ENGINEERING TRIPOS PART IIA

Monday 19 April 2010 2.30 to 4.00

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer ***one*** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

Supplementary page with colour versions of Figs. 3 and 4 for Question 5.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A *Medical Image Acquisition*

*Answer **one** question from this section*

1 (a) Describe the additive algebraic reconstruction technique (AART) for computed tomography and give the equation that governs the algorithm. Explain the role of the relaxation factor. [25%]

(b) An X-ray phantom consists of twelve homogeneous rods with unknown X-ray linear attenuation coefficients. The phantom is exposed to X-rays from four different directions. Let $Q = -\ln(I/I_0)$, where I_0 is the X-ray intensity incident on the phantom and I is the transmitted intensity. Q is recorded for various paths through the phantom and the values observed are shown in Fig. 1. For clarity, the phantom has been drawn four times in the same orientation.

(i) Explain how to choose the order in which the Q values are used to give the fastest convergence. [10%]

(ii) Show how the additive algebraic reconstruction technique can be used to calculate the relative X-ray linear attenuation coefficients of the rods. Start by assuming that all the rods have an attenuation coefficient of 5.0, work to 3 decimal places and use a relaxation factor of 1. Ensure you show all your working for the first update in one direction. You should update the estimates of the rod attenuation coefficients once based on each value of Q provided in Fig. 1. [65%]

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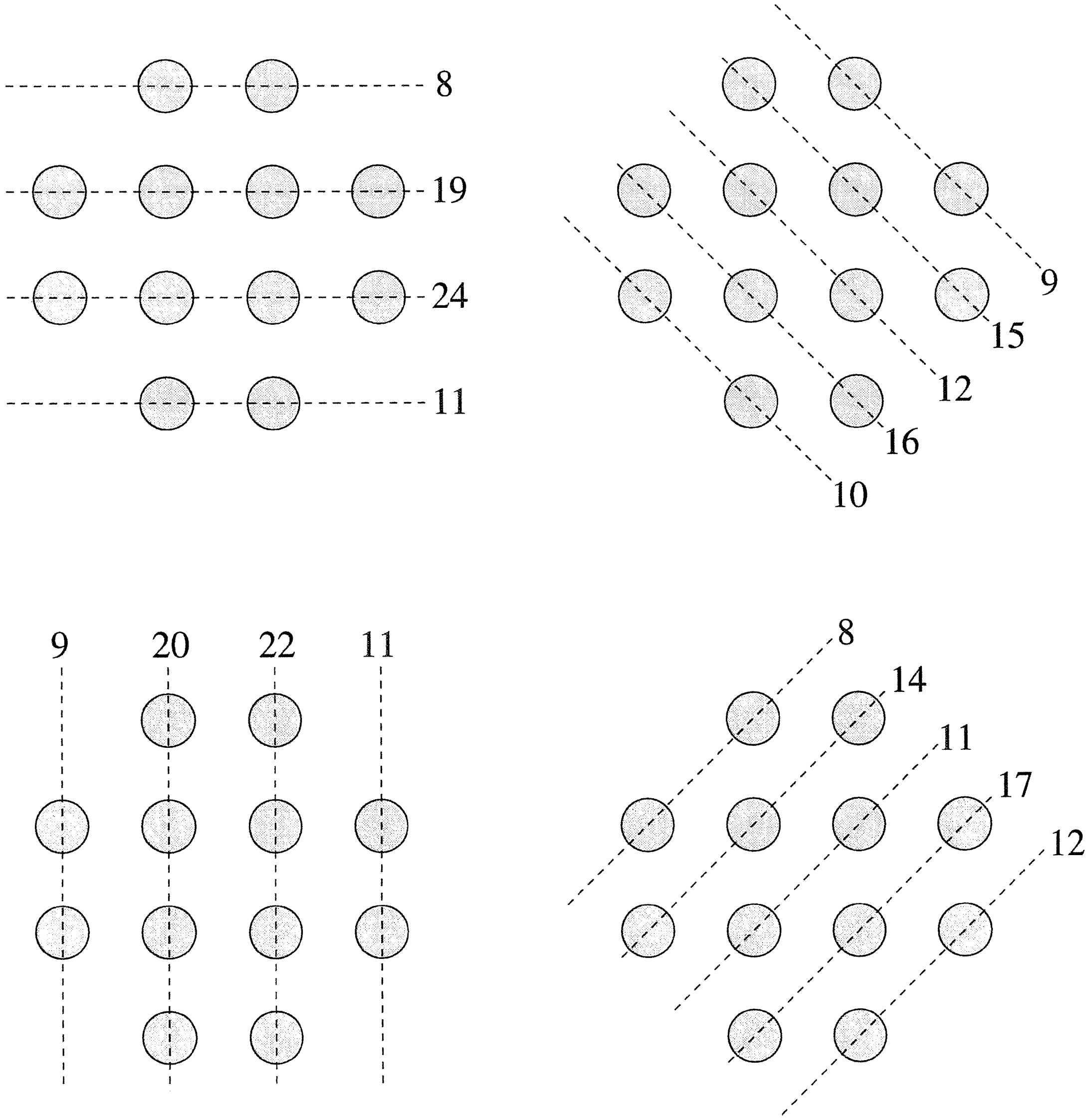

Fig. 1

2 (a) Calculate the Radon transform of the two-dimensional function

$$f(x,y) = \begin{cases} 1 & \text{if } \sqrt{x^2+y^2} \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$ [20%]

(b) Describe the three material properties, T_1, T_2 and PD, that are measured using magnetic resonance imaging. [20%]

(i) Explain why it is hard to determine these properties by measuring the output of *free induction decay.* [20%]

(ii) Describe how the *spin-echo* sequence addresses this problem. [20%]

(iii) Explain how the timing of the *spin-echo* sequence can be configured to produce a T_2-weighted image. [20%]

SECTION B *Curves, Surfaces and Interpolation*

Answer ***one*** *question from this section*

3 (a) List the advantages and disadvantages of using Nearest Neighbour interpolation, Tri-linear interpolation, B-spline approximation or Radial Basis Functions for interpolating or approximating three-dimensional scalar data. [30%]

(b) A one-dimensional data set has values $F(x) = (0,0,0,0,1,1,1,1)$ at discrete locations $x = (0,1,2,3,4,5,6,7)$. A piecewise function S is fitted to this data, such that each segment from x to $x+1$ is defined by:

$$S(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \mathbf{M} \begin{bmatrix} F(x-1) \\ F(x) \\ F(x+1) \\ F(x+2) \end{bmatrix}$$

where $0 \le t \le 1$. Both the B-spline ($\mathbf{M} = \mathbf{M}_s$) and Catmull-Rom spline ($\mathbf{M} = \mathbf{M}_{cr}$) are investigated, where:

$$\mathbf{M}_s = \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}, \quad \mathbf{M}_{cr} = \frac{1}{2} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

(i) By considering the $2 \le x \le 3$ segment, or otherwise, find the maximum and minimum values of S for each type of spline. [20%]

(ii) What are the gradients of S at the centre of the step ($x = 3.5$) for each type of spline, and how do these compare with the gradient if linear interpolation had been used? [20%]

(iii) On a single graph, sketch S over the range $1 \le x \le 6$, for both types of spline and for linear interpolation. Ensure you also mark the original sampled data $F(x)$. [20%]

(c) Comment on the relative merits of using B-splines and Catmull-Rom splines for interpolating scalar data. [10%]

4 (a) Marching Cubes can be used to extract an accurate polygonal surface from a three-dimensional medical data set. Describe, with appropriate diagrams, the topological problems associated with this technique. [30%]

(b) The surface of a real object can be digitised by scanning it with a laser beam and observing the reflection with a digital camera. List four sources of error when using this technique, briefly suggesting what can be done to reduce each error. [20%]

(c) Figure 2 shows a flat surface which is being digitised by the technique outlined in (b). The laser beam has a direction orthogonal to the surface, with uniform intensity over a thickness t which is just over 1 mm. The reflected light to the camera is at an angle θ to this direction, and the camera is far enough from the surface to assume θ is constant over x. The surface has uniform reflectance except for a patch of width 1 mm which does not reflect laser light. The location of the reflected beam is assumed to be at the centre of the light observed by the camera.

(i) Sketch the error in the z coordinate of the surface with x when scanning over the surface patch and surrounding surface. Mark the location of the patch on your sketch. What is the maximum error in terms of t and θ? [20%]

(ii) Repeat the sketch in (i) with t reduced to 0.5 mm. How feasible is this modification? [15%]

(iii) Repeat the sketch in (i) with t remaining at just over 1 mm, but instead increasing θ by a factor of two. What are the disadvantages of this new modification? [15%]

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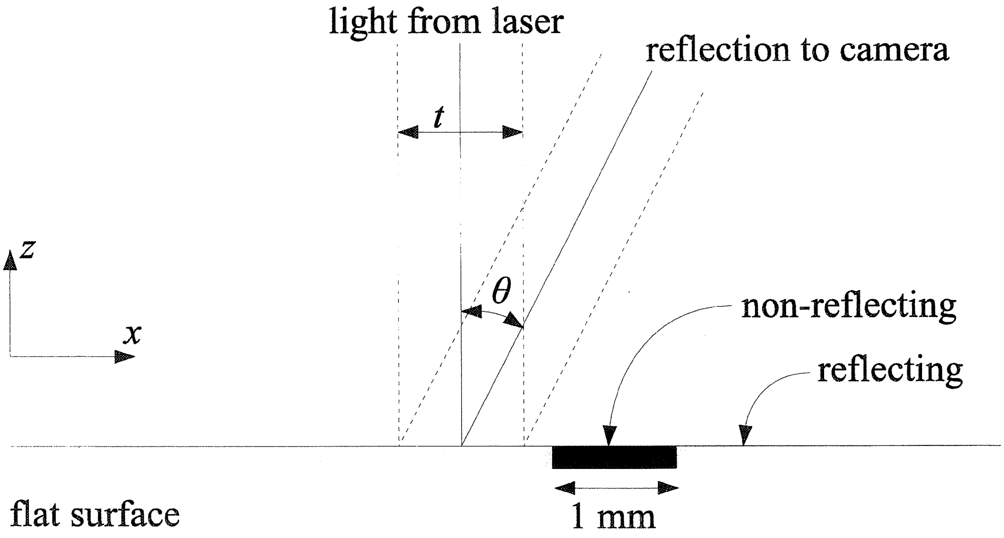

Fig. 2

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SECTION C *3D Graphical Rendering*

Answer ***one*** *question from this section*

5 (a) Draw a block diagram of the surface rendering pipeline. Show in each block the operations performed at that stage, and the coordinate systems used. [25%]

(b) Distinguish between *hidden surface removal* and *back face culling.* When is it appropriate to employ the latter? [25%]

(c) Figure 3 shows some snapshots of an OpenGL Mars Lander simulator developed for Part I of the Engineering Tripos. A colour version of this figure is provided in the attachment. Figure 3 (a) is a correct rendering, showing the lander (bright cone) firing its thruster (tip-down cone beneath the lander) as it approaches the surface. A four-panel parachute is deployed above the lander. What graphics programming errors most likely led to the incorrect renderings in:

(i) Fig. 3 (b) — two of the parachute panels are missing? [10%]

(ii) Fig. 3 (c) — the shading of the lander is incorrect? [10%]

(iii) Fig. 3 (d) — only the back of the lander is rendered? [10%]

(d) Figure 4 (a) is another snapshot from the simulator, this time showing the planet from a distance. A colour version of this figure is provided in the attachment. Explain carefully how the image in Fig. 4 (b) is used in the rendering of Fig. 4 (a). [20%]

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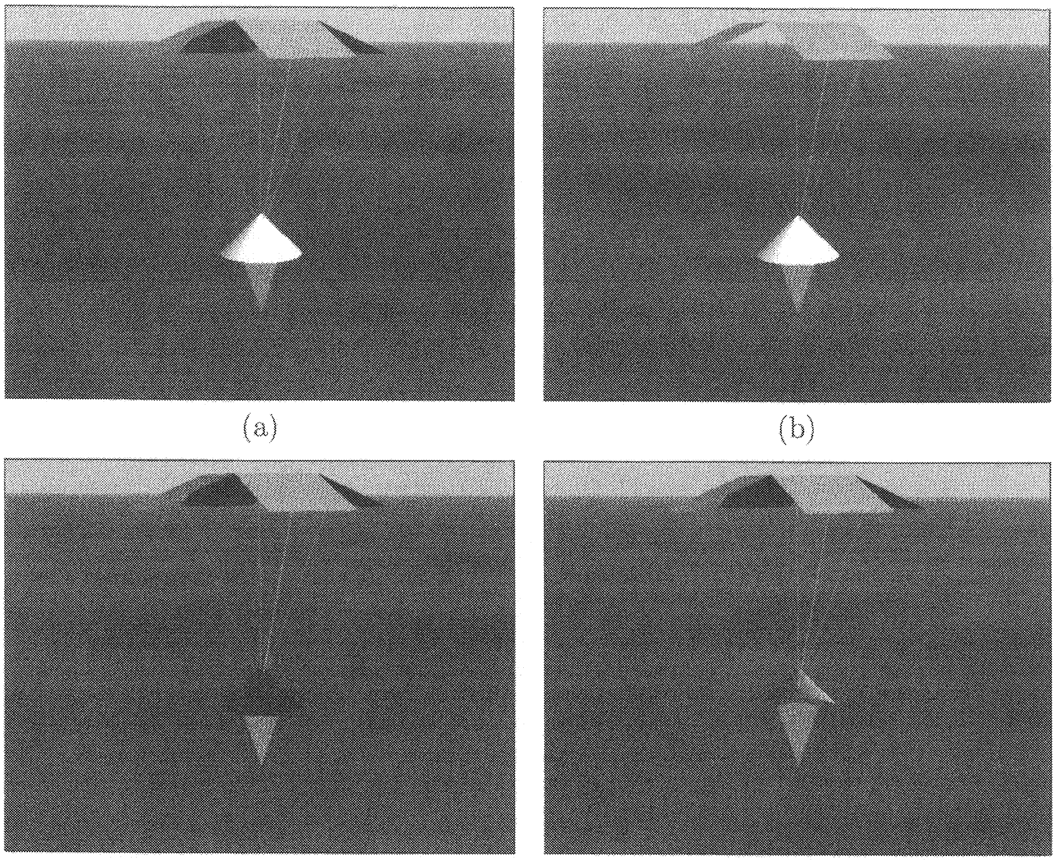
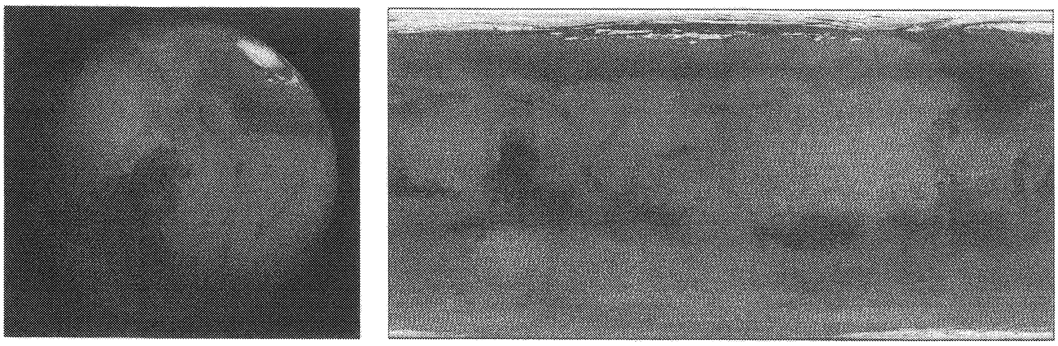


(a) (b)

(c) (d)

Fig. 3



(a) (b)

Fig. 4

(TURN OVER

6 (a) The basic Phong reflection model can be written as

$$I_\lambda = c_\lambda(I_a k_a + I_p k_d \mathbf{L}.\mathbf{N}) + I_p k_s \left(\mathbf{R}.\mathbf{V}\right)^n$$

Explain the meanings of the various terms. [20%]

(b) Distinguish between *Gouraud shading* and *Phong shading.* What are the relative strengths and weaknesses of the two techniques? [30%]

(c) One way to speed up Phong shading is to use unnormalised surface normals in the lighting calculations. Will this result in pixels that are too bright or too dark (justify your answer)? Comment on the likely acceptability of the results for both the diffuse and specular terms. [20%]

(d) A particular graphics hardware card features a rasterisation unit that interpolates four scalar values from vertices to pixels. Additionally, parallel processing units (referred to as *pixel shaders*) perform user-programmable operations on these interpolated values to calculate intensity values at each pixel. Describe how you might program the pixel shaders to achieve real-time Phong shading. What assumptions would you need to make about the illumination and the viewpoint? [30%]

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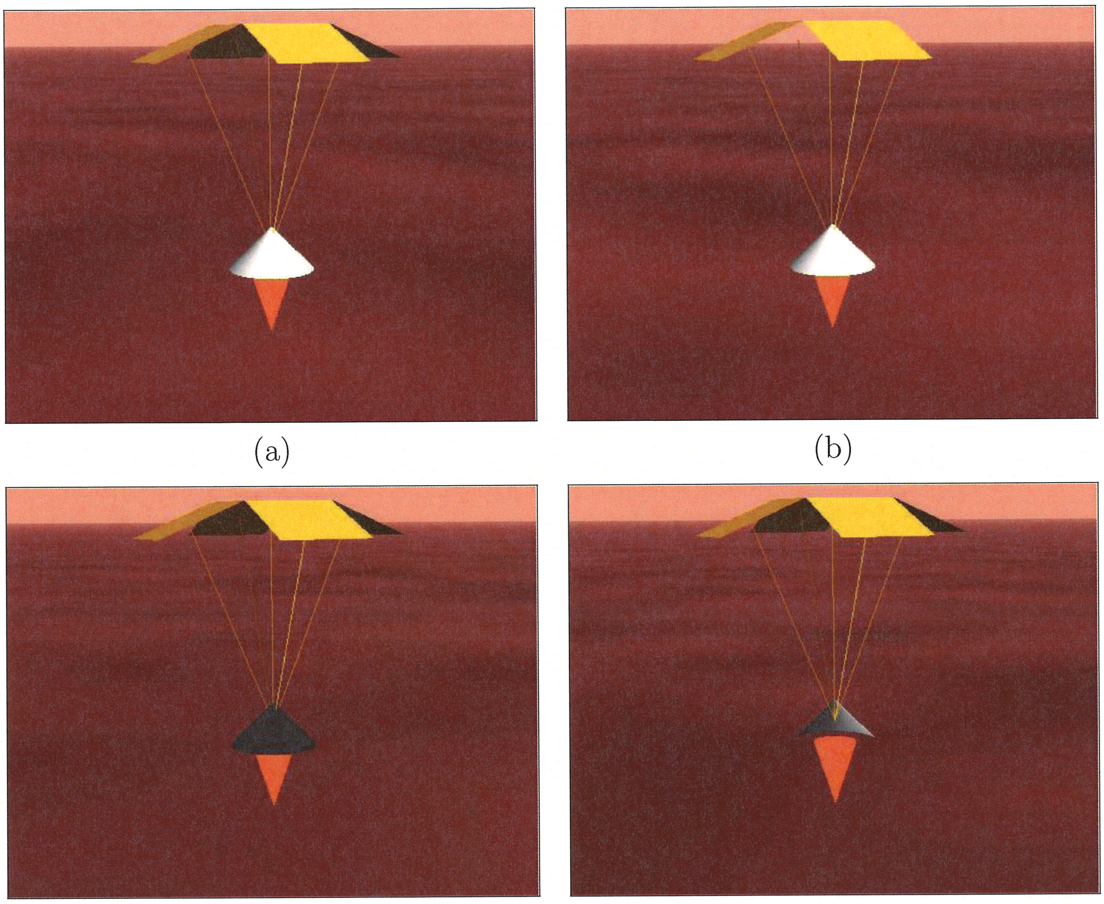
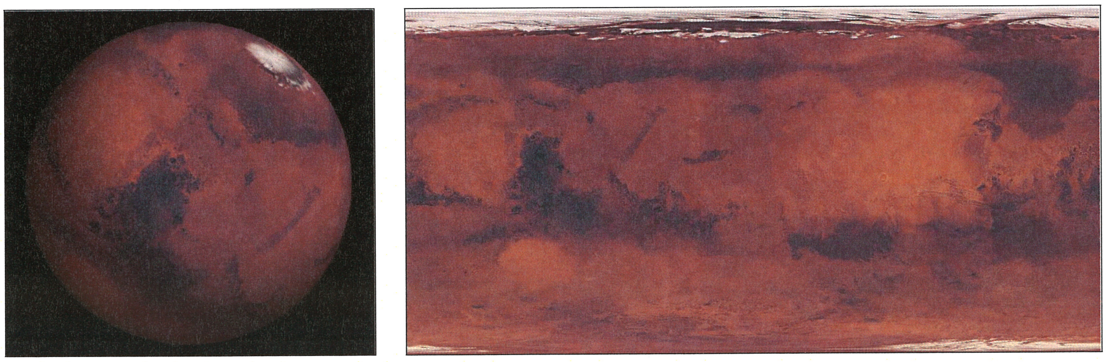
Supplementary page with colour versions of Figs. 3 and 4 for Question 5.



(a) (b)

(c) (d)

Fig. 3



(a) (b)

Fig. 4

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