ENGINEERING TRIPOS PART IIA

20th April 2010 9 to 10.30

Module 3D1

GEOTECHNICAL ENGINEERING I

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Geotechnical Engineering Supplementary Data Book (19 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 A building resting on shallow foundations has been observed to suffer damage due to excessive settlements. The building was constructed over 100 years ago on the surface of a 5 m thick clay layer, underlain by permeable sandstone. It can be assumed that there is no infiltration at the ground surface. Construction induced settlements ceased many years ago, but recently settlements have recommenced. Measurement of settlements relative to a fixed datum gave the following data:

1st April 2008	46 mm
1st April 2010	62 mm

Oedometer data from the clay suggests that it has a coefficient of consolidation c_v of 1 m^2 $year^{-1}$.

(a) It has been suggested that the settlement of the building may have been caused by extraction of water from the sandstone below the clay layer. Describe the mechanism by which extraction of water could lead to settlement of the building. [20%]

(b) Sketch isochrones of excess pore pressure in the clay layer beneath the building for:

(i) Phase I consolidation;

(ii) Phase II consolidation. [20%]

(c) From the measured settlements, calculate when water extraction appears to have started. It can be assumed that both measurements relate to phase II consolidation. [50%]

(d) Calculate the ultimate settlement that will occur if water extraction continues in the long term? [10%]

2 A flood embankment is to be constructed from compacted silty-clay fill. A Proctor standard compaction test on the fill material, which is available with two natural water contents of 11% and 20%, gives the data shown in the table below. The fill material has a specific gravity, G_s, of 2.65.

Water content, w	%	11	14	17	20	23
Bulk density, ρ	kg m^{-3}	1850	1920	1960	1960	1950

(a) Calculate the optimum water content for Proctor standard compaction and comment on the suitability of the soil at its two natural water contents as a material for construction of the flood embankment. [30%]

(b) The soil below the embankment consists of a normally consolidated layer of clay 10 m thick and has the water table coincident with the ground surface. The properties of the clay can be taken to be those given for kaolin in the Geotechnical Engineering Data Book. The clay can be taken to have a constant bulk unit weight of 15 kN m^{-3} and properties at the centre of the clay layer can be assumed to represent the entire clay layer. If a 5 m high embankment is constructed using the fill material at a water content of 20% compacted to Proctor optimum conditions, calculate the settlement of the fill surface that might be expected once consolidation is complete. [50%]

(c) Calculate what initial embankment height would be required such that after consolidation has occurred, the final embankment crest is 5 m above the initial ground surface. [20%]

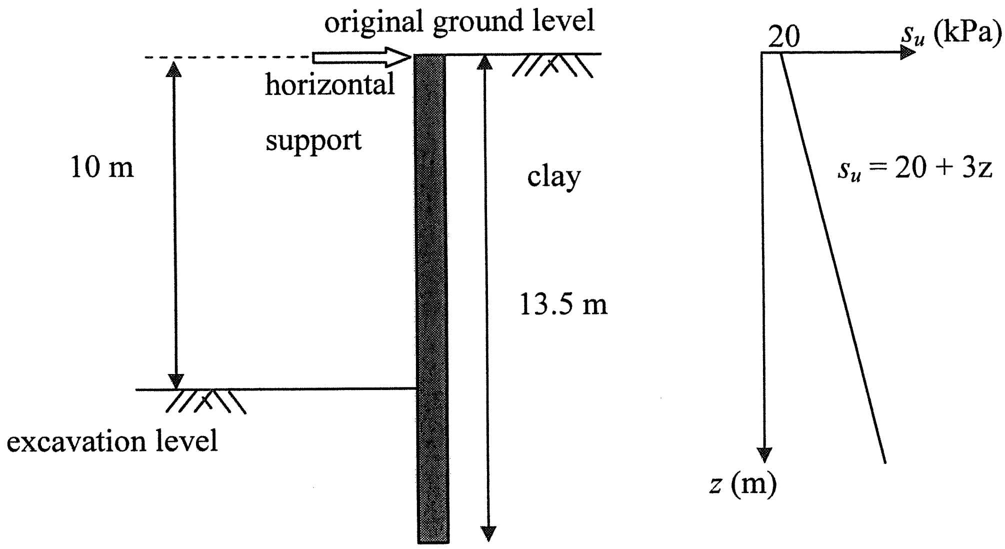
3 An excavation of 10 m depth is made in clay and a smooth retaining wall of 13.5 m depth with a horizontal support at the ground surface is constructed, as shown in Fig. 1(a). It is assumed that the wall rotates at the support when it fails. The undrained strength of the clay increases with depth, as shown in Fig. 1(b). The friction angle of the clay is 25 degrees and the saturated unit weight is 17 kN m^{-3}. The water table is located at the ground surface.

(a) Show the ultimate horizontal total stress distributions on the active and passive sides of the wall in undrained conditions. Will the wall fail in undrained conditions? State clearly any assumptions made. [30%]

(b) In the long-term, the water table on the excavation side will be kept at the original ground level. Show the ultimate horizontal total stress distributions on the active and passive sides of the wall in drained conditions. Will the wall fail? State clearly any assumptions made. [30%]

(c) If the water table is kept at the level of the bottom of the excavation on the excavation side in the long term, estimate the pore pressure distribution along the wall. What changes would you make from the calculations made in part (b) above? The stability of the wall does not need to be quantified, but describe how you would carry out the calculation. [25%]

(d) How could you improve the stability of the wall? List three possible solutions. [15%]


original ground level
horizontal
support
10 m
clay
13.5 m
excavation level
20
s_u (kPa)
$s_u = 20 + 3z$
z (m)

(a) (b)

Fig. 1.

4 A rigid strip foundation of 5 m width is embedded into clay as shown in Fig. 2. The embedment depth is 2 m. The undrained shear strength of the clay is 30 kPa and the friction angle of the clay is 25 degrees. The total unit weight of the clay is 16 kN m^{-3}. The water table is located at the surface. A vertical load V (including the foundation weight) is applied at a location 0.75 m away from the centreline of the foundation, as shown in the figure.

(a) Evaluate the maximum vertical load that can be applied to the foundation in undrained conditions using the stress fan concept. Make clear any assumption you made for the derivation. [30%]

(b) Evaluate the maximum vertical load that can be applied to the foundation in drained conditions. You may use the formula given in the Geotechnical Engineering Data Book. [20%]

(c) Which conditions (undrained or drained) are more critical in design? Why? [10%]

(d) 75% of the critical maximum vertical load determined from either part (a) or (b) above is now applied. Under this condition, a horizontal load H is applied to the foundation. Using the stress fan concept, derive an equation that could be used to evaluate H. There is no need to solve for H, but the form should be simplified as much as possible. For simplicity, ignore the active and passive earth pressures acting on the sides of the foundation. [40%]

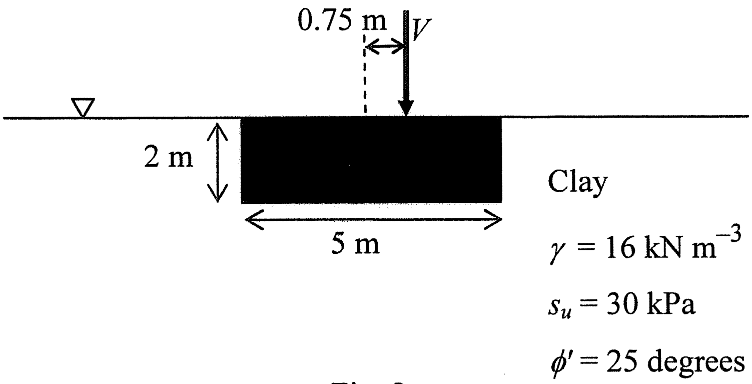


Fig. 2.

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Geotechnical Engineering

Data Book 2007-2008

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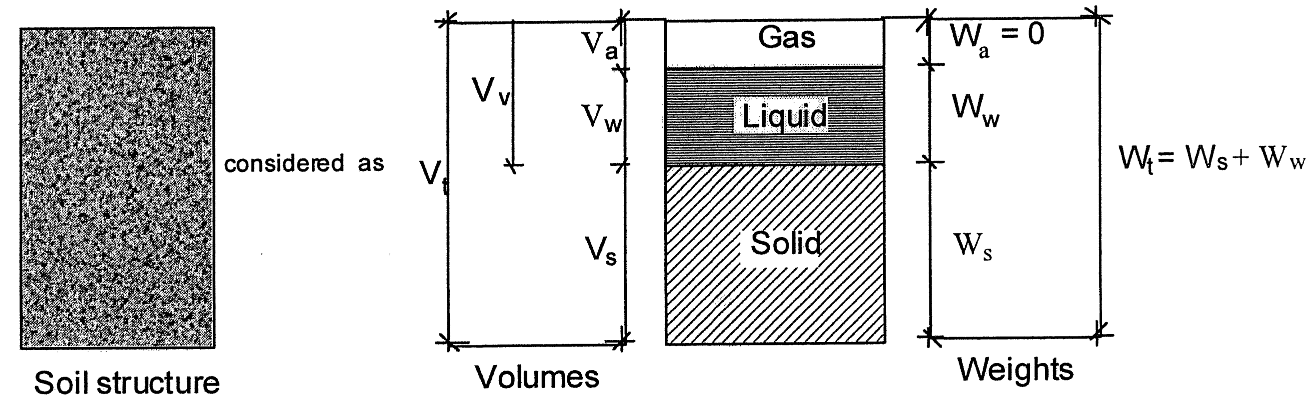
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81$ kN/m^3
Unit weight of soil	$\gamma = W_t / V_t = \left(\dfrac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\dfrac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\dfrac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\dfrac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit	w_L
Plastic Limit	w_P
Plasticity Index	$I_P = w_L - w_P$
Liquidity Index	$I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than 2 } \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

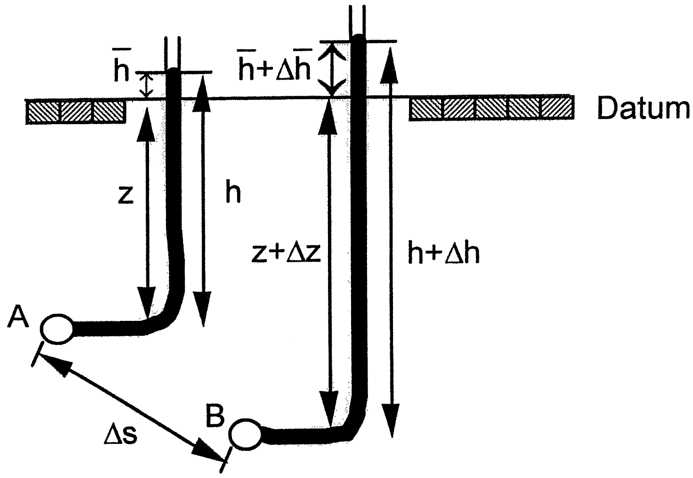
Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc.	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.
C_U	uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta\bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla\bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
10 mm $> D_{10} > 1\mu$m	:	$k \cong 0.01\,(D_{10}$ in mm$)^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

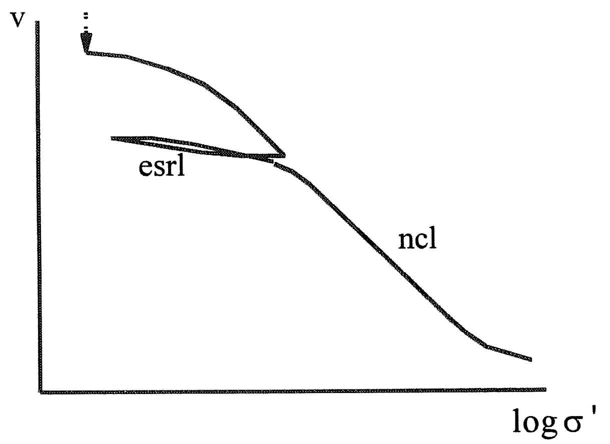
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3\times10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = -\gamma_w h_c$

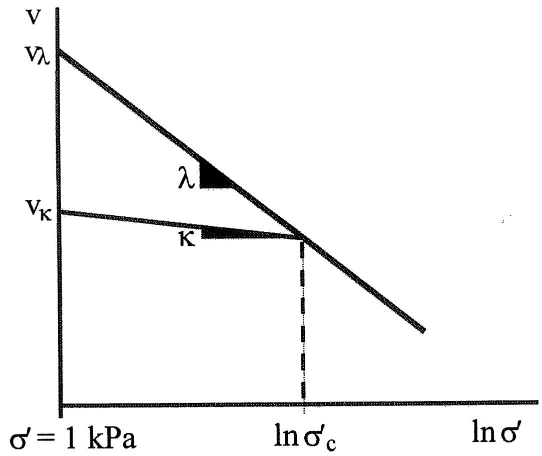
One-Dimensional Compression

• Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

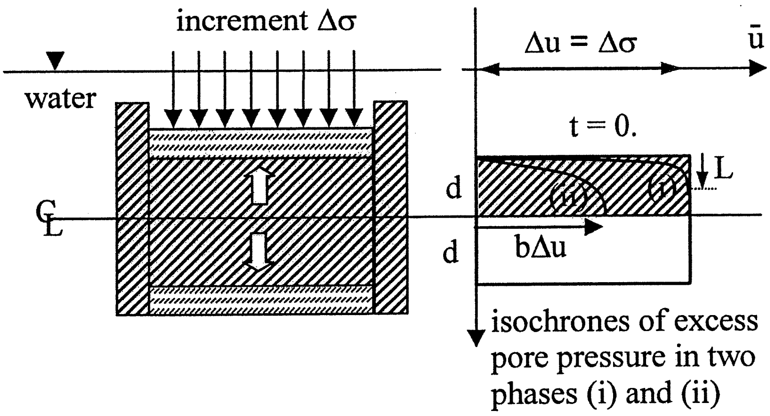
Tangent 1D plastic compression modulus $E_o = v\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz$	$= \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w}$	$= \dfrac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$	
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$	

• Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

total stress σ = effective stress σ' + pore water pressure u

- **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma_1', \sigma_2', \sigma_3'$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

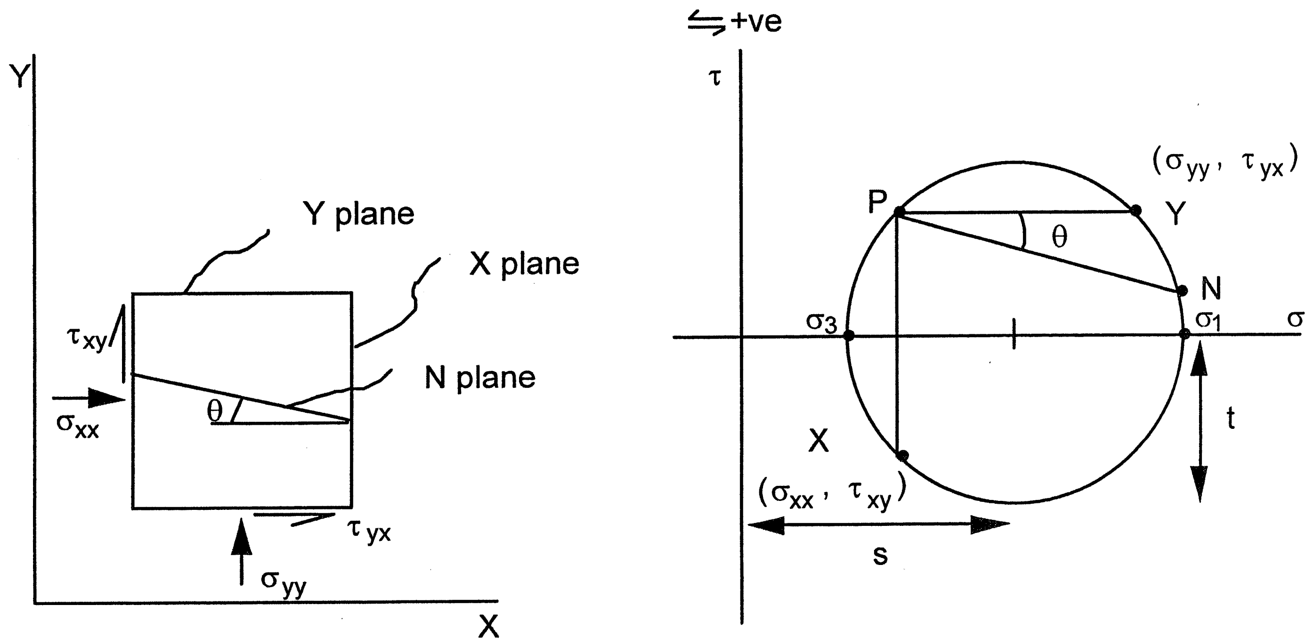
isotropic compression in which p' increases at zero q

triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive



Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility	$m_v = d\varepsilon / d\sigma'$
constrained modulus	$E_o = 1 / m_v$

Physically fundamental parameters

shear modulus	$G' = dt / d\varepsilon_\gamma$
bulk modulus	$K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus	$G_u = G'$
undrained bulk modulus	$K_u = \infty$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli	E' (effective), E_u (undrained)
Poisson's ratios	ν' (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2\,(1+\nu)}$$

$$K = \frac{E}{3\,(1-2\nu)}$$

$$E_o = \frac{E\,(1-\nu)}{(1+\nu)\,(1-2\nu)}$$

Cam Clay

- **Interchangeable parameters for stress combinations at yield, and plastic strain increments**

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- **General equations of plastic work**

Plastic work and dissipation $\qquad \sigma^* \delta\varepsilon^* + \tau^* \delta\gamma^* = \mu^*_{crit}\sigma^* \delta\gamma^*$

Plastic flow rule – normality $\qquad \dfrac{d\tau^*}{d\sigma^*} \cdot \dfrac{d\gamma^*}{d\varepsilon^*} = -1$

- **General yield surface**

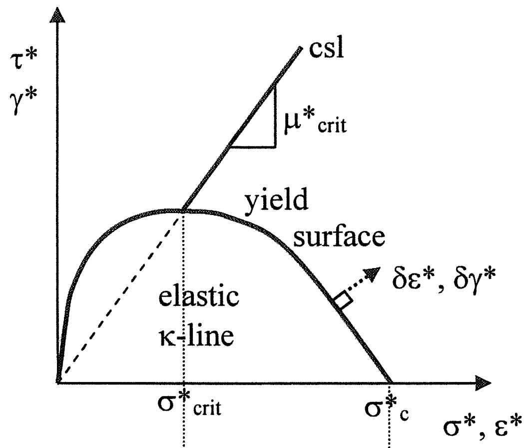
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

- **Parameter values which fit soil data**

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	-------------	-------------
w_P	0.26	0.18	0.42	-------------	-------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

- **The yield surface in (σ*, τ*, v) space**



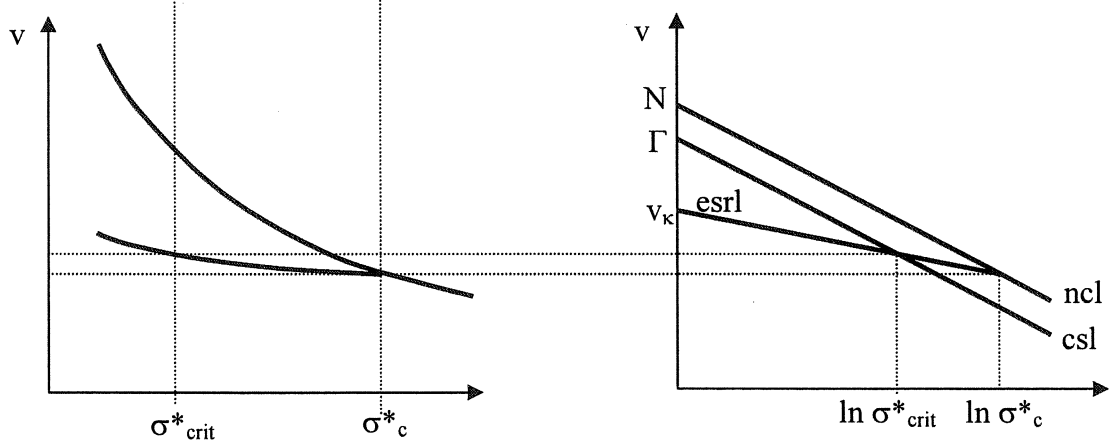
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

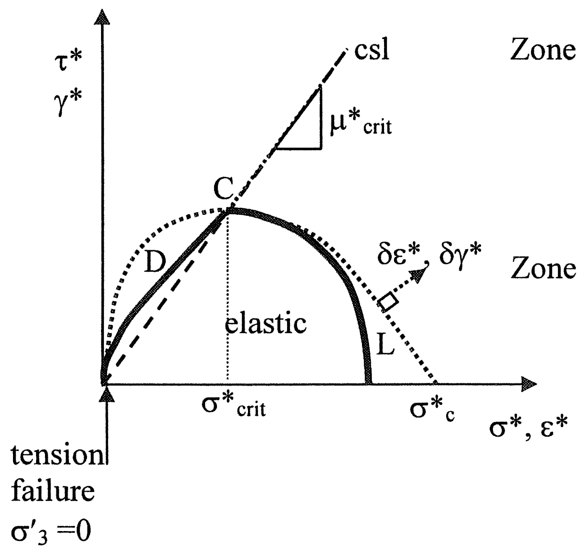
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



- **Regions of limiting soil behaviour**



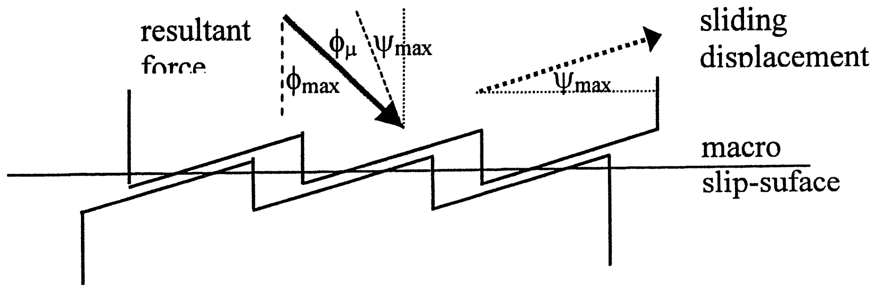
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- **Friction and dilatancy: the saw-blade model of direct shear**

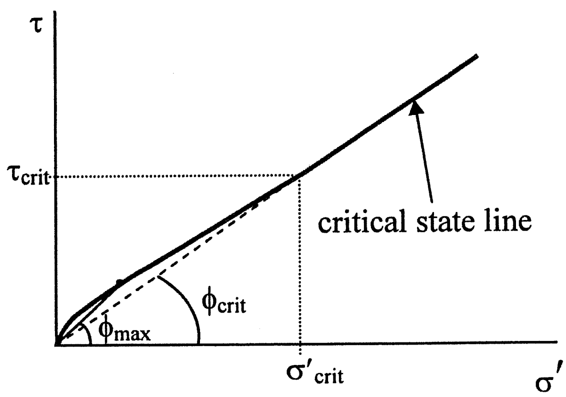


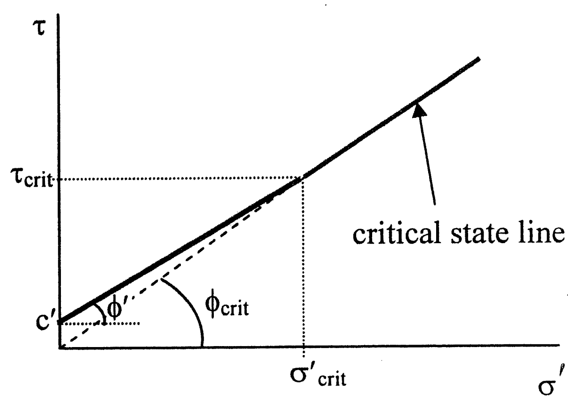
Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

- **Friction and dilatancy: secant and tangent strength parameters**





Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

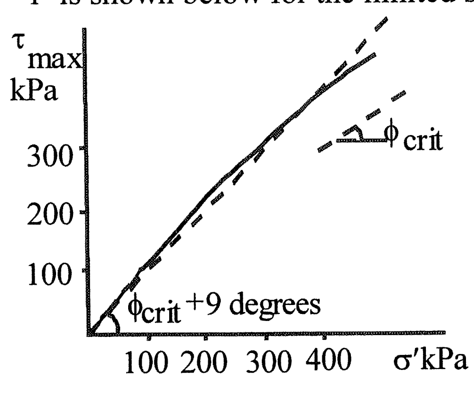
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

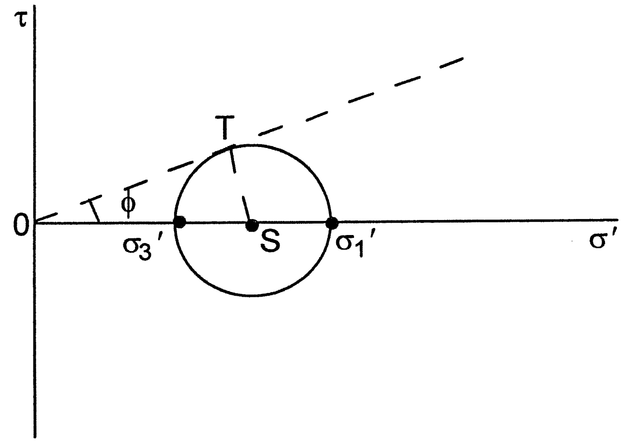
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1, \sigma' = 400$ kPa

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin\phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

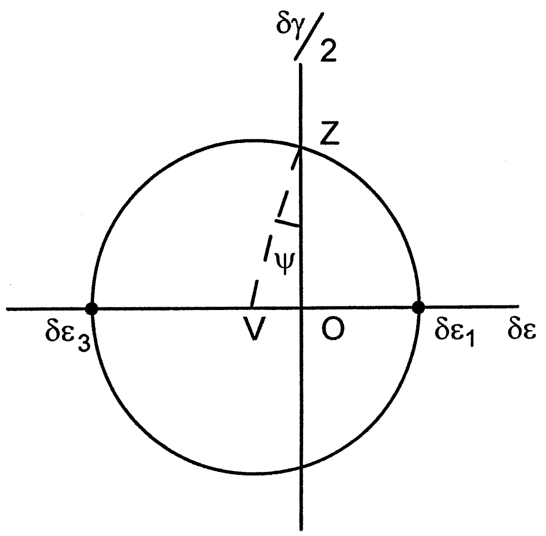
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin\psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

• Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

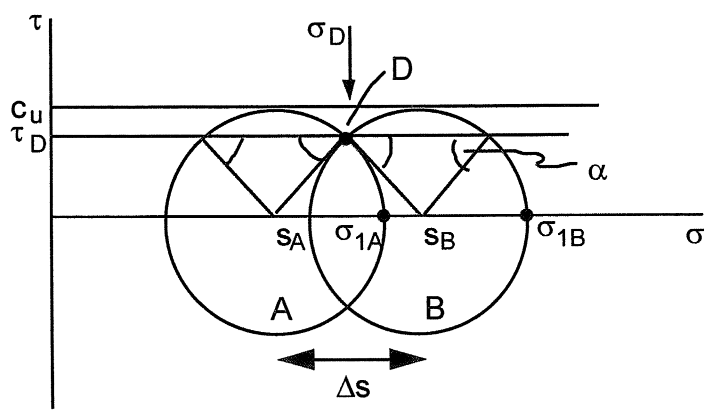
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

• Stress conditions across a discontinuity

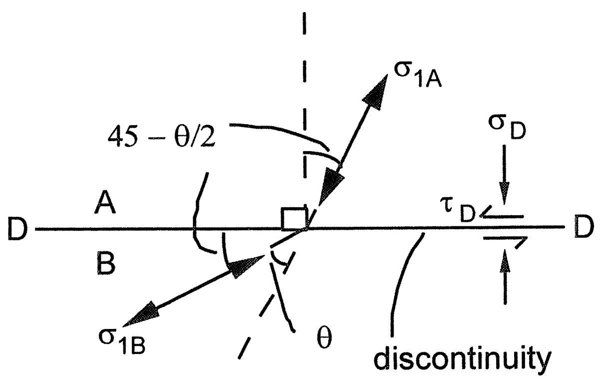


Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin\theta$$
$$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$$

In limit with $\theta \rightarrow 0$

$$ds = 2c_u \, d\theta$$



Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

- **Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

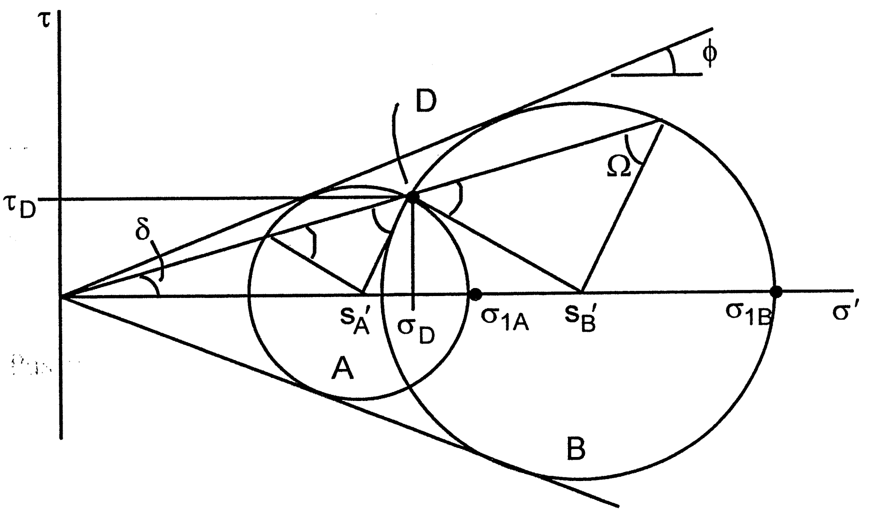
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

- **Stress conditions across a discontinuity**



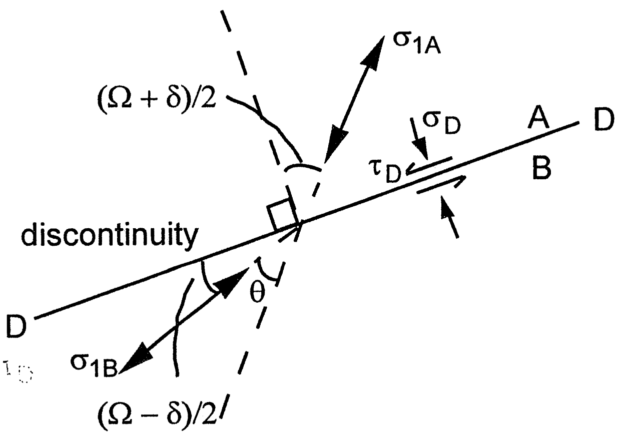
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

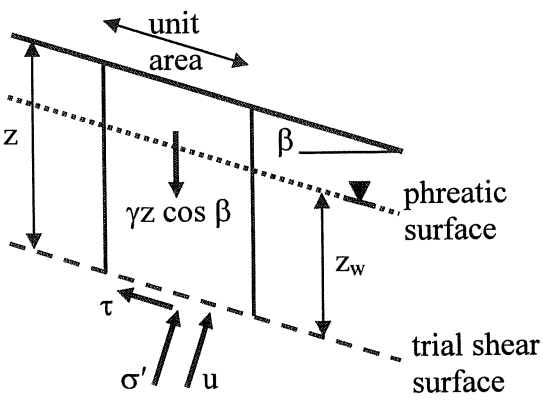
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Infinite slope analysis



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w) \cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \qquad \text{(Prandtl, 1921)}$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$$d_c = 1 + 0.33 \tan^{-1}(h/B) \qquad \text{(or h/D for a circular foundation)}$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $$H = H_{ult} = B s_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$$ (Taiebet & Carter 2000)

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

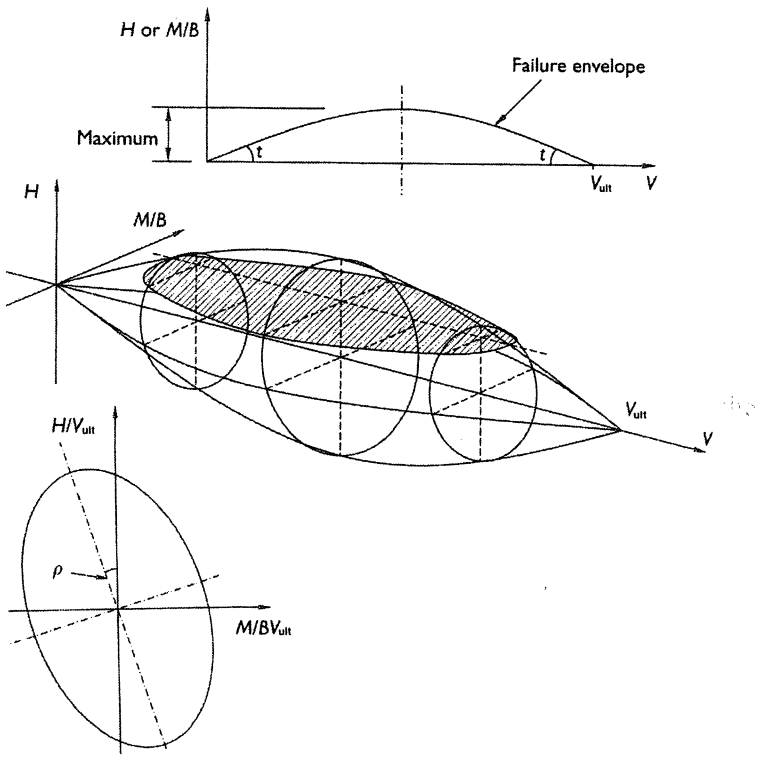
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15°$. Note that t_h is the friction coefficient, $H/V = \tan\phi$, during sliding.

Numerical Answers for 3D1 exam 2010-05-21

1) c) 2.68 years, August 2005
   d) 121 mm

2) b) 1.23m
   c) 6.4m

4) a) 651 kN/m
   b) 848 kN/m