ENGINEERING TRIPOS PART IIA

Module 3C6 Examination, 2011

Answers

1. (b) $u_0(x) = \sqrt{\frac{1}{\rho LA}}; \quad u_n(x) = \sqrt{\frac{2}{\rho LA}} \cos\frac{n\pi x}{L}, \quad n = 1,2,3...$

(c) $Fh(L,0,t) = \frac{t^2}{2}\left(\frac{F}{\rho AL}\right) + F\sum_{n=1}^{\infty} \frac{u_n(0)u_n(L)}{\omega_n^2}(1-\cos\omega_n t)$ (d) $f_n = 50n \quad Hz.$

2. (a) $\omega_n = \left(\frac{n\pi}{L}\right)\sqrt{\frac{P}{m}}; \quad u_n(x) = \sqrt{\frac{2}{mL}} \sin\frac{n\pi x}{L}, \quad n = 1,2,3...$

(b) $H(x_o, x_o, \omega) = \left(\frac{2}{mL}\right)\sum_{n=1}^{\infty} \frac{\sin^2(\frac{n\pi x_o}{L})}{\omega_n^2 - \omega^2}$

3. (a)
$$V = \frac{1}{2}k\left\{[z + 2a\phi + 2b\theta]^2 + [z + 2a\phi - 2b\theta]^2 + [z - 2a\phi + 2b\theta]^2 + [z - 2a\phi - 2b\theta]^2\right\}$$
$$T = \frac{1}{2}m\left\{[\dot{z} + a\dot{\phi}]^2 + [\dot{z} - a\dot{\phi} - b\dot{\theta}]^2 + [\dot{z} - a\dot{\phi} + b\dot{\theta}]^2\right\}$$

(b)
$$\omega^2 = \frac{8k}{m}; \quad [z \quad \theta \quad \phi]^T = [0 \quad 1 \quad 0]^T$$
$$\omega^2 = \frac{1.29k}{m}; \quad \frac{z}{a\phi} = 9.9; \quad \omega^2 = \frac{6.21k}{m}; \quad \frac{z}{a\phi} = -0.42$$

4. (a)
$$V = \frac{1}{2}k\left\{[\theta_2 - \theta_1]^2 + [\theta_3 - \theta_2]^2 + [\theta_4 - \theta_3]^2\right\} + \frac{1}{2}S[\theta_5 - \theta_4]^2$$
$$T = \frac{1}{2}J\left\{\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2 + \dot{\theta}_4^2 + \dot{\theta}_5^2\right\}$$

(b) $\frac{k}{J}\frac{\left(2\alpha^2 - 2\alpha + 1\right)}{\left(1+\alpha^2\right)}$ (c) $\alpha = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}; \quad \omega = 0.618\sqrt{\frac{k}{J}}, \quad \omega = 1.618\sqrt{\frac{k}{J}}$ (d) 0.686%

Final Version