ENGINEERING TRIPOS PART IIA

Monday 3 May 2010 9.00-10.30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 (a) A company is considering the introduction of a new product that is believed to have a 50-50 chance of being successful. One option is to try out the product in a test market, at a cost of £2 million, before making the introduction decision. Past experience shows that ultimately successful products are approved in the test market 80 percent of the time, whereas ultimately unsuccessful products are approved in the test market only 25 percent of the time. If the product is successful, the net profit to the company will be £40 million; if unsuccessful, the net loss will be £15 million.

(i) Discarding the option of trying out the product in a test market, develop a decision analysis formulation of the problem by identifying the decision alternatives, states of nature, and payoff table. Then find the optimal decision based on the expected value criterion. [10%]

(ii) Again, discarding the option of trying out the product in a test market, construct the regret payoff table, and find the optimal decision based on the minimax regret rule. [10%]

(iii) Calculate the Expected Value of Perfect Information (EVPI) for the above problem. [10%]

(iv) Calculate the posterior probabilities of the product being successful contingent on whether the product is approved or not in the test market. [20%]

(v) Now include the option of trying out the product in a test market. Construct an appropriate decision tree to determine the optimal course of action for this problem. What is the Expected Value of Sample Information (EVSI)? [20%]

- Hint: recall Bayes' theorem: $P(A_i|B) = P(A_i) \times P(B|A_i) / (\Sigma [P(A_i) \times P(B|A_i)])$

(b) Explain briefly the following concepts:

(i) Risk aversion. [10%]
(ii) Certainty Equivalent. [10%]
(iii) Risk Premium. [10%]

2 (a) Consider the Markov chain $\{X_n\}$ on the state space $S=\{0,1,2\}$ running according to the transition probability matrix

$$\mathbf{P} = \begin{Vmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \\ 3/4 & 1/4 & 0 \end{Vmatrix}$$

(i) If, initially, we are equally likely to start in any state, find $P(X_1=0)$. [10%]
(ii) Find $P(X_2=1 | X_0=0)$. [10%]
(iii) What is the long run percentage of time that the chain is in state 1? [15%]
(iv) If the chain starts in state 0, what is the expected number of steps until the chain hits state 1? [15%]

(b) Suppose that the probability it rains today is 0.3 if neither of the last two days was rainy, but 0.6 if at least one of the last two days was rainy.
(i) Set this problem up as a four state Markov chain. [20%]
(ii) What is the probability that it will rain on Wednesday given that it did not rain on Sunday or Monday? [15%]
(iii) Find the stationary distribution for this chain. [15%]

3 (a) A computer manufacturer wants to better understand how much consumers will value its new computer systems. More specifically, the management team would like to get an estimate about consumers' willingness to pay to purchase the new computer model contingent on some of its technical specifications (processing speed, weight, number of processors, type). After randomly selecting and interviewing 180 potential consumers, you run a multiple regression analysis on this data using the variables SPEED, WEIGHT, PROC, TYPE to predict PRICE, which produces the following report.

Definition of variables:

PRICE = consumer's willingness to pay (in £)
SPEED = the total processing speed of the computer (in MHz)
WEIGHT = the weight of the computer (in Kg)
PROC = number of processors
TYPE = 0 if laptop, 1 if desktop.

Multiple R	0.881
R Square	0.776
Standard Error	86.772
Observations	180

	Coefficients	Standard Error	t-Stat	Lower 95%	Upper 95%
Intercept	180.833	531.862	0.340	-871.169	1232.834
SPEED	0.057	0.015	3.799	0.028	0.086
WEIGHT	-35.71	20.62	-1.732	-76.50	5.08
PROC	45.421	48.840	0.930	-51.183	142.024
TYPE	-175.341	74.171	-2.364	-322.048	-28.634

(i) What evidence, if any, suggests that this is a good regression model? In other words, what would you consider as the strengths of this model? [15%]

(ii) What evidence, if any, suggests that there might be problems with the model? In other words, what seem to be the weaknesses of this model? What might you do to improve the current model if you think such improvement is necessary and possible? [20%]

(iii) The coefficient for SPEED is estimated to be 0.057. What is your interpretation of this number? [15%]

(iv) The company is considering developing a laptop with three processors, which will weigh 4 Kg and will have a processing speed of 3000 MHz. The marketing department suggested a price of £400. Do you find their suggestion consistent with the prediction of the regression model? [20%]

(b) (i) Standard deviation is often used as an indication of the risk associated with an investment. Give an example where you would consider it a good indication and also an example where you would consider it a bad indication. Explain briefly why for each case. [10%]

(ii) Suppose you observe the following situation:
Stock A: β_A=1.8 and expected return E[A]=22.00%
Stock B: β_B=1.6 and expected return E[B]=20.44%
If the risk-free rate of return is 7%, calculate the slope of the Security Market Line (SML) for each stock (also called the reward-to-risk ratio) and interpret their meaning. What would you expect to observe about those SMLs in such a situation?

[20%]

4 (a) A queuing system with 4 servers is observed for a long period of time, and data are collected on the proportion of time the system is in each of its possible states. The capacity of the system is limited, so whenever there is an arrival when 6 customers are present in the system (four customers being served and two waiting in queue), the arriving customer balks and goes somewhere else for service. For n=0, 1, 2, 3, 4, 5, 6 the steady state probability π_n that exactly n customers are present in the system is π_0=0.05, π_1=0.15, π_2=0.25, π_3=0.25, π_4=0.15, π_5=0.10, π_6=0.05. Assuming an infinite calling population.

(i) What is the probability that all servers are idle? [10%]

(ii) What is the probability that a customer will not have to wait? [15%]

(iii) What is the expected number of customers in the queue? [15%]

(b) A service station has one gasoline pump. Cars wanting gasoline arrive according to a Poisson process at a mean rate of 15 per hour. If the pump is already being used, however, these potential customers may balk (drive on to another service station). In particular, if there are n cars already at the service station, the probability that an arriving potential customers will balk is n/3 for n=1,2,3. The time required to fill a car has exponential distribution with a mean of 4 minutes.

(i) Construct the rate diagram for this queuing system. That is, a diagram that shows the possible states with the corresponding transition rates. [15%]

(ii) Develop the balance equations. [15%]

(iii) Solve these equations to find the steady state probability distribution of the number of cars at the station. [15%]

(iv) Find the expected number of cars in the station. [15%]

END OF PAPER