ENGINEERING TRIPOS PART IIA

Monday 26 April 2010 9 to 10.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: None

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final version

1 (a) Show that the restrained warping torsion constant Γ for an I-beam is given by:

$$\Gamma = \frac{I_f d^2}{2}$$

Identify the meaning of each of the variables in this expression. [20%]

(b) A footbridge is made from a symmetric I-beam on its side, supported at each end of each flange as shown in Figure 1. It is made from concrete with a Young's Modulus of 30 kN mm^{-2} and a Poisson's Ratio of 0.2. Assume the concrete remains uncracked.

The beam is loaded by a force of 10 kN vertically downwards in the centre of one flange at midspan. Calculate the resulting rotation at midspan:

(i) if restrained warping effects are ignored; [40%]

(ii) if restrained warping effects are taken into account. [40%]

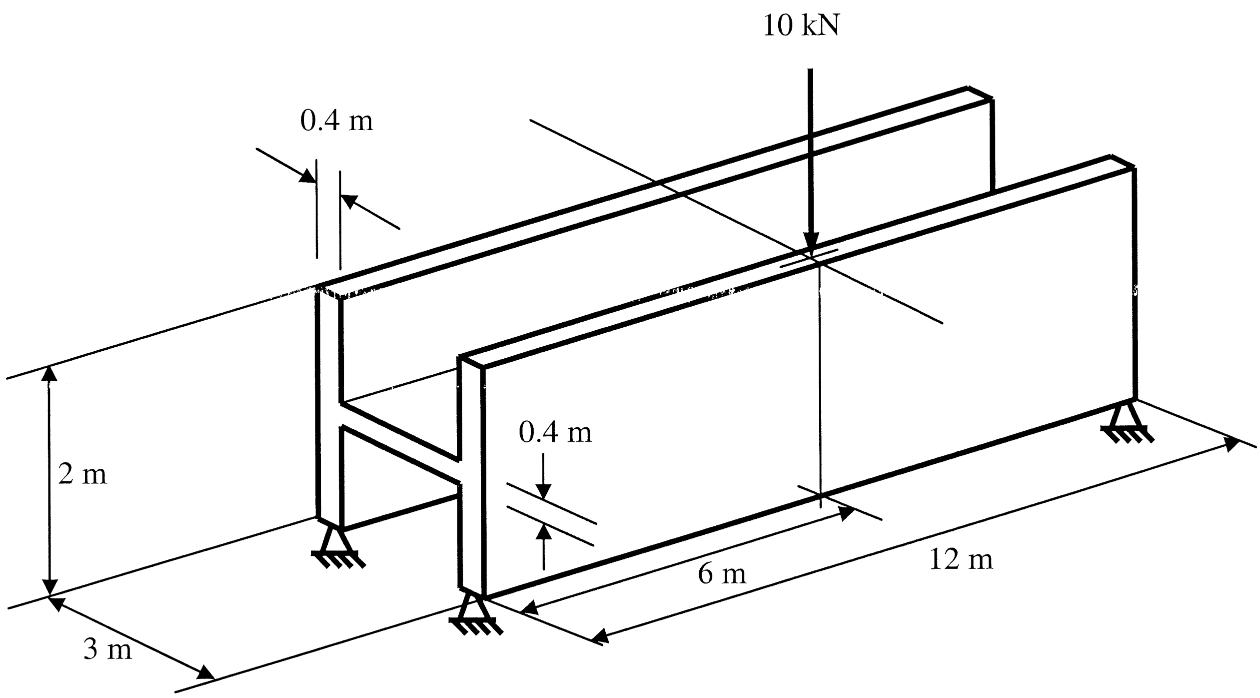

Fig.1

2 Consider the light continuous two-span beam carrying a uniformly-distributed load of 10 kN m^{-1} over its right-hand span, as shown in Fig. 2. The relevant flexural rigidity EI is 10^5 kN m^2 throughout.

(a) Show that the reaction at A is 13 kN upwards. [30%]

(b) Find an expression for the deflection everywhere in the beam. [30%]

(c) Show that the maximum downward deflection occurs at approximately $x = 11.34$ m and determine the magnitude of the deflection there. [40%]

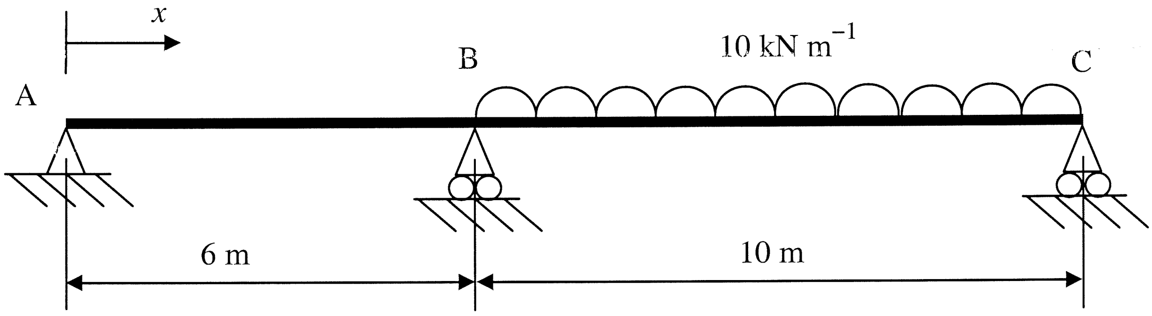

Fig. 2

(TURN OVER

3 (a) Explain why general elastic stability problems lead to eigenvalue problems, and explain the physical significance of the eigenvalues and eigenvectors. Explain why such static eigenvectors may differ from the dynamic eigenvectors encountered in structural vibration theory. [20%]

(b) What assumptions are necessary for a Rayleigh Quotient to give an upper bound on the buckling load of a structure? Sketch a simple structure where these assumptions would not be appropriate. [20%]

(c) A light, straight cantilever of height L (see Fig. 3) is loaded by an axial force P acting vertically at its tip.

(i) If the flexural rigidity decreases linearly with height x from EI at the base to $(1-\alpha)EI$ at the top (where $0 \leq \alpha < 1$), find an upper bound on the critical axial load P_{cr} by assuming that the lateral deflected shape during buckling is a quadratic (i.e. $w = ax^2$). [30%]

(ii) If the flexural rigidity is EI everywhere (i.e. $\alpha = 0$) but the foundation is not rigid, having instead a rotational stiffness $G = 3EI/L$ per unit rotation, find the least upper bound on P_{cr} associated with lateral deflections assumed to be of the form $w = ax(1 + bx/L)$, where a and b are dimensionless constants. [30%]

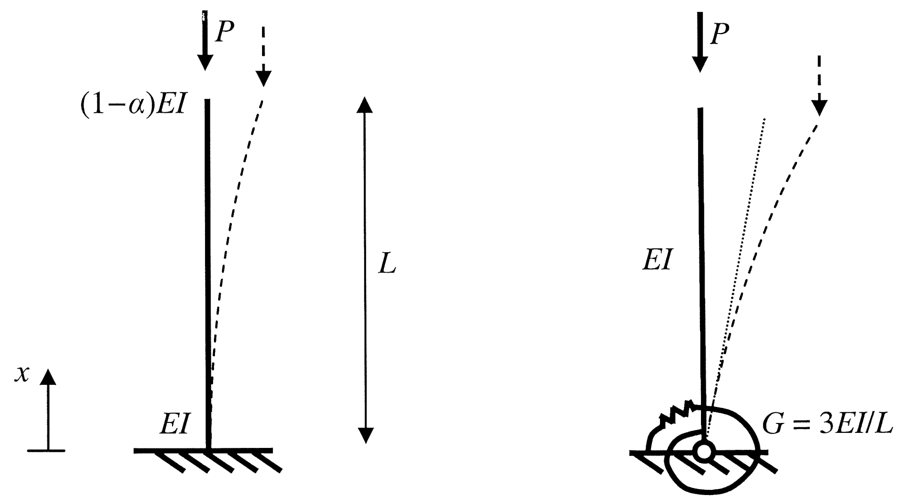

Fig. 3

4 (a) Define the *effective length* of a column, and explain how this may be affected by construction details in distant parts of the building. [20%]

(b) In the rigid-jointed steel frame shown in Figure 4, all beams are 610×305×149 UB and all columns are 305×305×118 UC. Stiff cross-bracing is provided in the right-hand bay. All webs are in the plane of the diagram. A table of *s* and *c* stability functions is provided in Fig. 5 overleaf.

(i) Determine the effective length of the left-hand column AB for major axis buckling in the plane of the diagram by assuming that the two beams that attach to it are pinned at their far ends (C and D). Determine the corresponding maximum axial load *P* that column AB can carry before it undergoes major axis buckling. [60%]

(ii) Without further calculation, explain what further effects must be considered in the design of column AB if the axial load that it carries is a result of loading along the adjacent upper beam AC. [10%]

(iii) Explain how loads applied to the lower beam BD can affect the buckling behaviour of the column AB. [10%]

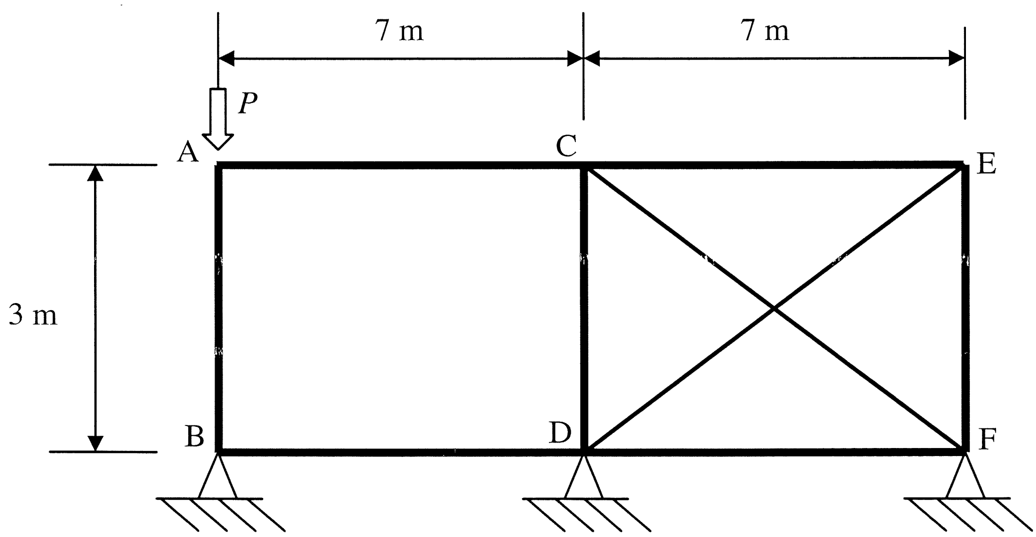

Fig. 4

(TURN OVER

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

Fig. 5. *s* and *c* stability functions

END OF PAPER