ENGINEERING TRIPOS PART IIA

Monday 3 May 2010 2.30 – 4.00

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of the question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper.	Engineering Data Book. CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final version

1 In this question the dead-state temperature T_0 and pressure p_0 are taken as 300 K and 1 bar respectively.

Figure 1 is a schematic diagram of the heater in a steady-flow, closed-cycle gas turbine. Gas enters at state 1 and leaves at state 2 and the mass flowrate is $\dot{m}$. Defining the specific steady-flow exergy as $e = h - T_0 s$, the exergy equation for the device can be written,

$$\dot{m}(e_2 - e_1) = \int_1^2 \left(1 - \frac{T_0}{T}\right) d\dot{Q} - T_0 \dot{S}_{irrev}$$

(a) Briefly, explain the physical meaning of the three terms in the equation. [15 %]

(b) If v is specific volume, show that for any gas flowing through the heater,

$$T_0 \dot{S}_{irrev} = -\dot{m} \int_1^2 \frac{T_0}{T} v \, dp$$ [25 %]

(c) Suppose the gas behaves as a perfect gas with $R = 2.08$ kJ kg^{-1} K^{-1} and $c_p = 5.19$ kJ kg^{-1} K^{-1}. Taking $\dot{m} = 10$ kg s^{-1}, $T_1 = 800$ K, $T_2 = 1500$ K, $p_1 = 5$ bar and $p_2 = 4.5$ bar, calculate the maximum shaft power that could be supplied by the gas turbine cycle and the corresponding exergy supply rate to the heater. [25 %]

(d) If instead the gas is such that its characteristic equation of state written in terms of the specific Gibbs function g is,

$$g = c_p(T - T_0) - c_p T \ln\left(\frac{T}{T_0}\right) + RT \ln\left(\frac{p}{p_0}\right) + BRT(p - p_0)$$

where R and c_p are constants having the same values as in part (c) and $B = -0.1$ bar^{-1}. Find expressions for the specific enthalpy and entropy of the gas as functions of pressure and temperature. Hence find the maximum shaft power that could be supplied by the gas turbine cycle for the conditions in part (c). [35 %]

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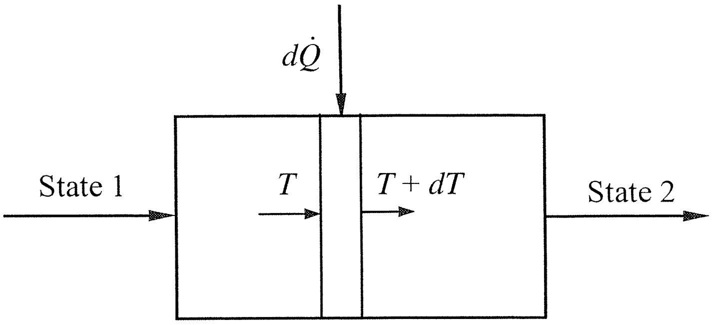

Fig. 1 Schematic diagram of the heater in Question 1.

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2 (a) Write down the condition for the equilibrium of a gas-phase chemical reaction in terms of the chemical potentials of the reacting species. Hence show that $\Delta\overline{G}_T^0$ (the Gibbs function change of the reaction at temperature T and standard pressure $p_0 = 1$ bar) is related to the equilibrium constant K_p by,

$$\Delta\overline{G}_T^0 = -\overline{R}T\ln(K_p)$$

where the overbar denotes quantities evaluated on a molar basis. It may be assumed that all species behave as semi-perfect gases and that the chemical potential per mole of species-i at partial pressure p_i and temperature T is given by,

$$\overline{\mu}_i(p_i,T) = \overline{\mu}_i^0(T) + \overline{R}T\ln\left(\frac{p_i}{p_0}\right)$$ [25 %]

(b) In the production of methanol (CH_3OH) from coal, a gas mixture consisting of 50% CO and 50% H_2 by volume leaves a coal gasifier and enters a catalytic converter where a reaction occurs. A gas mixture of CH_3OH, CO and H_2 leaves the converter at a temperature and pressure of 200 °C and 10 bar. The mixture is at equilibrium subject to the chemical reaction,

$$-CO - 2H_2 + CH_3OH = 0$$

(i) Using the data below, show that the equilibrium constant for the chemical reaction at 200 °C is 0.0368. It may be assumed that $\Delta\overline{H}_T^0$ is constant in the range 25 – 200 °C. [25 %]

(ii) Determine the mole fractions of the components of the mixture leaving the converter. [50 %]

DATA :

The standard enthalpy and Gibbs function changes at 25 °C for the chemical reaction as written above are,

$$\Delta\overline{H}_{298}^0 = -90.8 \text{ MJ per kmol of CO}, \qquad \Delta\overline{G}_{298}^0 = -25.4 \text{ MJ per kmol of CO}.$$

Van't Hoff's equation is,

$$\frac{d}{dT}(\ln K_p) = \frac{\Delta\overline{H}_T^0}{\overline{R}T^2}$$

3 A gas turbine cycle has a single stage of intercooling and a recuperator to transfer heat from the turbine exhaust to the working fluid at the exit of the compressor. The compressor inlet temperature is T_1 and the isentropic temperature ratio across the low pressure compressor is $\sqrt{r_t}$, where r_t is the isentropic temperature ratio across both compressors. After intercooling, the temperature of the working fluid is reduced to T_1 and the heat extracted is rejected to the environment. The effectiveness of the recuperator is unity and pressure losses within the heat exchangers and the combustor may be neglected. The working fluid is a perfect gas with constant values of c_p and γ throughout. All the turbomachinery may be modelled as isentropic and turbine cooling can be neglected.

(a) Sketch the cycle on a temperature-entropy diagram. Show that the cycle efficiency is given by

$$\eta_c = 1 - \frac{2r_t}{\theta\,(\sqrt{r_t}+1)}$$

where θ is the ratio of the turbine inlet temperature T_3 to the compressor inlet temperature T_1. If w is the specific work output of the cycle, derive an expression for $w/(c_pT_1)$ in terms of r_t and θ. [35 %]

(b) For $\gamma = 1.4$ and $\theta = 5$, show that the intercooled and recuperated cycle only gives a benefit in cycle efficiency (relative to the simple Joule cycle) at an overall pressure ratio below 26.4. It may be assumed without proof that the cycle efficiency of the simple Joule cycle is given by $(1-r_t^{-1})$. [20 %]

(c) If q_{ex} is the heat exchanged per unit mass of working fluid passing through the recuperator, derive an expression for $q_{ex}/(c_pT_1)$ in terms of θ and r_t. For $\gamma=1.4$ and $\theta = 5$, find the overall pressure ratio at which no heat will be exchanged. Explain why this value is significantly greater than the overall pressure ratio found in part (b). [25 %]

(d) State qualitatively how each of the following features of a real intercooled, recuperated gas turbine will affect $w/(c_pT_1)$, $q_{ex}/(c_pT_1)$ and η_c:

(i) a non-isentropic turbine;
(ii) a recuperator effectiveness of less than unity; and
(iii) pressure losses in the recuperator. [20 %]

4 *Where necessary, use the properties of steam given in Table 1 below.*

(a) The steam cycle in a coal-fired power station consists of a feed pump, boiler, turbine and condenser. The turbine entry pressure and temperature are 150 bar and 550 °C respectively. The condenser pressure is 0.04 bar and the water leaving the condenser is saturated liquid. The cycle efficiency is 0.39 and the feed pump work can be neglected. Calculate the dryness fraction at exit from the turbine and the turbine isentropic efficiency. [30 %]

(b) The steam cycle in part (a) is used in a power plant with an electricity output of 600 MW. In order to reduce the emissions from the plant, it is proposed that 50% of the carbon dioxide produced will be captured and stored underground. For the combustion of the coal with air, $\Delta H_{298}^{0} = -24.0\,\text{MJ/kg}$ and the air-to-fuel ratio by mass is 20.0. The exhaust products leaving the boiler have $c_p = 1.10\ \text{kJ kg}^{-1}\ \text{K}^{-1}$ and the stack temperature is 140 °C. The environment is at 25 °C and the mass of carbon per kg of coal is 0.8 kg. Determine the mass supply rate of coal, the overall efficiency of the power plant and the total mass of carbon dioxide that must be captured and stored annually. [40 %]

(c) Explain, briefly, how adding a direct contact feed heater to the steam cycle in part (a) would increase the cycle efficiency and thus reduce the amount of carbon dioxide generated. How else could the plant be modified to improve the cycle efficiency? [15 %]

(d) Briefly, describe the main challenges associated with capturing and storing a large amount of carbon dioxide from a power station. [15 %]

	Specific enthalpy kJ kg^{-1}	Specific entropy $\text{kJ kg}^{-1}\ \text{K}^{-1}$
Steam at 150 bar, 550 °C	3450.4	6.523
Saturated liquid at 0.04 bar	121.4	0.422
Saturated vapour at 0.04 bar	2553.7	8.473

Table 1 Properties of water and steam for Question 4.

END OF PAPER

ENGINEERING TRIPOS PART IIA 2010

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

1. (c) 25.88 MW, 26.54 MW

   (d) $s = c_p \ln\left(\frac{T}{T_0}\right) - R\ln\left(\frac{p}{p_0}\right) - RB(p - p_0)$, $h = c_p(T - T_0)$, 26.19 MW

2. (b) (ii) $X_{H2} = 0.316$, $X_{CO} = 0.500$, $X_{CH3OH} = 0.184$

3. (c) 42.75

4. (a) 0.835, 0.874

   (b) 72.1 kg/s 0.347, 3.33MtCO_2

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