ENGINEERING TRIPOS PART IIA

Friday 23 April 2010 2.30 to 4

Module 3D5

WATER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

The values of relevant parameters are listed at the end of the data sheet unless otherwise noted in the question.

Attachment: 3D5 data sheet (5 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final version

1 (a) Briefly explain what evapotranspiration (ET) and potential evapotranspiration (PET) mean and what factors they depend on. Why can the evapotranspiration be ignored in some rainfall-runoff analyses? [20%]

(b) It is known that three hours of uniform rainfall over a small catchment produces the following distribution of runoff:

Duration	0-3 hours	3-6 hours	6-9 hours	9-12 hours	12-15 hours
Runoff distribution (%)	15	45	30	10	0

Another rainfall over the catchment lasted for 1 hour and had an intensity of 30 mm h^{-1}. Prior to this rainfall, the infiltration-related coefficients were f_0 = 20 mm h^{-1}, f_c = 5 mm h^{-1}, and K_f = 0.2 h^{-1}. If the area of the catchment is 15 km^2, estimate the peak flow rate generated by this rainfall event. [50%]

(c) For a two hour rainfall of 20 mm in total, the average flow rate over successive two hour periods at the outlet of an impervious catchment varies according to:

Duration	0-2 hours	2-4 hours	4-6 hours	6-8 hours	8-10 hours	10-12 hours
Average flow rate (m^3 s^{-1})	30	80	50	20	10	0

(i) Estimate the area of the catchment. [10%]

(ii) Estimate the flow rate (in m^3 s^{-1}) variation at the outlet of this catchment for another rainfall event given in Fig. 1. [20%]

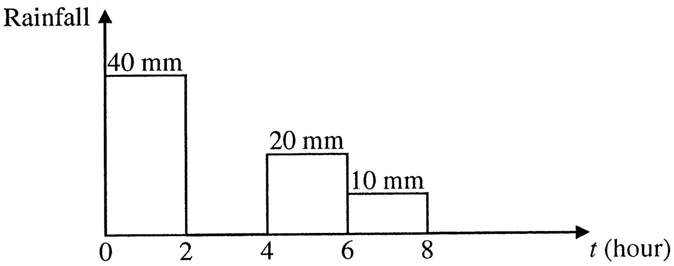


Fig. 1

2 (a) What are the units of Chézy coefficient? [10%]

(b) A flow rate of 10 m^3 s^{-1} occurs in a rectangular channel 6 m wide, lined with concrete (Manning's $n = 0.013$ s $m^{-1/3}$) and laid on a slope of 0.0001.

(i) If the flow is uniform, show that the water depth is 1.94 m. [10%]

(ii) If the flow is gradually varied and the water depth at one section is 1.5 m, calculate the distance between this section and an upstream section with the water depth of 1.6 m. Assume that the water depth changes linearly between these two sections. [40%]

(c) A very long rectangular channel has a width of 10 m and a roughness height of 0.02 m. The flow is uniform in the channel, with velocity 0.5 m s^{-1} and water depth 2 m.

(i) Calculate the bed slope. [10%]

(ii) A very high sluice gate in a mid-section of the channel is suddenly closed. Neglecting the influence of the bed slope and friction in unsteady flows, show that the water depth is 1.78 m at 50 m downstream of the sluice gate at 1 min after the closure of the sluice gate. [30%]

(TURN OVER

3 (a) Explain how the Rouse profile of suspended sediment concentration is derived. All the terms involved need to be explained fully. [30%]

(b) A very wide stream has a bed slope of 0.0008, and its bed consists of sediments with size d = 0.8 mm. The flow is uniform with water depth 0.6 m and velocity 1.0 m s^{-1}.

(i) Assuming the grain-related roughness height to be three times the sediment diameter, compute the bedload transport rate in kg s^{-1} m^{-1} using van Rijn's formula. [35%]

(ii) A factory continuously discharges water at a rate of 0.1 m^3 s^{-1} into the stream through a vertical line source. The discharged water contains a pollutant at a concentration of 50 kg m^{-3}. As shown in Fig. 2, the vertical line source is 2 m away from one river bank. Neglecting the discharge momentum and the influence of the other bank of the river, calculate the pollutant concentration 20 m downstream of the opening and 0.5 m from the river bank. [35%]

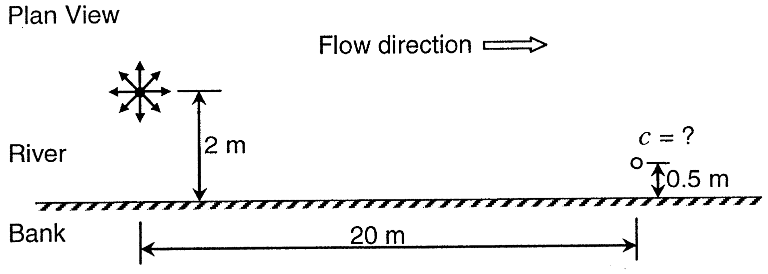


Fig. 2

4 (a) The distance, Z_W (m), from the water level in an abstraction well to the ground surface can be calculated from the following equation:

$$Z_W = 5.0 + 10 \times Q \times \ln(25000 \times Q)$$

where Q is the abstraction rate ($m^3\ s^{-1}$). Water is pumped from the well to a reservoir that is 10 m above the ground. The pipeline is 800 m long in total, and has a diameter of 200 mm and a roughness height of 0.2 mm. Allow $10 \cdot \left[U^2/(2g)\right]$ for all the local head losses, including inlet, bends and outlet, *etc.* The pump characteristics are as follows.

Q_p (litre s^{-1})	0	20	40	60	80	100
H_p (m)	60	58	52	41	25	7
η_p (%)	0	44	65	64	48	20

If such a pump is used to abstract water from this well to the reservoir, show that the flow rate is around 63 litre s^{-1} and calculate the power consumption. [55%]

(b) In the system described in part (a), a second identical pump is installed.

(i) Determine the unregulated discharge produced by connecting the pumps in parallel. [15%]

(ii) If the target discharge is 70 litre s^{-1} and the discharge is regulated by a valve, calculate the power consumption when two pumps are connected in series. [20%]

(c) Briefly describe the process of water treatment. [10%]

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

Engineering Tripos Part IIA

Third Year

Module 3D5: Water Engineering
Data Sheet (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity)

$$f = f_c + (f_0 - f_c)e^{-K_f t}$$

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flow with a free surface (uniform flow in a very wide channel):

Eddy viscosity coefficient $$\nu_t = \kappa u_* z\left(1 - \frac{z}{h}\right)$$

Velocity in hydraulically smooth regime ($u_* k_s/\nu < 5$) $$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$$

Velocity in hydraulically rough regime ($u_* k_s/\nu > 70$) $$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$$

Open Channel Flow

Chézy coefficient in large Reynolds-number flows $$C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$$

Froude number for rectangular channels $$Fr = \frac{U}{\sqrt{gh}}$$

Steady flow momentum equation $$\sum F = \rho Q(U_{out} - U_{in})$$

Bed shear stress $$\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$$

Uniform flows:

Chézy formula $$U = C\sqrt{R_h S_b}$$

Manning formula $$U = \frac{1}{n} \cdot R_h^{1/6}\sqrt{R_h S_b} = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2}$$

Gradually varied flows: $$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

or $$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \dfrac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \dfrac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$$

Characteristics for unsteady flows in rectangular channels:

$$\frac{d}{dt}\left(U+2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U+\sqrt{gh}$$

$$\frac{d}{dt}\left(U-2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U-\sqrt{gh}$$

Pollutant Transport

Analytical values of the mixing coefficients: $D_{tx}=D_{ty}=0.15hu_*$, $D_{tz}=0.067hu_*$, $D_L=5.86hu_*$

For instantaneous release from origin at $t = 0$ in uniform flows along x direction:

One-dimensional $$c(x,t)=\frac{M/A}{\sqrt{4\pi D_x}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)$$

Two-dimensional $$c(x,y,t)=\frac{M/h}{4\pi t\sqrt{D_x D_y}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}-\frac{y^2}{4D_y t}\right)$$

Three-dimensional $$c(x,y,z,t)=\frac{M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}\exp\left(-\frac{(x-Ut)^2}{4D_x t}-\frac{y^2}{4D_y t}-\frac{z^2}{4D_z t}\right)$$

For continuous release from origin in uniform flows along x direction:

Two-dimensional $$c(x,y)=\frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Three-dimensional $$c(x,y,z)=\frac{\dot{M}}{4\pi x\sqrt{D_y D_z}}\exp\left(-\frac{y^2}{4D_y\, x/U}-\frac{z^2}{4D_z\, x/U}\right)$$

Sediment Transport

Definitions of Shields parameter, non-dimensional grain size and transport stage parameter:

$$\theta=\frac{\tau_b}{g(\rho_s-\rho)d},\quad d_*=d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3},\quad T=\frac{\tau_b'-\tau_{bc}}{\tau_{bc}}=\frac{\theta'-\theta_c}{\theta_c}$$

Critical Shields parameter $$\theta_c=\frac{0.30}{1+1.2d_*}+0.055\left[1-\exp(-0.02d_*)\right]$$

Fall velocity $$w_s=\frac{\nu}{d}\left[\sqrt{10.36^2+1.049\cdot d_*^3}-10.36\right]$$

Shear stress partition: $$C'=7.8\ln\left(\frac{12h}{k_s'}\right),\quad \tau_b'=\rho g\frac{U^2}{C'^2}$$

$$C=7.8\ln\left(\frac{12h}{k_s}\right),\quad \tau_b=\rho g\frac{U^2}{C^2}$$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=8\left[\left(\frac{C}{C'}\right)^{1.5}\theta-0.047\right]^{1.5}$$

van Rijn $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=0.053\frac{T^{2.1}}{d_*^{0.3}}$$

Rouse profile of suspended sediment concentration $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

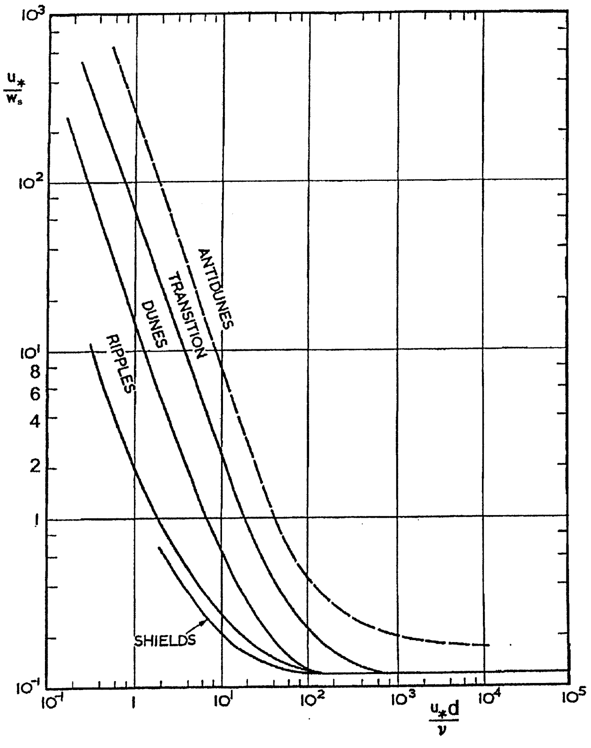
Reference volumetric concentration close to the bed:

Zyserman and Fredsøe $$\bar{c}(2d) = \frac{0.331\cdot(\theta'-0.045)^{1.75}}{1+0.72\cdot(\theta'-0.045)^{1.75}}$$

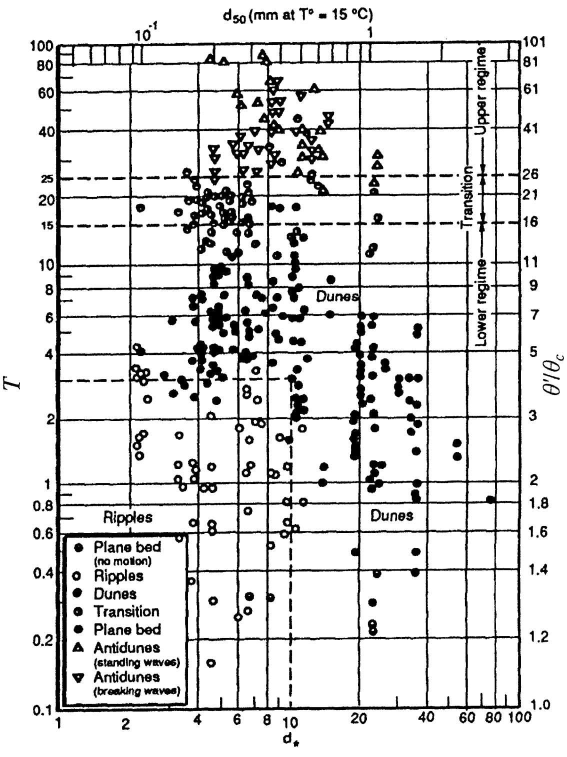
van Rijn $$\bar{c}(a) = 0.015\frac{d\cdot T^{1.5}}{a\cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right)+I_2\right]$$

a/h	$w_s/(\kappa u_*) = 0.2$		$w_s/(\kappa u_*) = 0.6$		$w_s/(\kappa u_*) = 1.0$		$w_s/(\kappa u_*) = 1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

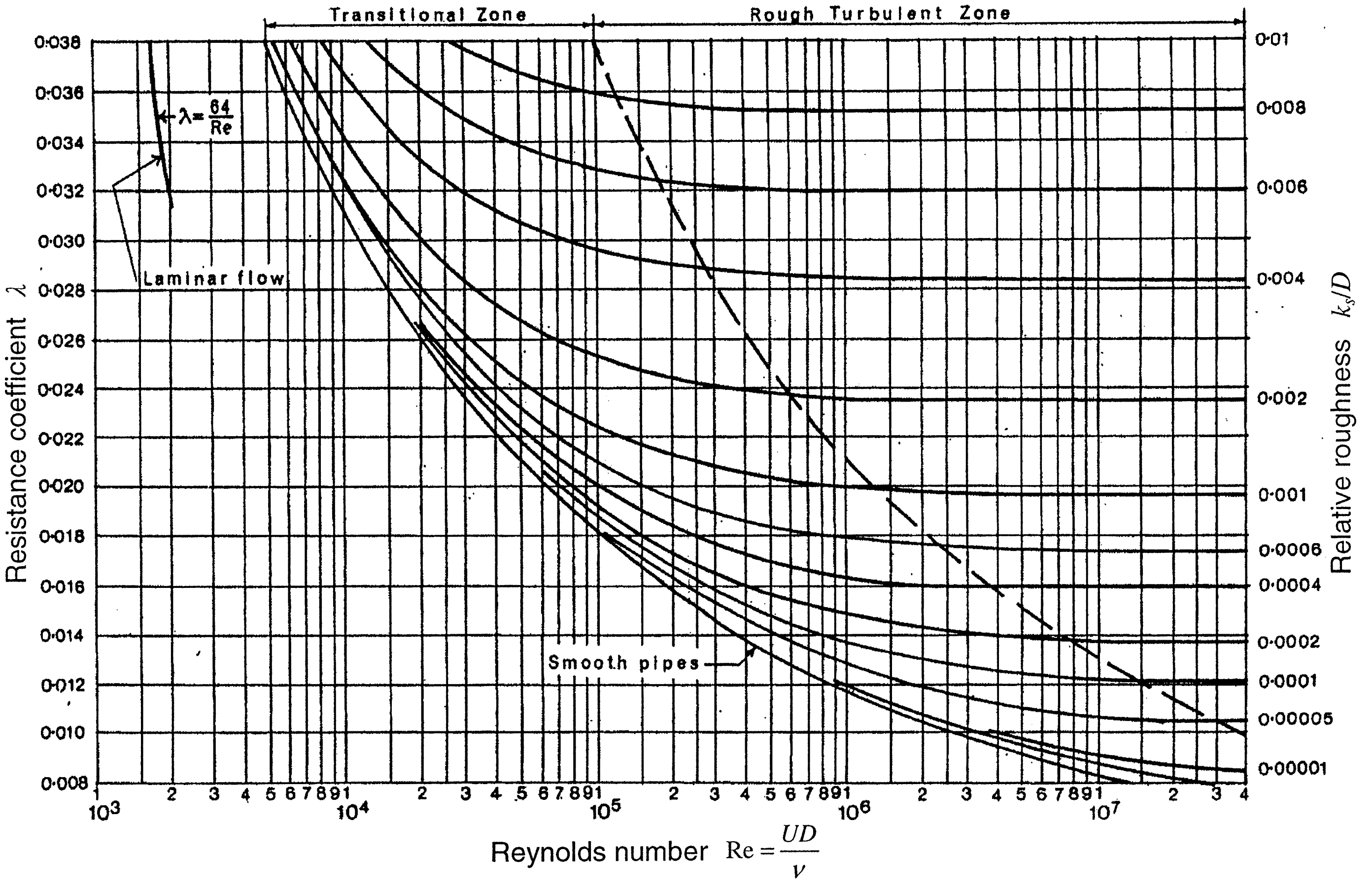
Darcy-Weisbach Equation $H_f = \lambda \frac{L}{D}\frac{U^2}{2g}$

Colebrook-White formula $\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$ with $\mathrm{Re} = \frac{UD}{\nu}$

Power consumption $P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\frac{Q_p}{N_p \cdot D_p^{\,3}},\ \frac{gH_p}{N_p^{\,2} \cdot D_p^{\,2}},\ \frac{P_p}{\rho \cdot N_p^{\,3} \cdot D_p^{\,5}}$

Specific speed $N_S = \frac{N_p \cdot Q_p^{\,1/2}}{H_p^{\,3/4}}$



Symbols

A	area
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient
c	concentration

D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x,D_y,D_z	diffusion coefficients in x, y and z directions respectively
D_{tx},D_{ty},D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.8 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	rate of decrease of f capacity
k_s	roughness height (also called equivalent or Nikuradse's sand roughness height)
M	amount of the pollutant released
$\dot{M}$	rate of the pollutant release
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity
u_*	shear velocity
w_s	fall velocity
x,y,z	spatial coordinates
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
$\overline{}$	Reynolds-averaged value
$'$	effective shear-stress component (also called grain-related shear-stress component)
$_{\mathrm{p}}$	pump

D Liang, Jan 2010

2010 IIA 3D5 WATER ENGINEERING
List of Numerical Answers

Q1. (b) 7.6 $\mathrm{m^3\ s^{-1}}$
(c) (i) 68.4 $\mathrm{km^2}$
(ii)

Time Interval (hour)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Flow rate at outlet ($\mathrm{m^3\ s^{-1}}$)	60	160	130	135	110	45	20	5	0

Q2. (a) $\mathrm{m^{1/2}\ s^{-1}}$
(b) (ii) 1021 m
(c) (i) 6.3×10^{-5}

Q3. (b) (i) 0.17 $\mathrm{kg\ s^{-1}\ m^{-1}}$
(ii) 7.15×10^{-2} $\mathrm{kg\ m^{-3}}$

Q4. (a) 38.5 kW
(b) (i) 81 litre $\mathrm{s^{-1}}$
(ii) 79.9 kW