - e^{-jm\pi}\right) = 2$
and for even m
$\left(1 - e^{-jm\pi}\right) = 0$
This allows $S_{xx}(w)$ to be rewritten as

$$S_{xx}(w) = \frac{1}{T}\left(0.75 + \frac{2\pi}{T}\sum_{m=-\infty}^{\infty} 0.5\,\delta\left(w - (2m+1)\frac{2\pi}{8T}\right)\right)$$

$$S_{xx}(w) = \frac{1}{T}(0.75) + \frac{2\pi}{T^2}\sum_{m=\infty} 0.5\,\delta\left(w - \frac{(2m+1)\pi}{4T}\right)$$

ie, a flat spectrum plus line components of

$\ldots -\frac{1}{8}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8} \ldots$

symbol rate.

(8)

(5)

(ii) Advantages

Stops long runs of 1s and zeros - good for channels with a.c. coupling

Gives some timing information.

Still zero mean.

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Disadvantages.

Uses more bandwidth.

2.) (a) When a single pulse is transmitted through a communication channel, the pulse response at the receiver is determined by the characteristics of the channel and may last for many symbol periods. When many such pulses are transmitted in adjacent symbol slots their individual pulse responses add together (in a linear channel), leading to intersymbol interference, since the received signal in a given symbol period is composed of a mixture of responses from pulses in the current and surrounding symbol periods.

Equalisers are filters that process the received signal in order to reduce intersymbol interference prior to the data slicer (decision threshold).

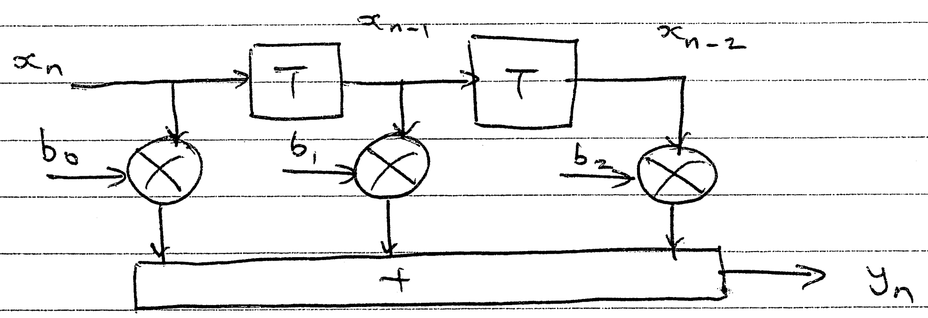
Zero forcing (ZF). Forces the pulse response to zero at all times except for the symbol of interest. Takes no account of channel noise and hence channel noise can be enhanced, thus reducing the BER performance.

Minimum mean squared error (MMSE). Minimises the expected error squared between the equalised symbol value and the actual symbol value. Thus explicitly takes in to account the noise statistics of the channel. Essentially trades-off ISI reduction against noise enhancement.

(2)

Decision Feedback (DF). Includes a slicer in the feedback loop of an IIR based filter equaliser. This strips noise from the detected data before it is used to cancel ISI, i.e, 'noise-free' equalisation. However, incorrect decisions can lead to long bursts of errors at the equaliser output. 8

(b) The required FIR filter has the form:



so,

$$y_n = x_n b_0 + x_{n-1} b_1 + x_{n-2} b_2$$

or in general for a N-tap filter:

$$y_n = \sum_{i=0}^{N-1} x_{n-i} b_i \quad .$$

1

In this case the input $x(n)$ is the single pulse $p(n)$, i.e,

$x_0 = p_0 = 1$

$x_1 = p_1 = -0.5$

$x_2 = p_2 = 0.2$

The zero forcing (ZF) constraint is:

$y_0 = 1, \quad y_1 = 0, \quad y_2 = 0 \quad .$

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So, writing out the system output equations:

$n = 0$, $y_0 = x_0 b_0 = p_0 b_0$

$\therefore b_0 = y_0 / p_0$

$b_0 = 1 / 1$

$b_0 = 1$.

$n = 1$, $y_1 = x_1 b_0 + x_0 b_1$

$y_1 = p_1 b_0 + p_0 b_1$

$0 = (-0.5 \times 1) + (1 \times b_1)$

$b_1 = 0.5$

$n = 2$, $y_2 = x_2 b_0 + x_1 b_1 + x_0 b_2$

$y_2 = p_2 b_0 + p_1 b_1 + p_0 b_2$

$0 = (0.2 \times 1) + (-0.5 \times 0.5) + 1 \times b_2$

$b_2 = 0.05$

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(c) With the equaliser:

First need to calculate the 'residual' equaliser outputs in response to the single pulse.

$n = 3$ $\quad y_3 = x_2 b_1 + x_1 b_2$

$y_3 = p_2 b_1 + p_1 b_2$

$y_3 = (0.2 \times 0.5) + (-0.5 \times 0.05)$

$y_3 = 0.075$

$n = 4$ $\quad y_4 = x_2 b_2$

$y_4 = p_2 b_2$

$y_4 = 0.2 \times 0.05$

$y_4 = 0.01$

4

The equaliser output noise voltage (rms) is,

$$\sigma_w = \sigma_v \sqrt{b_0^2 + b_1^2 + b_2^2}$$

$$\sigma_w = 0.2\sqrt{1^2 + (0.5)^2 + (0.05)^2}$$

$$\sigma_w = 0.2238 \text{ V}.$$

We now need to find the minimum eye opening for this unipolar scheme:

Worst case '1' is 1

" " '0' is $0.075 + 0.01 = 0.085$

$\therefore h = 1 - 0.085$

$h = 0.915$.

$$P_e = Q\left(\frac{h}{2\sigma_w}\right)$$

$$= Q\left(\frac{0.915}{2 \times 0.2238}\right) = Q(2.044)$$

$$= 0.02.$$

without equaliser:

Worst case '1' is $1 - 0.5 = 0.5$

" " '0' is $0 + 0.2 = 0.2$

$\therefore h = 0.5 - 0.2$

$h = 0.3$

$$P_e = Q\left(\frac{h}{2\sigma_v}\right) = Q\left(\frac{0.3}{2 \times 0.2}\right) = Q(0.75)$$

$$P_e = 0.224.$$

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(d) With an ideal DFE, there is no noise enhancement and zero ISI.

So,

$$P_e = Q\left(\frac{h}{2\sigma_v}\right) \quad \text{i.e., eye opening } h = 1.$$

$$P_e = Q\left(\frac{1}{2\times 0.2}\right) = Q(2.5)$$

$$P_e = 6.23 \times 10^{-3}$$

[2]

3F4

3. a) If $s(t) = a(t) . \cos(\omega_c t + \phi(t))$

then $a(t)$ represents amplitude modulation and $\phi(t)$ represents phase (or frequency) modulation.

We may also write this as

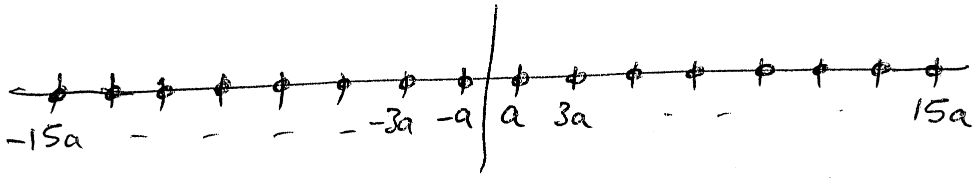
$$s(t) = Re\{a(t)\, e^{j(\omega_c t + \phi(t))}\}$$
$$= Re\{a(t)\, e^{j\phi(t)} . e^{j\omega_c t}\}$$
$$= Re\{p(t)\, e^{j\omega_c t}\}$$

where $p(t) = a(t)\, e^{j\phi(t)}$

Hence $a(t) = |p(t)|$ and $\phi(t) = \angle p(t)$.

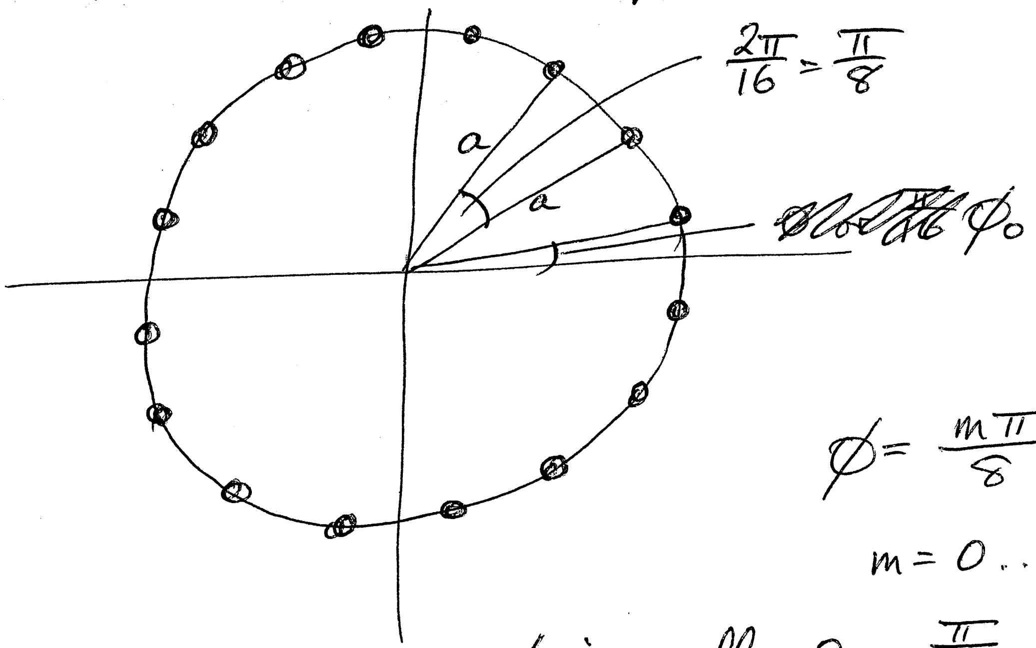
Thus $p(t)$ ~~rep~~ represents both amplitude & phase modulation.

b) 16-ASK: 16 amplitude states at constant-phase (say zero phase).



This is the zero-mean or suppressed-carrier form.
It can also have a non-zero mean or carrier component.

b) (cont.) 16-PSK: 16 phase states at constant amplitude. The phases are normally equally distributed from 0 to 2π.

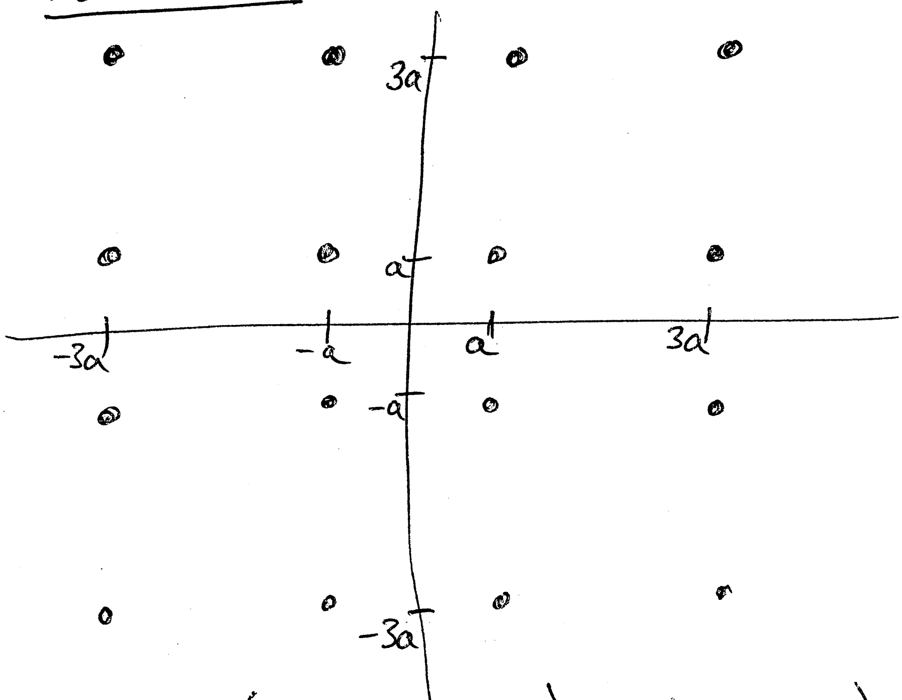


$$\phi = \frac{m\pi}{8} + \phi_0$$

$$m = 0 \ldots 15$$

ϕ_0 is usually 0 or $\frac{\pi}{16}$.

16-QAM: 4-ASK states on two quadrature carriers



$$p_k(t) = \left(S_{2k} + j\,S_{2k+1}\right) \cdot g(t - kT_s)\, e^{j\phi_0}$$

$S_{2k}, S_{2k+1} = -3, -1, +1, +3$. ϕ_0 is usually 0.

b) (cont.) Noise phasor calculations.

~~the 16ASK: 2 [illegible]~~

Decision thresholds are midway between adjacent constellation states. Hence for 16-ASK & 16-QAM, the min noise amplitudes are $\underline{a}$.

For 16-PSK the states are $2.a\sin\frac{\pi}{16}$ apart, so the min noise amplitude is $a\sin\frac{\pi}{16}$.

But all need to be normalised for unit mean squared amplitude (MSA).

For $\underline{16\text{-ASK}}$, MSA $= \frac{1}{8}\sum_{k=1}^{8}\left((2k-1)a\right)^2$

$= a^2.\frac{1}{8}(1+9+25+49+81+121+169+225)$

$= a^2.\frac{680}{8} = 85a^2$

If the MSA $=1$, then $a = \frac{1}{\sqrt{85}} = 0.1085$

Hence min noise ampl. for 16-ASK = $\underline{0.1085}$.

For $\underline{16\text{-PSK}}$, MSA $= a^2$ since all phasors are of magn. a.

$\therefore$ If MSA $=1$, then $a=1$

Hence min noise ampl for 16-PSK $= \sin\frac{\pi}{16} = \underline{0.1951}$

For $\underline{16\text{-QAM}}$, MSA $= \frac{a^2}{2}(1^2+9^2+1^2+9^2)$

$= 10a^2$,

If the MSA $=1$, then $a = \frac{1}{\sqrt{10}} = 0.3162$

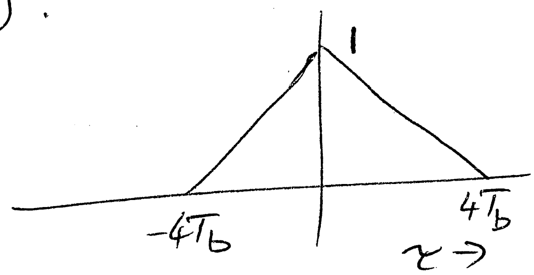
Hence min noise ampl for 16-QAM $= a = \underline{0.3162}$

c) PSD of 16-ASK:

Symbol period = 4 bits = $4T_b$

Each symbol is rectangular of duration $4T_b$ & uncorrelated with adjacent symbols (assuming the carrier is suppressed).

Hence the ACF ~~is~~ of $p(t)$ is:

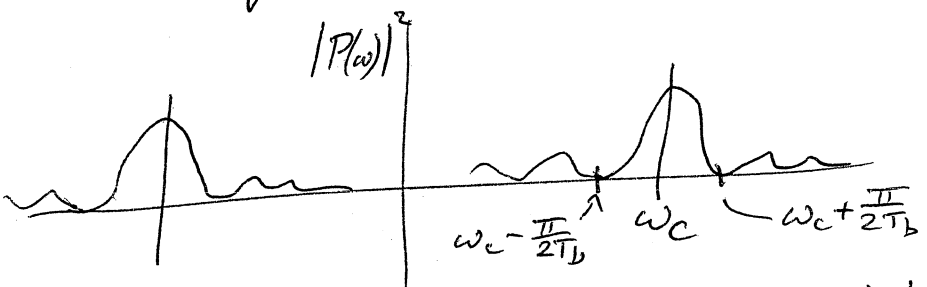


The ACF has unit magnitude at $\tau = 0$ because the mean square amplitude of the signal $p(t)$ is unity.

The power spectrum $|P(\omega)|^2$ is the Fourier Transform of the ACF, so, from the E&I data book

$$|P(\omega)|^2 = 4T_b \operatorname{sinc}^2\left(\frac{\omega . 4T_b}{2}\right) = \underline{\underline{4T_b \operatorname{sinc}^2(2\omega T_b)}}$$

When modulated onto a carrier ω_c, this spectrum will be shifted ~~to~~ by $\pm\omega_c$ to give



1st zeros are when argument of sinc^2 is $\pm\pi$,

ie when $\omega = \frac{\pm\pi}{2T_b}$ or $f = \frac{\pm 1}{4T_b}$ Hz.

Hence the bandwidth $= 2 . \frac{1}{4T_b} = \underline{\underline{\frac{1}{2T_b}}}$ Hz to first zeros.

d) Both 16-PSK + 16-QAM use the same number of signal states as 16-ASK, and hence they will have the same symbol rate = $\frac{1}{4}$ of the bit rate. Since they both are zero-mean modulation processes (like 16-ASK) they will have the same ACF + hence the same PSD as 16-ASK. So the bandwidths will be the same, and equal $\underline{\underline{\frac{1}{2T_b}\text{ Hz}}}$

~~(Their main advantage over 16-ASK is the improved resilience to noise as calculated in part (b).)~~

e) All 3 methods have the same bandwidth for a given bit rate $\frac{1}{T_b}$ bit/s. However we saw in part (b) that the minimum noise amplitude to cause errors is 0.1085, 0.1951 + 0.3162 of the signal rms level at the detector, for 16-ASK, 16-PSK + 16-QAM respectively. Hence 16-QAM is the most resilient to noise (and will therefore have the lowest error rate at a given SNR) while 16-ASK is the least resilient.

3F4

4. a) COFDM:

Main features:

a) Uses error-correction coding

b) Splits the data into many lower-bit-rate schemes, and uses many parallel carrier waves to convey the separate bit-streams. The discrete Fourier transform (DFT) is used to achieve the multi-carrier demodulation & preserve low intersymbol interference.

c) The low modulation rates, and the use of a guard period between symbols, allows for multiple signal paths & the different-path delays that each will have.

The main practical problem is multi-path, and OFDM allows for a much greater range of path delays than could be tolerated with a single serial-modulated carrier wave. Multi-path occurs due to reflections from building & other large obstacles (eg hills). Coding is needed with OFDM to allow the system to accomodate spectral nulls (or notches) caused by interfering multi-path components of a signal, which effectively 'knock out' some of the multiple carriers of OFDM.

b) User bit rate = 30 Mb/s

Coded bit rate $= \frac{5}{3} \times 30 = 50$ Mb/s

As explained above, coding can correct errors that are caused by poor SNR on some of the OFDM carriers which occurs when the multi-path waves destructively interfere and tend to cancel each other ~~other~~ out at certain frequencies. In between these frequencies, the signal strength is much better and provides good quality data for the error correction coder to use.

c) Path difference of 6Km gives a delay difference of $\frac{6000}{c} = \frac{6000}{3.10^8} = 20.10^{-6}$ s $= 20\,\mu s$.

Hence the guard period between symbols $= 20\mu s$.

If the carriers are 5kHz apart, the analysis period in the receiver DFT must be $\frac{1}{5000} = 200\mu s$. ~~/ symbol~~,

Hence the interval between consecutive symbols $= 200+20 = 220\mu s$.

64-level modulation can carry 6 bits per symbol.

$\therefore$ Bit rate per carrier $= \frac{6}{220.10^{-6}} = 27.2727$ kHz.

∴ No. of carrier to carry 50Mb/s

$$= \frac{50.10^6}{27.2727 \,.\, 10^3} = 1833.33 = 1834 \text{ when rounded up}$$

Total no. of carriers, including pilot tones

$= 1834 + 200 = \underline{\underline{2034}}$.

Carriers are spaced 5kHz apart.

∴ Approx bandwidth for 2034 carriers $= 2034 \times 5000$
$= 10.17$ MHz

In practice, more bandwidth would be needed to allow for finite filter transition bands, so approx <u>11 MHz</u> would be required.

d) Bandwidth efficiency is very important in digital TV systems, so 64-QAM is a good modulation scheme that has high efficiency, while its bit error rate vs. SNR performance is not too much worse than the best error-rate schemes like QPSK which are not very bandwidth efficient. The 3:5 code rate is chosen to provide ~~good~~ error correction performance that is good enough to correct 5 to 10% of bit error rate due to multipath spectral nulls, while not degrading the spectral efficiency too much (only by $\frac{3}{5}$). Finally the 5KHz tone spacing is selected so that the required guard period (20μs) is significantly less than the analysis period of $\frac{1}{5K} = 200\mu s$, so that the loss of spectral efficiency due to the guard period is only about 10%. Tone spacings of less than 5KHz would be even more efficient but the increased number of tones needed would result in additional hardware complexity (& also would require improved ~~osc~~ receiver oscillator stability & noise levels). Hence 5kHz is a good tradeoff.

Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2010 - Answers

1.
a)
b)
c)

2.
a)
b) $b_0=1$, $b_1=0.5$, $b_2=0.05$
c) BER (no equaliser) = 0.224, BER (equaliser) = 0.02
d) BER = $6.23*10^{-3}$

3.
a)
b) (i) 0.1085 (ii) 0.1951 (iii) 0.3162
c)
d)
e)

4.
a)
b) 2034, 10.17MHz min so approx 11MHz practical
c)
d)

I. J. Wassell May 2010