ENGINEERING TRIPOS PART IIA

Monday 19 April 2010 9 to 10.30

Module 3D2

GEOTECHNICAL ENGINEERING II

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Geotechnical Engineering Data Book (19 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 An infinitely long sandy slope with an angle of 30 degrees is shown in Fig. 1. The soil is uniform sub-angular quartz sand, which has maximum and minimum void ratios of 0.85 and 0.40. The slope has a 3 m thick surface layer, which is well compacted, and the in situ void ratio of the soil is estimated to be 0.50. Underneath the surface layer, the soil is less well compacted. The in situ void ratio at the top of the less compacted layer is estimated to be 0.65. The specific gravity of the sand is 2.65.

(a) Find the dry and saturated unit weights of the surface layer soil. [10%]

(b) If the slope is dry, find the stress state of the soil at the top of the less compacted layer. Plot the stress state on $\sigma' - \tau$ space and discuss whether the slope is stable or not by considering the failure strength of the soil. [30%]

(c) The slope is subjected to heavy rain and the water table rises to a depth of 0.5 m as shown in Fig. 2. The water table is parallel to the ground surface. Find the total and effective stress of the soil at the top of the less compacted layer. Plot the stress state on $\sigma' - \tau$ space and discuss whether the slope is stable or not. [45%]

(d) If the soil of the less compacted layer is lightly overconsolidated clay rather than sand, what evaluation would you carry out to check the stability of the slope? [15%]

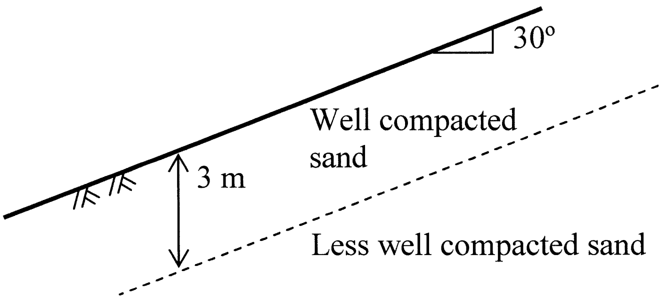

Fig. 1

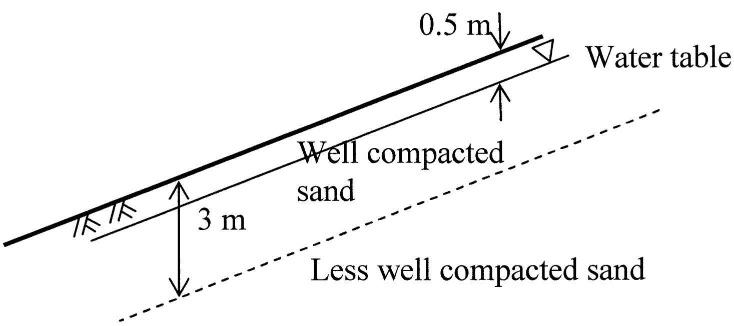

Fig. 2

2 A clay sample was retrieved from the ground in order to perform triaxial tests on specimens made from the sample. The sampling depth was 10 m. The unit weight of the clay was 17 kNm^{-3} and the site investigation showed that it was almost constant throughout the depth. The water table was at the ground surface. Use the Cam clay model to answer the following questions.

(a) One of the specimens was isotropically consolidated to a stress equal to the in situ vertical effective stress. The specimen was then subjected to drained axial compression.

(i) The specimen yielded when the deviator stress was 45 kPa and it failed at critical state when the deviator stress was 90 kPa. Evaluate the plastic normal stress p'_c at the initial yield and the critical state ratio M. Plot the stress path, initial yield surface and failure line on $q - p'$ space. [30%]

(ii) The specific volume after the isotropic consolidation was 2.5. During the drained compression, the change in specific volume was also monitored. Results show that the specific volumes at yield and at critical state were 2.489 and 2.419, respectively. Plot the state path on v – lnp' space. Evaluate the κ value. [15%]

(iii) An oedometer test was performed on another clay specimen and the slope of the normal compression line was found to be $\lambda = 0.161$. Plot the isotropic normal compression line and the critical state line on the same v – lnp' graph used in part (ii) above. [15%]

(b) Another specimen was isotropically consolidated to the same stress as part (a) above, but this time an undrained compression test was performed. Estimate the undrained shear strength. [20%]

(c) An undrained shear strength of 100 kPa is required at 10 m depth to construct a structure safely on this clay. It is proposed to apply a temporary surcharge load at the ground surface in order to consolidate the clay, which in turn will increase its undrained strength. Estimate the magnitude of isotropic consolidation stress that would give the required strength in an undrained triaxial compression test. Comment on the result in relation to the feasibility of the proposal if the surcharge is made by earth fill. [20%]

(TURN OVER

3 A large cylindrical oil tank is to be constructed on a site underlain by soft clay at which the water table is 3 m below the ground surface. It has been established from in situ and laboratory testing that the unit weight of the clay γ = 17.5 kNm^{-3}, the critical state parameter M = 1.0, at a depth of 10 m the coefficient of earth pressure at rest K_0 = 0.8 and the overconsolidation ratio, OCR = 2.0.

(a) Assuming that the coefficient of earth pressure at rest for the clay in its original normally consolidated state, K_{nc} = 0.6, calculate the previous maximum vertical and horizontal effective stresses and the present day values. [20%]

(b) Two triaxial tests are undertaken on samples of clay taken from a depth of 10 m. In both cases the present day total and effective stresses were imposed on the samples prior to additional shearing.

(i) Test 1: an undrained test to simulate rapid filling of the oil tank with water to test the tank and its foundation. This was done by increasing the axial stress, while maintaining the cell pressure constant, until failure occurred. The pore pressure in the sample was measured throughout the test. Yielding occurred after an increase in total vertical stress of 40 kPa and at ultimate failure the undrained shear strength was found to be 40 kPa. Assuming the soil behaves as linear elastic and isotropic prior to yield, plot the total and effective stress paths in $q-p$ and $q-p'$ space for the test. What was the measured excess pore pressure at yield? What was the measured excess pore pressure at ultimate failure? [40%]

(ii) Assume that Test 1 simulated the response of the ground beneath the centre line of the tank at a depth of 10 m during the tank test and that the water height in the tank was just sufficient to cause ultimate failure of the soil locally at a depth of 10 m. Without changing the water level in the tank, the excess pore water pressure in the ground was then allowed to dissipate to zero, so that the pore pressure returned to its value at the beginning of the tank test. Sketch the likely effective stress path. What would happen to the oil tank during this stage? [15%]

(cont.

(iii) Test 2: a drained test to simulate slow filling of the tank with water to test the tank and its foundation. This was done by increasing the axial stress, while maintaining the cell pressure constant, and the pore water pressure constant and equal to the in situ value. Plot the total and effective stress paths in $q - p$ and $q - p'$ space for the test. What was the measured drained strength at failure? What is the significance of this test? [25%]

(TURN OVER

4 A 4 m diameter tunnel is constructed in a stiff clay with its axis at a depth of 25 m. The clay has a unit weight of 20 kNm^{-3} and a constant undrained shear strength of 125 kPa. The inward radial movement measured by sub-surface instrumentation in the ground at the tunnel boundary was 20 mm. Pressure cells incorporated in the tunnel lining, which is assumed to be smooth, measured an average radial stress of 150 kPa. Assume the tunnel construction process to be an axisymmetric cylindrical cavity contracting under undrained conditions.

(a) A self-boring pressuremeter is to be used to measure the in situ properties of the ground. Outline briefly some of its advantages over laboratory testing of soil samples, giving examples of three important ground properties that could be measured. [15%]

(b) Estimate the elastic shear modulus of the clay from the tunnel measurements, showing clearly your method. [40%]

(c) A larger tunnel of 10 m diameter is to be constructed in ground with the same properties at the same depth. The average radial stress in the tunnel lining is estimated to be 200 kPa. What would the radial ground movement be at the tunnel boundary? [25%]

(d) Estimate the settlement of the ground surface for the larger tunnel. [20%]

END OF PAPER

Engineering Tripos Part IIA

3D1 & 3D2
Geotechnical Engineering

Data Book 2007-2008

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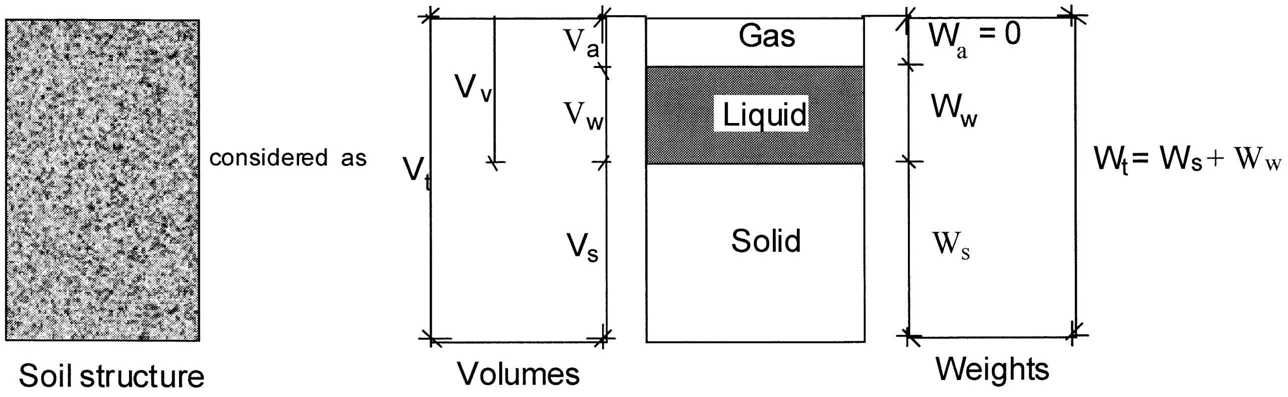
General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Porosity	$n = V_v / V_t = e/(1 + e)$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w\,G_s/e)$
Unit weight of water	$\gamma_w = 9.81\ \text{kN/m}^3$
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e}\right) \gamma_w$
Buoyant saturated unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e}\right) \gamma_w$
Unit weight of dry solids	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e}\right) \gamma_w$
Air volume ratio	$A = V_a/V_t = \left(\frac{e(1 - S_r)}{1+e}\right)$

Soil classification (BS1377)

Liquid limit w_L

Plastic Limit w_P

Plasticity Index $I_P = w_L - w_P$

Liquidity Index $I_L = \dfrac{w - w_P}{w_L - w_P}$

$$\text{Activity} = \frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2\ \mu\text{m}}$$

$$\text{Sensitivity} = \frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$$ (at the same water content)

Classification of particle sizes:–

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

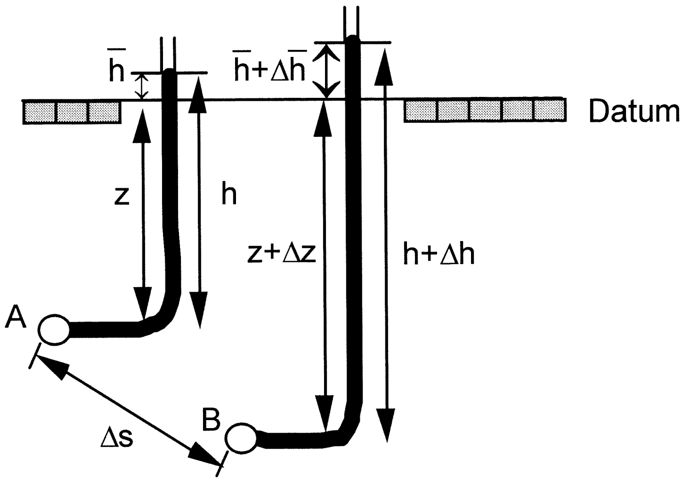
D equivalent diameter of soil particle

D_{10}, D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

C_U uniformity coefficient D_{60} / D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w(\bar{h} + z)$

B: $u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})$

Hydraulic gradient A → B $\quad i = -\dfrac{\Delta \bar{h}}{\Delta s}$

Hydraulic gradient (3D) $\quad i = -\nabla \bar{h}$

Darcy's law $\quad V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10$ mm	:	non-laminar flow
$10 \text{ mm} > D_{10} > 1 \mu\text{m}$	:	$k \cong 0.01\ (D_{10} \text{ in mm})^2$ m/s
clays	:	$k \cong 10^{-9}$ to 10^{-11} m/s

Saturated capillary zone

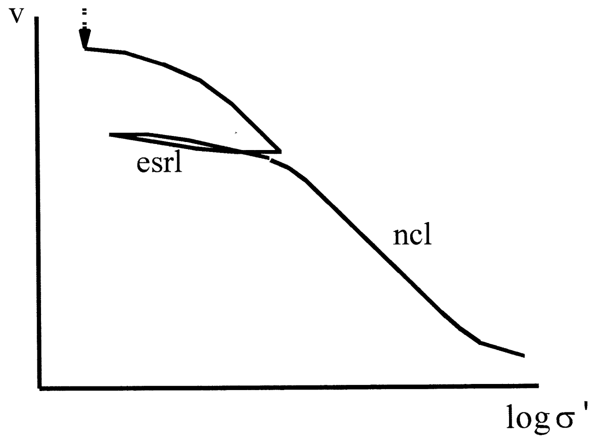
$h_c = \dfrac{4T}{\gamma_w d}$: capillary rise in tube diameter d, for surface tension T

$h_c \approx \dfrac{3 \times 10^{-5}}{D_{10}}$ m : for water at 10°C; note air entry suction is $u_c = - \gamma_w h_c$

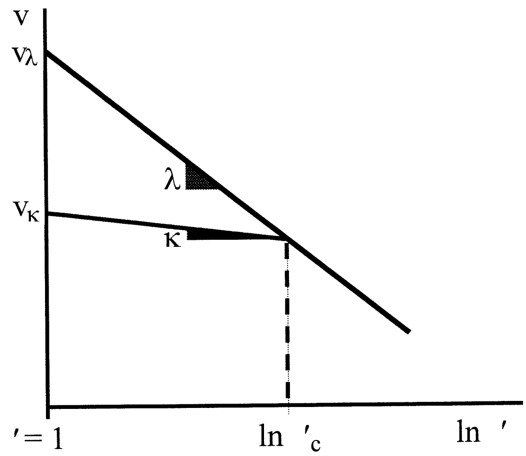
One-Dimensional Compression

• Fitting data

Typical data (sand or clay)



Mathematical model



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl): $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$

$= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for $\log_{10}$ stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta\sigma' / \Delta\varepsilon)_o$

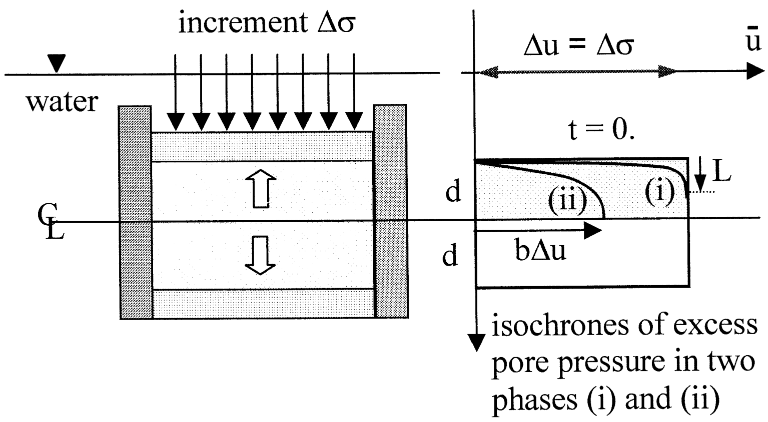
Tangent 1D plastic compression modulus $E_o = v\,\sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v\,\sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u})\, dz \quad = \int (\Delta u - \bar{u}) / E_o\, dz$
Coefficient of consolidation	c_v	$= \dfrac{k}{m_v \gamma_w} \quad = \dfrac{k E_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \dfrac{c_v t}{d^2}$
Relative settlement	R_v	$= \dfrac{\rho}{\rho_{ult}}$

- **Solutions for initially rectangular distribution of excess pore pressure**



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12\, c_v t$

$R_v = \sqrt{\dfrac{4T_v}{3}}$ for $T_v < {}^1/_{12}$

Phase (ii) $b = \exp(\tfrac{1}{4} - 3T_v)$

$R_v = [1 - {}^2/_3 \exp(\tfrac{1}{4} - 3T_v)]$ for $T_v > {}^1/_{12}$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

• Principle of effective stress (saturated soil)

total stress σ = effective stress σ' + pore water pressure u

• Principal components of stress and strain

sign convention	compression positive
total stress	σ_1, σ_2, σ_3
effective stress	σ_1', σ_2', σ_3'
strain	ε_1, ε_2, ε_3

• Simple Shear Apparatus (SSA) ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

work increment per unit volume $\delta W = \tau\,\delta\gamma + \sigma'\delta\varepsilon$

• Biaxial Apparatus - Plane Strain (BA-PS) ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ_2', in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma_1' + \sigma_3')/2 = s - u$
shear stress	$t = (\sigma_1' - \sigma_3')/2 = (\sigma_1 - \sigma_3)/2$
volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$
work increment per unit volume	$\delta W = \sigma_1'\delta\varepsilon_1 + \sigma_3'\delta\varepsilon_3$
	$\delta W = s'\delta\varepsilon_v + t\delta\varepsilon_\gamma$

providing that principal axes of strain increment and of stress coincide.

• Triaxial Apparatus – Axial Symmetry (TA-AS) (cylindrical element with radial symmetry)

total axial stress	σ_a	=	$\sigma'_a + u$
total radial stress	σ_r	=	$\sigma'_r + u$
total mean normal stress	p	=	$(\sigma_a + 2\sigma_r)/3$
effective mean normal stress	p'	=	$(\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	q	=	$\sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	η	=	q/p'
axial strain	ε_a		
radial strain	ε_r		
volumetric strain	ε_v	=	$\varepsilon_a + 2\varepsilon_r$
triaxial shear strain	ε_s	=	$\frac{2}{3}(\varepsilon_a - \varepsilon_r)$
work increment per unit volume	δW	=	$\sigma_a'\delta\varepsilon_a + 2\sigma_r'\delta\varepsilon_r$
	δW	=	$p'\delta\varepsilon_v + q\delta\varepsilon_s$

Types of triaxial test include:

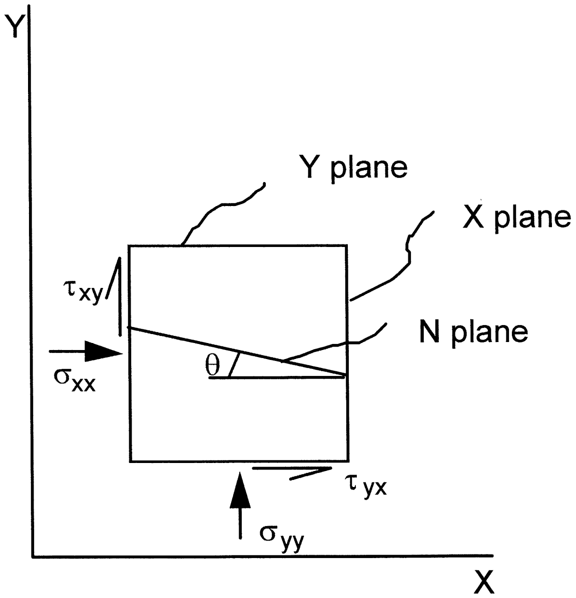
isotropic compression in which p' increases at zero q

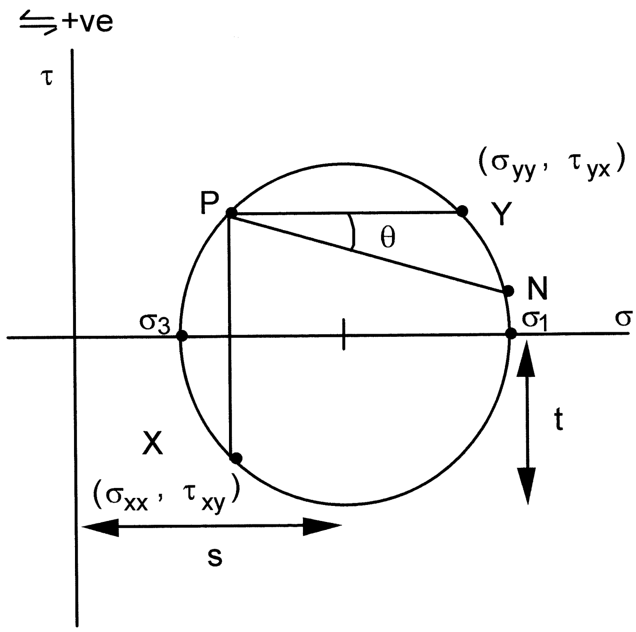
triaxial compression in which q increases *either* by increasing σ_a *or* by reducing σ_r

triaxial extension in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• Mohr's circle of stress (1–3 plane)

Sign of convention: compression, and counter-clockwise shear, positive





Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ-line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

compressibility $\quad m_v = d\varepsilon / d\sigma'$

constrained modulus $\quad E_o = 1 / m_v$

Physically fundamental parameters

shear modulus $\quad G' = dt / d\varepsilon_\gamma$

bulk modulus $\quad K' = dp' / d\varepsilon_v$

Parameters which can be used for constant-volume deformations

undrained shear modulus $\quad G_u = G'$

undrained bulk modulus $\quad K_u = \infty \quad$ (neglecting compressibility of water)

Alternative convenient parameters

Young's moduli $\quad E'$ (effective), E_u (undrained)

Poisson's ratios $\quad \nu'$ (effective), $\nu_u = 0.5$ (undrained)

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

Relationships:

$$G = \frac{E}{2\,(1+\nu)}$$

$$K = \frac{E}{3\,(1-2\nu)}$$

$$E_o = \frac{E\,(1-\nu)}{(1+\nu)\,(1-2\nu)}$$

Cam Clay

• Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan\phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin\phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

• General equations of plastic work

Plastic work and dissipation $\quad \sigma^*\,\delta\varepsilon^* + \tau^*\,\delta\gamma^* = \mu^*_{crit}\,\sigma^*\,\delta\gamma^*$

Plastic flow rule – normality $\quad \dfrac{d\tau^*}{d\sigma^*}\cdot\dfrac{d\gamma^*}{d\varepsilon^*} = -1$

• General yield surface

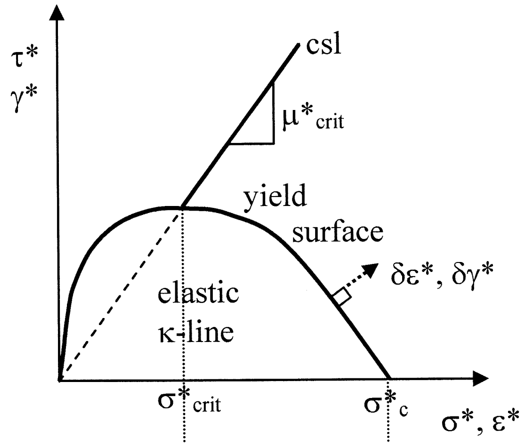
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit}\,.\ln\left[\frac{\sigma_c^*}{\sigma^*}\right]$$

• Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c,\,virgin}$ kPa	1	1	1	Loose 500	Loose 2500
				Dense 1500	Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	--------------	--------------
w_P	0.26	0.18	0.42	--------------	--------------
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^*, κ^*, Γ^*, σ^*_c should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• The yield surface in (σ*, τ*, v) space



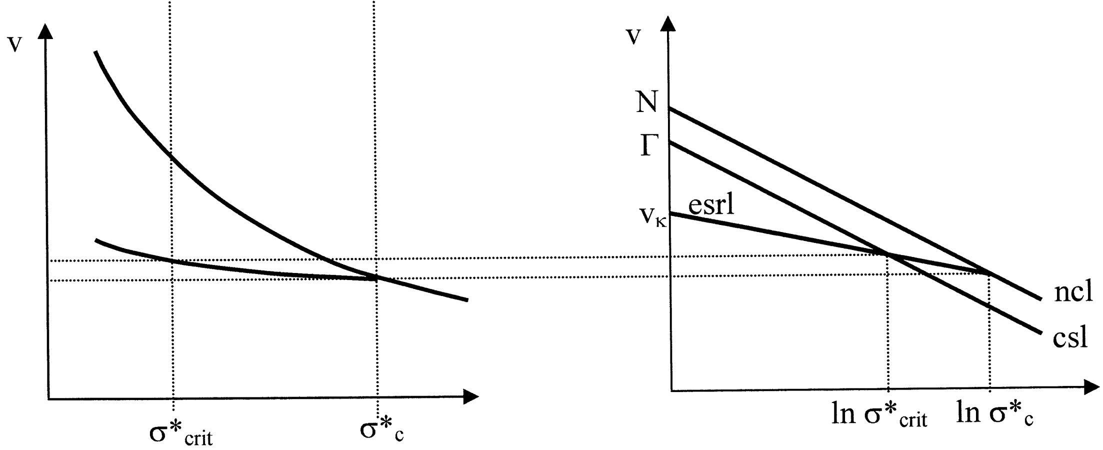
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

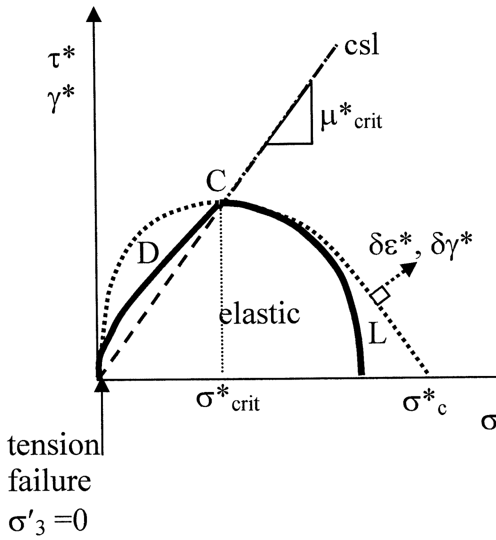
csl: critical state line

$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



• Regions of limiting soil behaviour



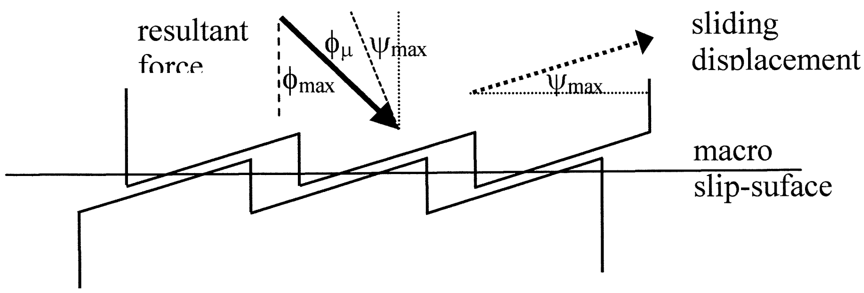
Variation of Cam Clay yield surface

Zone D: denser than critical, "dry",
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, "wet",
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

• Friction and dilatancy: the saw-blade model of direct shear

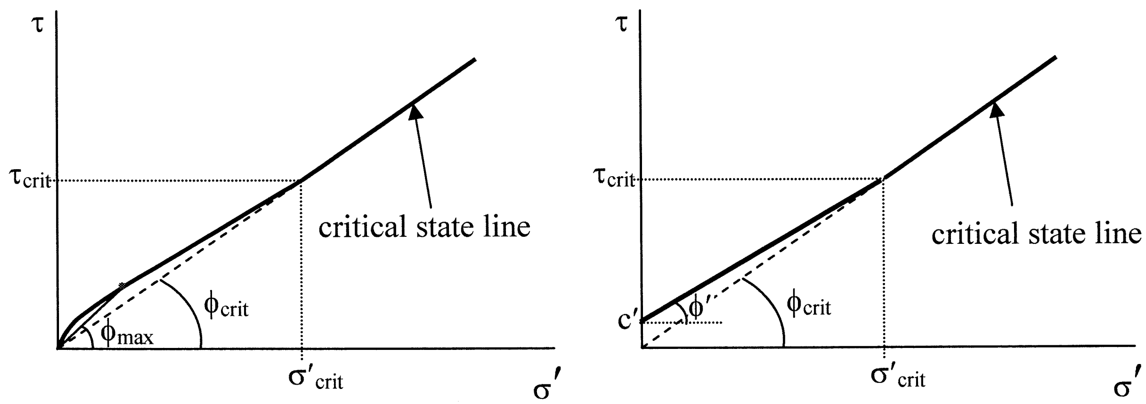


Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{max}

Angle of internal friction $\phi_{max} = \phi_\mu + \psi_{max}$

• Friction and dilatancy: secant and tangent strength parameters



Secant angle of internal friction

$\tau = \sigma' \tan \phi_{max}$
$\phi_{max} = \phi_{crit} + \Delta\phi$
$\Delta\phi = f(\sigma'_{crit}/\sigma')$

typical envelope fitting data:
power curve
$(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^\alpha$
with $\alpha \approx 0.85$

Tangent angle of shearing envelope

$\tau = c' + \sigma' \tan \phi'$
$c' = f(\sigma'_{crit})$

typical envelope:
straight line
$\tan \phi' = 0.85 \tan \phi_{crit}$
$c' = 0.15\, \tau_{crit}$

• Friction and dilation: data of sands

The inter-granular friction angle of quartz grains, $\phi_\mu \approx 26°$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of ϕ_{crit} ($\pm 2°$) are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \dfrac{(e_{max} - e)}{(e_{max} - e_{min})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test
e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{max} - \phi_{crit}) = f(I_R)$

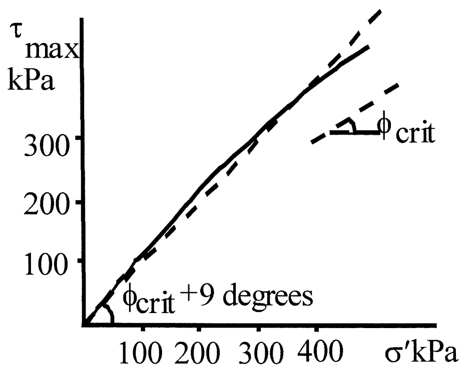
Relative dilatancy index $I_R = I_D I_C - 1$ where:

$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state
$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

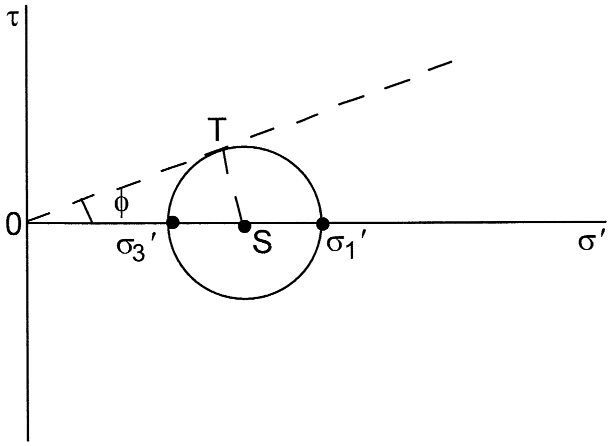
plane strain conditions	$(\phi_{max} - \phi_{crit})$	$= 0.8\ \psi_{max}$	$= 5\ I_R$ degrees
triaxial strain conditions	$(\phi_{max} - \phi_{crit})$	$= 3\ I_R$ degrees	
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{max}$	$= 0.3\ I_R$	

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$\phi_{max} > \phi_{crit} + 9°$ for $I_D = 1$, $\sigma' = 400$ kPa

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



$$\sin \phi = TS/OS = \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

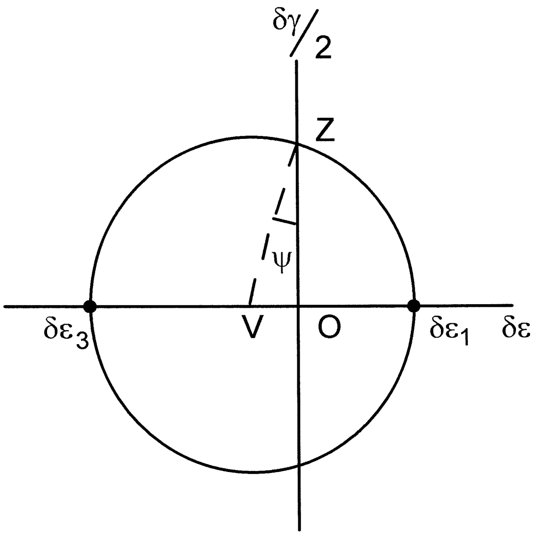
$$\left[\frac{\sigma_1'}{\sigma_3'}\right] = \frac{(1+\sin\phi)}{(1-\sin\phi)}$$

Angle of shearing resistance:

at peak strength ϕ_{max} at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\sin \psi = VO/VZ = -\frac{(\delta\varepsilon_1 + \delta\varepsilon_3)/2}{(\delta\varepsilon_1 - \delta\varepsilon_3)/2} = -\frac{\delta\varepsilon_v}{\delta\varepsilon_\gamma}$$

$$\left[\frac{\delta\varepsilon_1}{\delta\varepsilon_3}\right] = -\frac{(1-\sin\psi)}{(1+\sin\psi)}$$

at peak strength $\psi = \psi_{max}$ at $\left[\frac{\sigma_1'}{\sigma_3'}\right]_{max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

- **Limiting stresses**

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

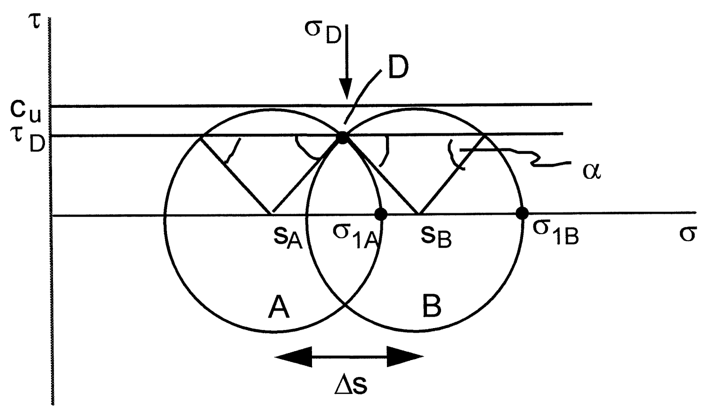
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \, \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u, this becomes

$$D = A c_u x$$

- **Stress conditions across a discontinuity**



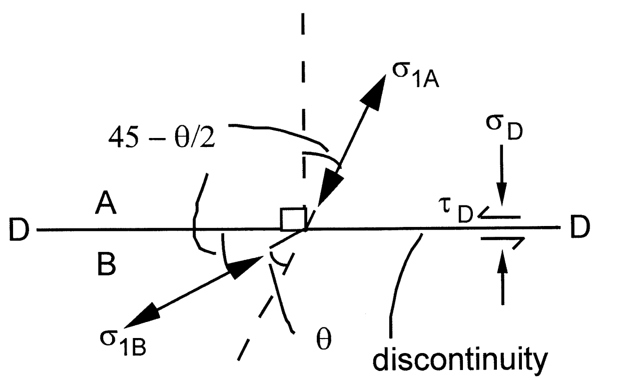
Rotation of major principal stress θ

$s_B - s_A = \Delta s = 2c_u \sin\theta$

$\sigma_{1B} - \sigma_{1A} = 2c_u \sin\theta$

In limit with $\theta \rightarrow 0$

$ds = 2c_u \, d\theta$



Useful example:

$\theta = 30^\circ$

$\sigma_{1B} - \sigma_{1A} = c_u$

$\tau_D / c_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{max} = \tan\phi$

- **Limiting stresses**

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$\sigma'_v > \sigma'_h$

$\sigma'_1 = \sigma'_v$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_h$

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

Passive pressure:

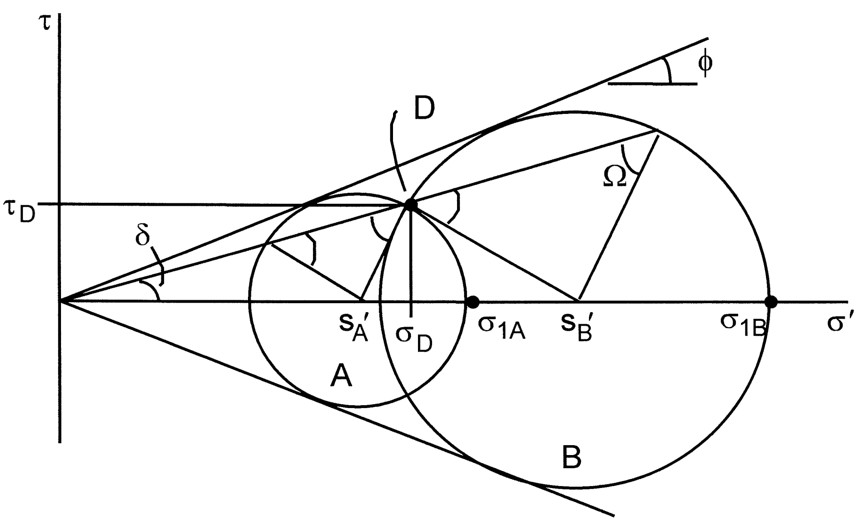
$\sigma'_h > \sigma'_v$

$\sigma'_1 = \sigma'_h$ (assuming principal stresses are horizontal and vertical)

$\sigma'_3 = \sigma'_v$

$K_p = (1 + \sin\phi)/(1 - \sin\phi) = 1/K_a$

- **Stress conditions across a discontinuity**



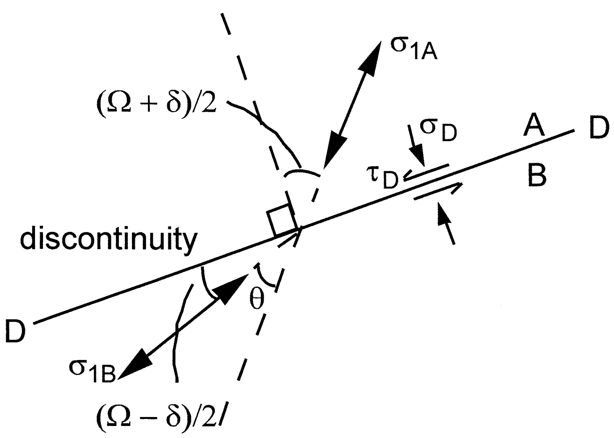
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \square\sin\delta / \sin\phi$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \to 0$ and $\delta \to \phi$

$ds' = 2s'.\, d\theta \tan\phi$

Integration gives $s'_B/s'_A = \exp(2\theta \tan\phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin\phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha}-1)}{(n_{max}-1)}\right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin\phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta\sigma_c = \sigma_c - \sigma_o$

At radius r: small displacement $\rho = \dfrac{\delta A}{2\pi r}$

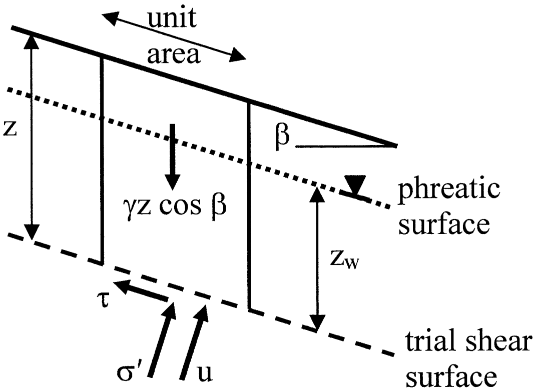
small shear strain $\gamma = \dfrac{2\rho}{r}$

Radial equilibrium: $r\dfrac{d\sigma r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta\sigma_c = G\dfrac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta\sigma_c = c_u\left[1 + \ln\dfrac{G}{c_u} + \ln\dfrac{\delta A}{A}\right]$

Infinite slope analysis



$$u = \gamma_w z_w \cos^2\beta$$
$$\sigma = \gamma z \cos^2\beta$$
$$\sigma' = (\gamma z - \gamma_w z_w)\cos^2\beta$$
$$\tau = \gamma z \cos\beta \sin\beta$$

$$\tan\phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan\beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \qquad \text{(Prandtl, 1921)}$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2\, B / L$$

The exact solution for a rough circular foundation (D = B = L) is $q_f = 6.05 s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$$d_c = 1 + 0.33 \tan^{-1}(h/B) \qquad \text{(or h/D for a circular foundation)}$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $$H = H_{ult} = B s_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0 \quad \text{(Taiebet \& Carter 2000)}$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

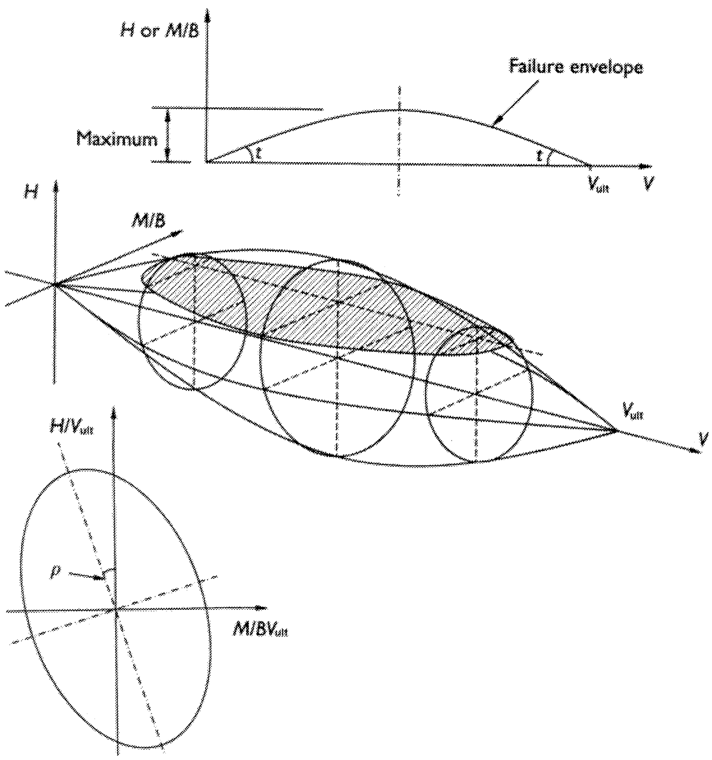
The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)} \qquad \text{(Prandtl 1921)}$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2\,(N_q - 1) \tan \phi$$



Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$
$$s_\gamma = 1 - 0.3\, B / L$$

For circular footings take L = B.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi, 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. Note that t_h is the friction coefficient, H/V= tanϕ, during sliding.

3D2 Geotechnical Engineering II

1 (a) 17.7 kN/m^3, 21 kN/m^3, assuming the unit weight of water is 10 kN/m^3

(b) $\sigma = 39.8$ kPa, $\tau = 23.0$ kPa, $\phi_{crit} = 36°$, $\phi_{peak} = 44.7°$

(c) $\sigma = 46.0$ kPa, $\tau = 26.6$ kPa, $u = 18.8$ kPa, $\sigma' = 27.2$ kPa, $\phi_{crit} = 36°$, $\phi_{peak} = 45.5°$

(d) –

2(a) (i) $M = 0.9$, $p_c = 153$ kPa, (ii) $\kappa = 0.057$, (iii) –

(b) $q_f = 54.3$ kPa

(c) $p = 423$ kPa

3 (a) $\sigma_v' = 105$ kPa, $\sigma_h' = 84$ kPa, $\sigma_{v0}' = 210$ kPa, $\sigma_{h0}' = 126$ kPa

(b) (i) Δu at yield = 13 kPa, Δu at ultimate failure = 31 kPa

(ii) –

(iii) drained strength at failure = 125 kPa

4(a) –

(b) $G = 37.8$ MN/m^2

(c) 33 mm

(d) 6.6 mm