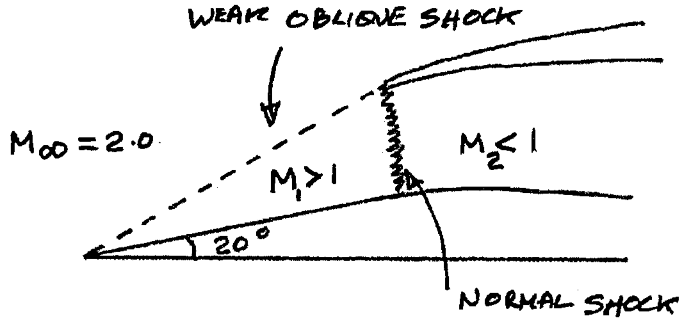
2011 3A3 Question 1

a) Slot is to bleed-off the fuselage boundary layer so that it does not enter the intake. High levels of flow non-uniformity will reduce engine performance and may lead to instability (engine surge). $\boxed{\text{2 MARKS}}$

b)



Oblique shock (weak) and normal shock are focussed on cowl lip

$\boxed{\text{5 MARKS}}$

c) OBLIQUE $\quad M_\infty = 2.00,\ \theta = 20°$ $\quad \dfrac{p_1}{p_\infty} = 2.8429 \quad \dfrac{p_{01}}{p_{0\infty}} = 0.89291$

$M_1 = 1.2102$.

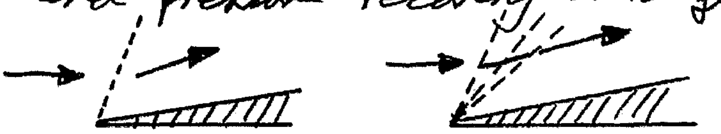
NORMAL $\quad M_1 = 1.2102 \quad \dfrac{p_2}{p_1} = 1.5415 \quad \dfrac{p_{02}}{p_{01}} = 0.9918$.

$$\Rightarrow \quad \frac{p_s}{p_\infty} = \frac{p_2}{p_\infty} = \frac{p_1}{p_\infty} \times \frac{p_2}{p_1} = 2.8429 \times 1.5415 = \underline{\underline{4.382}}$$

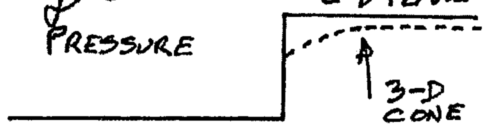
$$\& \quad \frac{p_{0s}}{p_{0\infty}} = \frac{p_{02}}{p_{0\infty}} = \frac{p_{01}}{p_{0\infty}} \times \frac{p_{02}}{p_{01}} = 0.89291 \times 0.9918 = \underline{\underline{0.8856}}$$

$\boxed{\text{6 MARKS}}$

d) Flow conditions downstream of 2-D planar shock are uniform: all flow is turned through 20°.
For the 3-D, half cone, there is a conical shock originating from the vertex (due to effective change of flow area). Thus flow turning is more gradual for cone geometry and pressure recovery is slightly higher.



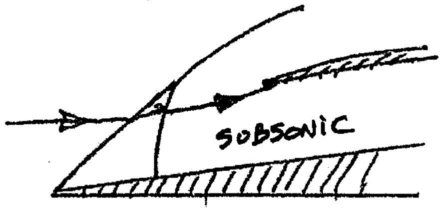
2-D PLANAR SHOCK. $\qquad$ 3-D CONE



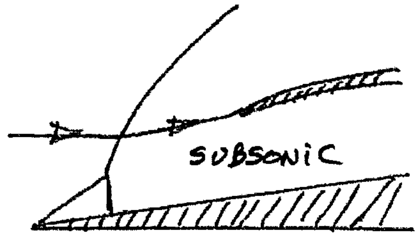
$\boxed{\text{4 MARKS}}$

2011 3A3 Q1 cont.

e)



Slight slowing: shock detaches and get subsonic turning ahead of cowl lip.



Further slowing moves shock system further upstream, more "pitot" intake like. Greater sub-sonic turning.

When the shock system detaches from the cowl lip the pressure recovery drops (even for lower Mach number) and more spill drag is caused. [4 MARKS]

f) Conical intakes can be more efficient than 2-D planar intakes at their design operation. However, 2-D planar systems are easier to design and manufacture and can easily have variable ramp geometry. This means that variable geometry 2-D ramp intakes are very popular for modern supersonic aircraft where acceleration and wide operating range are required. [3 MARKS]

Examiner's comments: Popular question, numerical part (c) done very well. Diagrams were not as good as was expected – many shock systems were impossible. Few candidates explicitly stated "weak oblique shock" and "subsonic downstream of normal".

2011 3A3 QUESTION 2

a) Databook : Oblique shock: $$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2 \sin^2\beta}{2\left(1 + \frac{\gamma-1}{2} M_1^2 \sin^2\beta\right)}$$

limit $M_1 \Rightarrow \infty$ $$\frac{\rho_2}{\rho_1} \rightarrow \frac{(\gamma+1) M_1^2 \sin^2\beta}{(\gamma-1) M_1^2 \sin^2\beta} = \frac{\gamma+1}{\gamma-1}$$ $\boxed{\text{4 MARKS}}$

b) Databook : Oblique shock: $$\tan\theta = \frac{2\cot\beta\,(M_1^2\sin^2\beta - 1)}{(\gamma+1)M_1^2 - 2(M_1^2\sin^2\beta - 1)}$$

limit $M_1 \Rightarrow \infty$ $$\tan\theta \rightarrow \frac{2\cot\beta\, M_1^2\sin^2\beta}{(\gamma+1)M_1^2 - 2M_1^2\sin^2\beta}$$

$$= \frac{2\cos\beta\sin\beta}{(\gamma+1) - 2\sin^2\beta}$$

$$\tan\theta = \frac{\sin 2\beta}{(\gamma+1) - 2\sin^2\beta}$$ $\boxed{\text{6 MARKS}}$

(Alternatives: $$\tan\theta = \frac{\sin 2\beta}{\gamma + \cos 2\beta} = \frac{2\tan\beta}{(\gamma+1) + (\gamma-1)\tan^2\beta}$$)

c) $$\tan\theta = \frac{\sin 2\beta}{\gamma + \cos 2\beta}$$

$$\frac{\partial(\tan\theta)}{\partial\beta} = \frac{2\cos 2\beta}{\gamma + \cos 2\beta} - \frac{\sin 2\beta(-2\sin 2\beta)}{(\gamma+\cos 2\beta)^2} = 0$$

$$0 = 2\cos 2\beta(\gamma + \cos 2\beta) + 2\sin^2 2\beta$$

$$\gamma\cos 2\beta + \cos^2 2\beta + \sin^2 2\beta = 0$$

$$\cos 2\beta = -\frac{1}{\gamma} \qquad \beta = 67.8^\circ \quad (\gamma = 1.4)$$ $\boxed{\text{3 MARKS}}$

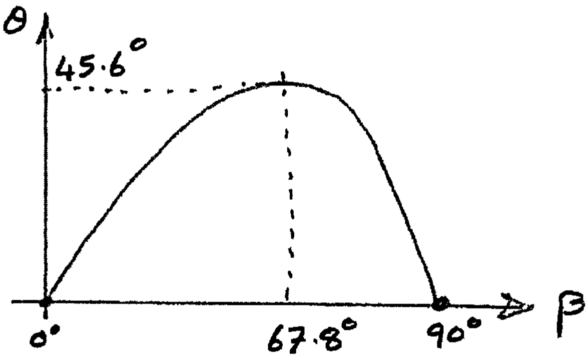
$$\cos 2\beta = -\frac{1}{\gamma} \Rightarrow \sin 2\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \frac{\sqrt{\gamma^2 - 1}}{\gamma}$$ (correct sign)

$$\tan\theta = \frac{\sin 2\beta}{\gamma + \cos 2\beta} = \frac{\sqrt{\gamma^2-1}}{\gamma(\gamma - 1/\gamma)} = \frac{\sqrt{\gamma^2-1}}{\gamma^2-1} = \frac{1}{\sqrt{\gamma^2-1}}$$

$$\tan\theta = \frac{1}{\sqrt{\gamma^2-1}} \qquad \theta = 45.6^\circ \quad (\gamma = 1.4)$$ $\boxed{\text{2 MARKS}}$

2011 3A3 Q2 cont.

c) cont.) $\tan\theta = \dfrac{\sin 2\beta}{\gamma + \cos 2\beta}$



$\boxed{\text{3 MARKS}}$

d) Tables $M_1 = 1.95$, $\theta = 22°$

$\Rightarrow M_2 = 0.9622$ (WEAK OBLIQUE)

or $M_2 = 0.8829$ (STRONG OBLIQUE)

This case is unusual as both weak and strong shocks have <u>subsonic</u> downstream conditions.

$\boxed{\text{4 MARKS}}$

e) Actual flow conditions in part (c) (ie ~~for~~ for large M_1) will depend upon the required deflection angle (ie. geometry) and the back pressure. $\boxed{\text{2 MARKS}}$

Examiner's Comments: Students who used the databook and could do elementary algebra did quite well in parts of (a), (b) and (c). Few students commented that subsonic weak and strong shocks are unusual oblique shocks.

2011 3A3 QUESTION 3

a) STILL AIR

$\leftarrow a$

P, ρ $V = 0$ | $P + \delta P$, $\rho + \delta\rho$, $v = \delta v$

FRAME MOVING SO WAVE AT REST

P, ρ $\rightarrow v = a$ | $P + \delta P$, $\rho + \delta\rho$, $\rightarrow a + \delta v$

CONTINUITY:
$$\rho a = (\rho + \delta\rho)(a + \delta v)$$
$$\Rightarrow \rho\,\delta v + a\,\delta\rho = 0$$
$$\Rightarrow \rho\,\delta v = -a\,\delta\rho \qquad [\text{EQN } 1]$$

MOMENTUM:
$$P + \rho a^2 = (P + \delta P) + (\rho + \delta\rho)(a + \delta v)^2$$
$$0 = \delta P + a^2\delta\rho + 2\rho a\,\delta v \qquad [\text{EQN } 2]$$

[1] & [2] $\Rightarrow$
$$\delta P + a^2\delta\rho + 2a(-a\,\delta\rho) = 0$$
$$\delta P = a^2\delta\rho$$
$$\Rightarrow \frac{\delta P}{\delta\rho} = a^2$$

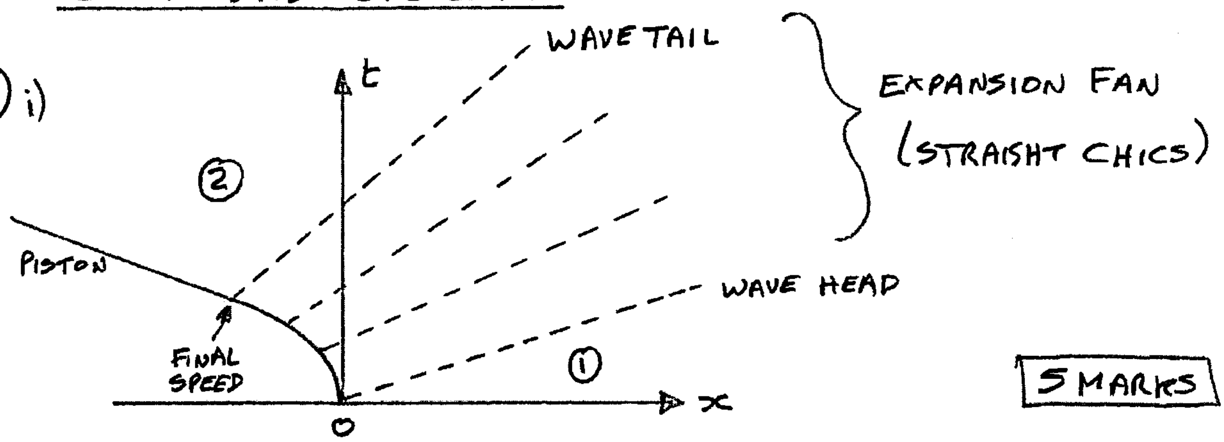
ABOVE IS FOR ISENTROPIC WAVE $\Rightarrow$ $\left.\dfrac{\partial P}{\partial\rho}\right|_s = a^2$ $\boxed{\text{6 MARKS}}$

NOTE: (NOT REQUIRED, BUT RELEVANT!)
$$T\,ds = c_p\,dT - \frac{1}{\rho}dP \Rightarrow \left.\frac{\partial T}{\partial\rho}\right|_s = \frac{1}{\rho c_p}\left.\frac{\partial P}{\partial\rho}\right|_s$$
$$P = \rho RT \Rightarrow \left.\frac{\partial P}{\partial\rho}\right|_s = RT + \rho R\left.\frac{\partial T}{\partial\rho}\right|_s = RT + \frac{R}{c_p}\left.\frac{\partial P}{\partial\rho}\right|_s$$
$$\Rightarrow \left.\frac{\partial P}{\partial\rho}\right|_s\left(1 - \frac{R}{c_p}\right) = RT \Rightarrow \left.\frac{\partial P}{\partial\rho}\right|_s = \gamma RT$$

Examiners Comment: Many candidates just quoted $a^2 = \gamma RT$ (from data book) and then used isentropic relations to obtain the result. This was not considered to be "show"!

2011 3A3 Q3 cont.

b) i)



$\boxed{\text{5 MARKS}}$

ii) RIEMANN INVARIANT : $V_1 - \frac{2a_1}{\gamma-1} = V_2 - \frac{2a_2}{\gamma-1}$

$V_1 = 0$ (STATIONARY AIR)

$V_2 = V_p$ (N.B. $V_p < 0$)

$$V_p = \frac{2}{\gamma-1}(a_2 - a_1)$$

$$\frac{V_p}{a_1} = \frac{2}{\gamma-1}\left(\frac{a_2}{a_1} - 1\right)$$

$$\frac{a_2}{a_1} = \sqrt{\frac{T_2}{T_1}} = \frac{\sqrt{3}}{2} \quad \Rightarrow \frac{V_p}{a_1} = \frac{1}{\gamma-1}(\sqrt{3}-2) = -0.670$$

$\boxed{\text{6 MARKS}}$

iii) $v_2 = -a_2$ (– SIGN DUE TO MOVING TO LEFT!)

$$\Rightarrow \quad -\frac{2a_1}{\gamma-1} = -a_2 - \frac{2a_2}{\gamma-1} = -\frac{\gamma+1}{\gamma-1}a_2$$

$$\Rightarrow \frac{a_2}{a_1} = \frac{2}{\gamma+1} = \sqrt{\frac{T_2}{T_1}}$$

UNSTEADY EXPANSION FAN IS ISENTROPIC $\Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{\gamma+1}\right)^{2\gamma/\gamma-1} = 0.2791$$

$$\Rightarrow P_2 = 150\,\text{kPa} \times 0.2791 = 41.87\,\text{kPa}$$

$\boxed{\text{7 MARKS}}$

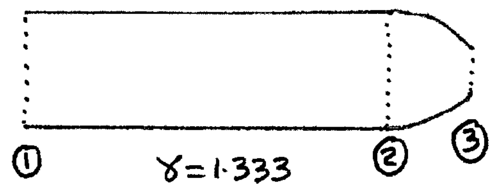
Examiner's Comments: Other than sign errors, common error was to assume that isentropic meant T_0, P_0 were constant. (Not true for unsteady flow!)

2011 3A3 QUESTION 4

a) i) $T_{01} = 600\,K$

$P_{01} = 3\,bar$

$M_1 = 0.4$



$\gamma = 1.333$ @ $M = 1$ $\quad P/P_0 = 0.5398$

NOW $P_3/P_{01} = 1/3 = 0.333 \Rightarrow$ CHOKED $\Rightarrow M_3 = 1$

$\gamma = 1.333 \qquad M_1 = 0.4 \Rightarrow \dfrac{\dot m \sqrt{c_p T_{01}}}{P_{01} A} = 0.8427$

$P_{03} = P_{01}$ etc. $\quad M_3 = 1.0 \Rightarrow \dfrac{\dot m \sqrt{c_p T_{03}}}{P_{03} A_N} = 1.3468$

$$\frac{A_N}{A} = \frac{\dot m \sqrt{c_p T_{01}}}{P_{01} A} \cdot \frac{P_{01} . A_N}{\dot m \sqrt{c_p T_{01}}} = \frac{0.8427}{1.3468} = \underline{\underline{0.6257}}$$

$\boxed{\text{5 MARKS}}$

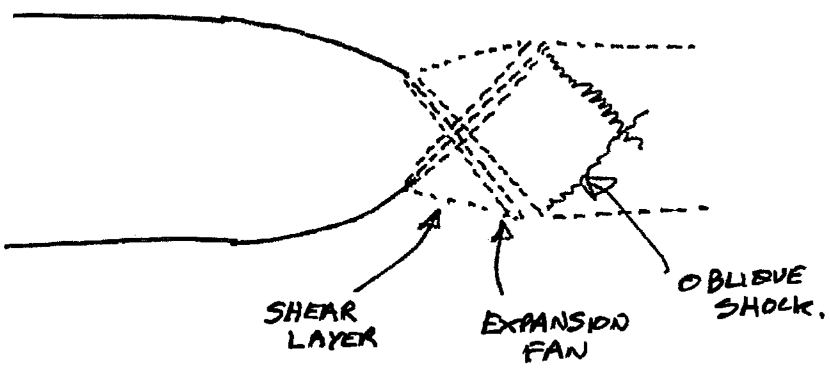
a) ii) SPECIFIC IMPULSE $\dfrac{F}{\dot m} = \dfrac{F}{\dot m \sqrt{c_p T_0}} \sqrt{c_p T_0}$

@ $M_3 = 1 \qquad \dfrac{F}{\dot m \sqrt{c_p T_{01}}} = 0.9351$

$$\Rightarrow \frac{F}{\dot m} = 0.9351 \sqrt{1149 \times 600} = \underline{\underline{776.4}}\ \frac{Ns}{kg}$$

$\boxed{\text{2 MARKS}}$

a) iii) UNDEREXPANDED JET, SO EXPECT "SHOCK DIAMONDS" SINCE $P_3 > P_{DOWNSTREAM}$ (CHOKED CONVERGENT NOZZLE).



$\boxed{\text{2 MARKS}}$

2011 3A3 Q4 cont.

b)i) HEAT INPUT ① → ② $\dot{Q}/\dot{m} = 300$ kJ/kg

$$T_{02} = T_{01} + \frac{300\times10^3}{c_p} = 600 + \frac{300\times10^3}{1149} = \underline{\underline{861.1\ K}} \quad \boxed{2\text{ MARKS}}$$

① → ② NO NET FORCE $\Rightarrow (F/\dot{m})|_1 = (F/\dot{m})|_2$

$$@\ M_1 = 0.4 \quad \frac{F}{\dot{m}\sqrt{c_pT_{01}}} = 1.2959$$

$$\Rightarrow \frac{F}{\dot{m}\sqrt{c_pT_{02}}} = \frac{F}{\dot{m}\sqrt{c_pT_{01}}}\sqrt{\frac{T_{01}}{T_{02}}} = 1.2959\sqrt{\frac{600}{861.1}} = 1.0817$$

TABLES ($\gamma = 1.333$) $\Rightarrow$ $\underline{\underline{M_2 = 0.546}}$ $\boxed{3\text{ MARKS}}$

$$@\ M_2 = 0.546 \quad \frac{\dot{m}\sqrt{c_pT_{02}}}{A\,P_{02}} = 1.0641$$

$$@\ M_3 = 1.0 \quad \frac{\dot{m}\sqrt{c_pT_{02}}}{A_N\,P_{02}} = 1.3468$$

(NOZZLE IS STILL CHOKED, SEE b) iii))

$$\left.\frac{A_N}{A}\right|_{REHEAT} = \frac{1.0641}{1.3468} = \underline{\underline{0.7901}}$$

(NOZZLE IS LARGER) $\boxed{2\text{ MARKS}}$

b)ii) $$\frac{F}{\dot{m}} = \frac{F}{\dot{m}\sqrt{c_pT_{03}}}\sqrt{c_pT_{03}} = 0.9351\sqrt{1149\times861.1} = \underline{\underline{930.1\ Ns/kg}}$$

(MORE THRUST) $\boxed{2\text{ MARKS}}$

b)iii) $$P_3 = \frac{P_3}{P_{03}}\times\frac{P_{03}}{P_{02}}\times\frac{P_{02}}{P_{01}}\times P_{01}$$

$P_{01} = 3.0$ bar

$M_3 = 1.0 \Rightarrow \frac{P_3}{P_{03}} = 0.5398$

ISENTROPIC NOZZLE $\Rightarrow \frac{P_{03}}{P_{02}} = 1.0$

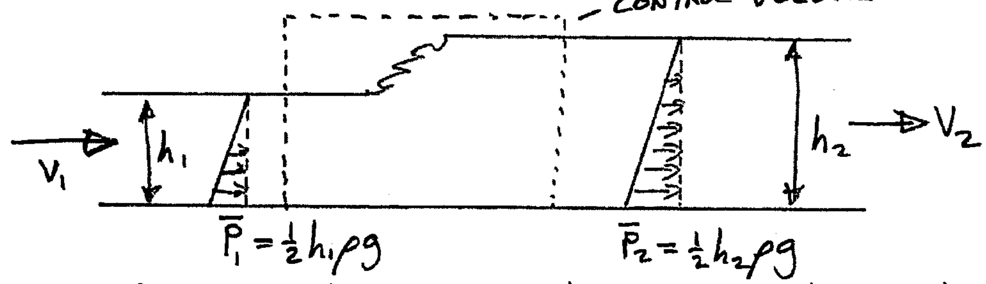
$$\frac{P_{02}}{P_{01}} = \underbrace{\frac{\dot{m}\sqrt{c_pT_{01}}}{A\,P_{01}}}_{M_1=0.4}\cdot\underbrace{\frac{A\,P_{02}}{\dot{m}\sqrt{c_pT_{02}}}}_{M_2=0.546}\sqrt{\frac{T_{02}}{T_{01}}} = \frac{0.8427}{1.0641}\sqrt{\frac{861}{600}} = 0.9487$$

$$P_3 = 0.5398\times1.0\times0.9487\times3.0 = 1.536\ \text{bar} \quad \boxed{6\text{ MARKS}}$$

Examiner's Comment: Question well done except for common mistakes of $\gamma = 1.4$ and assuming isentropic ①-② in (b).

2011 3A3 QUESTION 5

a) WIDTH w.



CONTINUITY: $\dot{m} = \rho v_1 w h_1 = \rho v_2 w h_2 \Rightarrow v_1 h_1 = v_2 h_2$

FORCE-MOM: $\bar{P}_1 h_1 w + \dot{m} v_1 = \bar{P}_2 h_2 w + \dot{m} v_2$

$$\frac{1}{2}g h_1^2 + v_1^2 h_1 = \frac{1}{2} g h_2^2 + v_2^2 h_2 = \frac{1}{2} g h_2^2 + v_1^2 h_1^2/h_2$$

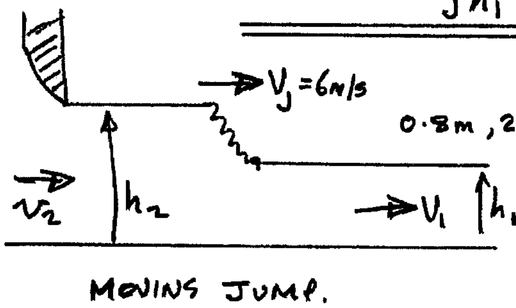
$$\Rightarrow v_1^2\left(h_1 - \frac{h_1^2}{h_2}\right) = \frac{1}{2} g\left(h_2^2 - h_1^2\right)$$

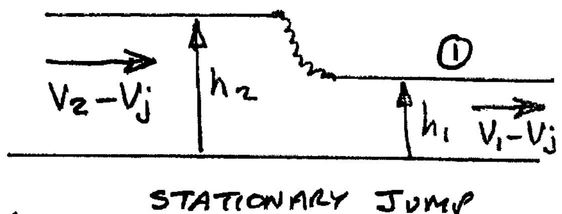
$$v_1^2 \frac{h_1}{h_2}(h_2 - h_1) = \frac{1}{2} g (h_2 - h_1)(h_2 + h_1)$$

$$\underline{\underline{(Fr_1)^2 = \frac{v_1^2}{g h_1} = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right)}}$$

$\boxed{7 \text{ MARKS}}$

b)





(NOTE: BOTH $V_1 - V_j$ & $V_2 - V_j < 0$)

$$(Fr_1)^2 = \frac{(V_1 - V_j)^2}{g h_1} = \frac{(2.3-6)^2}{9.81 \times 0.8} = 1.7444 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1}+1\right) = \frac{1}{2} r(r+1)$$

$r = \frac{h_2}{h_1} \quad \Rightarrow r^2 + r - 2 \times 1.7444 = 0$

$$r = \frac{-1 \pm \sqrt{1^2 + 4.2.1.7444}}{2} = 1.4336 \text{ or } \underbrace{-2.433}_{\text{NOT PHYSICAL.}}$$

$$h_2 = r h_1 = 1.4336 \times 0.8 = \underline{\underline{1.147\,m}}$$

CONTINUITY $(v_2 - v_j) h_2 = (v_1 - v_j) h_1$

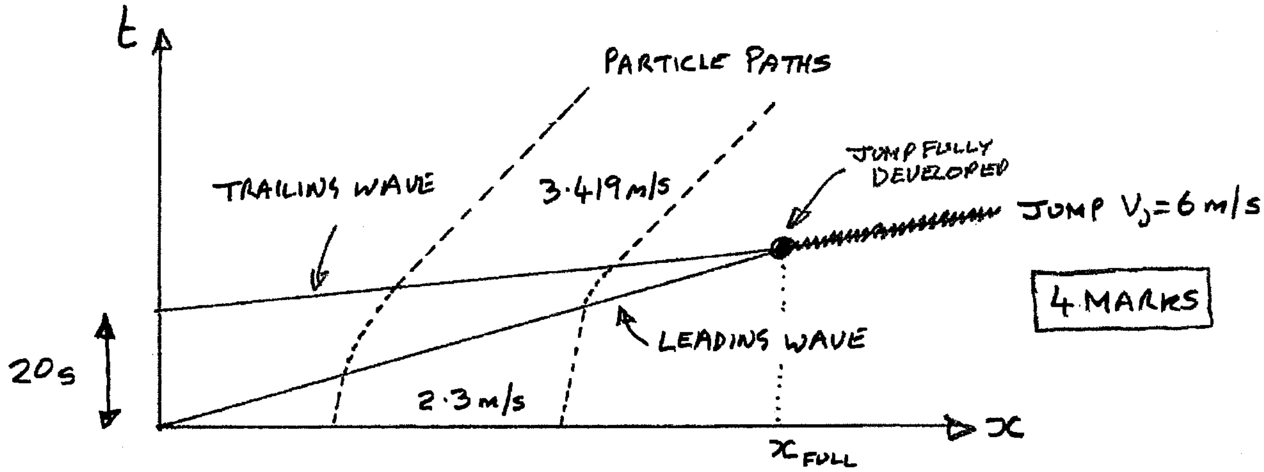
$$v_2 = v_j + (v_1 - v_j) h_1/h_2$$

$$= 6.0 + (2.3 - 6.0)/1.4336$$

$$\underline{\underline{v_2 = 3.419\ m/s}}$$

$\boxed{9 \text{ MARKS}}$

c)



$\boxed{\text{4 MARKS}}$

LEADING WAVE $\quad v_1 + c_1 = 2.3 + \sqrt{9.81 \times 0.8} = 5.10$ m/s

TRAILING WAVE $\quad v_2 + c_2 = 3.419 + \sqrt{9.81 \times 1.147} = 6.77$ m/s

WAVES MEET WHEN :

$$5.10t = 6.77(t - 20)$$

$$t = \frac{6.77 \times 20}{6.77 - 5.10} = 81.1 \text{ s}$$

$$\underline{\underline{x_{FULL} = 5.10 \times 81.1 = 413.6 \text{ m}}}$$

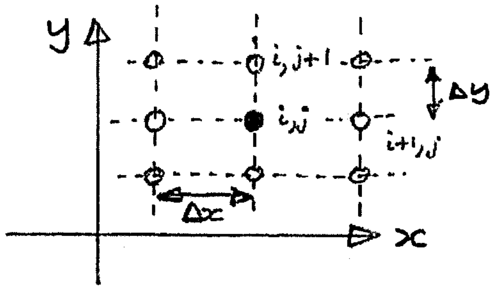
$\boxed{\text{4 MARKS}}$

Examiner's Comments: Part (a) done very well. In Part (b) many candidates had difficulty with change of frame of reference, not helped when their diagram had the depth of water <u>decreasing</u> when the sluice gate was raised!

Diagram for part (c) was done much better than the simple geometry.

2011 3A3 QUESTION 6

a)



SPACING Δx, index "i"
Δy, index "j"

$$\frac{\partial u}{\partial x} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} + O(\Delta x)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta y^2} + O(\Delta y^2)$$

Via Taylor or quote.

$$\Rightarrow \quad U\left(\frac{u_{i+1,j} - u_{i,j}}{\Delta x}\right) = \mu\left(\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta y^2}\right)$$

$\boxed{\text{7 MARKS}}$

b) SAW TOOTH

	i	$i+1$
$j+1$	$-\epsilon$	
j	$+\epsilon$	?
$j-1$	$-\epsilon$	

$u_{i,j} = \epsilon$

$u_{i,j\pm 1} = -\epsilon$

$$u_{i+1,j} = u_{i,j} + \frac{\mu \Delta x}{U \Delta y^2}\left(u_{i,j-1} - 2u_{i,j} + u_{i,j+1}\right)$$

$$u_{i+1,j} = \epsilon + \frac{\mu \Delta x}{U \Delta y^2}\left(-\epsilon - 2\epsilon - \epsilon\right) = \epsilon\left(1 - \frac{4\mu\Delta x}{U\Delta y^2}\right)$$

STABILITY: $\left|\dfrac{u_{i+1,j}}{\epsilon}\right| = \left|1 - \dfrac{4\mu\Delta x}{U\Delta y^2}\right| \leq 1$ $\boxed{\text{6 MARKS}}$

$$\Rightarrow \quad -1 \leq 1 - \frac{4\mu\Delta x}{U\Delta y^2} \leq 1$$

$$2 \geq \frac{4\mu\Delta x}{U\Delta y^2} \geq 0$$

TWO CONDITIONS: (i) $\Delta x \leq \dfrac{U\Delta y^2}{2\mu}$ "cell Re < $\frac{1}{2}$" $\boxed{\text{2 MARKS}}$

(ii) $\dfrac{\Delta x}{U} \geq 0$ FORWARD DIFFERENCE IN **FLOW** DIRECTION

$\boxed{\text{2 MARKS}}$

c) Assuming that the forward difference is in the flow direction ($\Delta x/U > 0$) then the main condition is:

$$\Delta x < \frac{U \Delta y^2}{2\mu}$$

For a turbulent boundary layer (i) μ large $\Rightarrow \Delta x$ small
(ii) U small $\Rightarrow \Delta x$ small
(iii) Δy^2 very small $\Rightarrow \Delta x$ very small.

All these require very small $\Delta x \Rightarrow$ large computational cost.

$\boxed{\text{3 MARKS}}$

If the flow separates, $U < 0$, scheme is unstable as $\Delta x/U < 0$.

$\boxed{\text{1 MARK}}$

Improved methods:

(i) Introduce pseudo time & time-march.
(ii) Implicit methods (eg backward difference).
(iii) More general solvers (elliptic) for full B.L. equation.

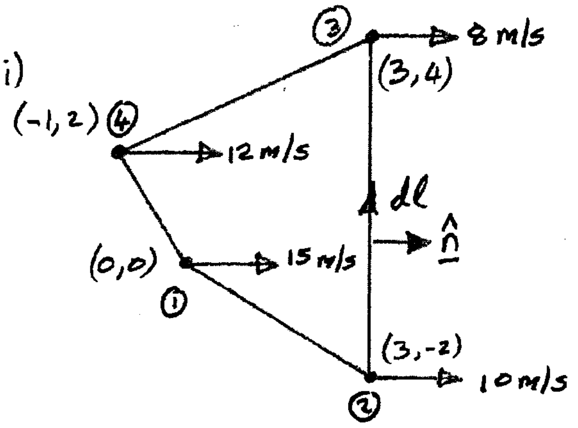
$\boxed{\text{3 MARKS}}$

Examiner's comments:

Part (a) done very well.
Part (b) done well except for missing $\Delta x/U > 0$.
Part (c) Many candidates mentioned everything they could ~~have~~ remember, rather than what was relevant.

2011 3A3 QUESTION 7

a) i)



$$\text{AREA} = \tfrac{1}{2}\left|(\underline{x}_3-\underline{x}_1)\times(\underline{x}_4-\underline{x}_2)\right|$$

$$= \tfrac{1}{2}\left|\left(\begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}\right)\times\left(\begin{pmatrix}-1\\2\end{pmatrix}-\begin{pmatrix}3\\-2\end{pmatrix}\right)\right|$$

$$= \tfrac{1}{2}\left|\begin{pmatrix}3\\4\end{pmatrix}\times\begin{pmatrix}-4\\4\end{pmatrix}\right| = 14 \text{ mm}^2$$

$$\underline{\underline{\text{AREA} = 14\times10^{-6}\ \text{m}^2}} \quad \boxed{\text{3 MARKS}}$$

a) ii) $\dfrac{\partial\rho}{\partial t} = -\underline{\nabla}\cdot(\rho\underline{u}) \Rightarrow \displaystyle\int \frac{\partial\rho}{\partial t}\,dA = -\int_A(\underline{\nabla}\cdot\rho\underline{u})\,dA = -\oint \rho\underline{u}\cdot\hat{n}\,dl$

MASS FLUX OUT $= \oint \rho\underline{u}\cdot\hat{n}\,dl = \oint \rho u\,dy - \rho v\,dx$ (GAUSS)

$v=0 \Rightarrow$ MASS FLUX OUT $= \rho\displaystyle\int u\,dy$ $\qquad u = \tfrac{1}{2}(u_i+u_{i+1})$, $dy = y_{i+1}-y_i$

$$\text{MASS FLUX OUT} = \rho\left[\frac{(10+15)}{2}(-2\times10^{-3}) + \left(\frac{8+10}{2}\right)(6\times10^{-3}) + \left(\frac{8+12}{2}\right)(-2\times10^{-3}) + \left(\frac{12+15}{2}\right)(-2\times10^{-3})\right]$$

$$= \rho[-25+54-20-27]\times10^{-3}$$

$$= 1.2[-18]\times10^{-3} \quad \underline{\underline{= -21.6\times10^{-3}\ \text{kg/ms}}} \quad \boxed{\text{7 MARKS}}$$

a) iii) $$\frac{\partial\rho}{\partial t} = -\frac{1}{A}\oint\rho\underline{u}\cdot\hat{n}\,dl = -\frac{(-21.6\times10^{-3})}{14\times10^{-6}} = \underline{\underline{1543\ \text{kg/m}^3\text{s}}}$$

$\boxed{\text{2 MARKS}}$

b) Euler Work $\quad h_{02}-h_{01} = U_2v_2 - U_1v_1 \qquad U = r\Omega$

$\Rightarrow h_{02} - r_2\Omega v_2 = h_{01} - r_1\Omega v_1$ ie $h_0 - r\Omega v = \text{const.}$

Now $h_0^{rel} = h + \tfrac{1}{2}(u^2 + {v^{rel}}^2 + w^2) \qquad v^{REL} = v - r\Omega$

$h_0^{REL} = h + \tfrac{1}{2}(u^2 + v^2 - 2vr\Omega + r^2\Omega^2 + w^2)$

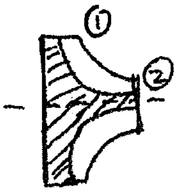
$= h + \tfrac{1}{2}(u^2+v^2+w^2) + \tfrac{1}{2}r^2\Omega^2 - r\Omega v$

$\Rightarrow \underline{\underline{h_0^{REL} - \tfrac{1}{2}r^2\Omega^2 = h_0 - r\Omega v = \text{const}}} \quad \boxed{\text{4 MARKS}}$

Valid for steady (or time average) adiabatic flow $\boxed{\text{1 MARK}}$

2011 3A3 Q7 cont.

c) $$Y_p = \frac{p_{02}^{REL,ISEN} - p_{02}^{REL}}{p_{02}^{REL} - p_2}$$



$\boxed{\text{2 MARKS}}$

$$T\Delta s = \Delta h_0^{REL} - \frac{1}{\rho_0}\Delta p_0^{REL} \qquad \Delta s = 0 \Rightarrow \frac{1}{\rho}\Delta p_0^{REL,ISEN} = \Delta h_0^{REL}$$

$$\Rightarrow \frac{1}{\rho}\left(p_{02}^{REL,ISEN} - p_{01}^{REL}\right) = h_{02}^{REL} - h_{01}^{REL} = \tfrac{1}{2}\left(r_2^2 - r_1^2\right)\Omega^2$$

$$\Rightarrow p_{02}^{REL,ISEN} = p_{01}^{REL} - \tfrac{1}{2}\rho\left(r_1^2 - r_2^2\right)\Omega^2$$

$\boxed{\text{2 MARKS}}$

$$500\text{rpm} \Rightarrow \Omega = \frac{2\pi . 500}{60} = 52.36 \text{ RAD/s}$$

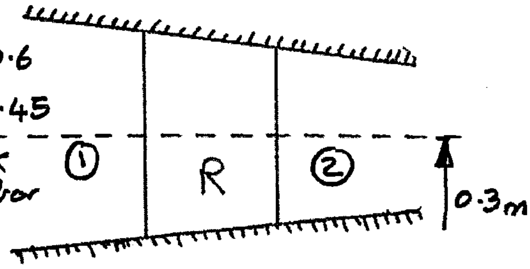
$$p_{02}^{REL,ISEN} = 60\times10^5 - \tfrac{1}{2}\cdot1000\left(1.9^2 - 0.6^2\right).\,52.36^2 = 15.45 \text{ bar}$$

$$Y_p = \frac{15.45 - 14.7}{14.7 - 7.4} = 0.103$$

$\boxed{\text{3 MARKS}}$

Examiner's comments: Most candidates managed part (a) with a few errors on the sign and $(\times10^{-3})$ factor. Part (b) was standard bookwork which was generally well done. Part (c) was challenging in that the isentropic change in the relative stagnation pressure had to be calculated — several candidates managed the calculation perfectly!

2011 3A3 QUESTION 8



a) $M_1 = 0.4$

$T_1 = T_{01}/(1+\frac{1}{2}(\gamma-1)M_1^2)$

$T_1 = 300/(1+0.2\times0.4^2) = 290.7\text{ K}$

$u_1 = M_1\sqrt{\gamma R T_1} =$

$= 0.4\sqrt{1.4.287.290.7}$

$\underline{\underline{u_1 = 136.7\text{ m/s}}}$ $\boxed{\text{2 MARKS}}$

$$\phi = \frac{u}{U} \Rightarrow U = \frac{u_1}{\phi} = \frac{136.7}{0.6} = 227.8\text{ m/s}$$

$$U = r\Omega \Rightarrow \Omega = \frac{U}{r} = \frac{227.8}{0.3} = \underline{\underline{759.3\text{ RAD/s}}} \quad \boxed{\text{2MARKS}}$$

$(= 7251\text{ rpm})$

b) $\psi = \frac{\Delta h_0}{U^2} \Rightarrow T_{02} = T_{01} + \frac{\psi U^2}{c_p}$

$$T_{02} = 300 + \frac{0.45\times 227.8^2}{1005} = \underline{\underline{323.2\text{ K}}} \quad \boxed{\text{2MARKS}}$$

$$\frac{\Delta h_0}{U^2} = \frac{Uv_2 - Uv_1}{U^2} = \frac{v_2}{U}$$

$$\Rightarrow v_2 = U.\frac{\Delta h_0}{U^2} = 227.9\times0.45 = \underline{\underline{102.5\text{ m/s}}} \quad \boxed{\text{2MARKS}}$$

$$\alpha_2 = \tan^{-1}\left(\frac{v_2}{u}\right) = \tan^{-1}\left(\frac{102.5}{136.7}\right) = \underline{\underline{36.9^\circ}} \quad \boxed{\text{1 MARK}}$$

c) $\eta_c = \frac{T_{02}^{ISEN} - T_{01}}{T_{02} - T_{01}} \Rightarrow T_{02}^{ISEN} = T_{01} + \eta_c(T_{02} - T_{01})$

$$T_{02}^{ISEN} = 300 + 0.93(323.2 - 300) = 321.6\text{ K}$$

$$P_{02} = P_{01}\left(\frac{T_{02}^{ISEN}}{T_{02}}\right)^{\gamma/\gamma-1} = 1.0\left(\frac{321.6}{300}\right)^{3.5} = \underline{\underline{1.276\text{ bar}}} \quad \boxed{\text{4MARKS}}$$

d) $T_2 = T_{02} - (u^2 + v_2^2)/2c_p$

$$= 323.2 - \frac{(136.7^2 + 102.5^2)}{2\times1005} = 308.7\text{ K}$$

2011 3A3 Q8 cont.

d) cont.

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 1.276\left(\frac{308.7}{323.2}\right)^{3.5} = 1.087 \text{ bar}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{1.087\times10^5}{287\times 308.7} = 1.227 \text{ kg/m}^3$$

$$r_H + r_T = 2\bar{r} \Rightarrow r_T = 2\bar{r}/(1+r_H/r_T) = 2\times0.3/(1+0.68) = 0.3571 \text{ m}$$

$$r_H = 2\bar{r} - r_T = 2\times0.3 - 0.3571 = 0.2429 \text{ m}$$

$$A_2 = \pi(r_T^2 - r_H^2) = \pi(0.3571^2 - 0.2429^2) = 0.21526 \text{ m}^2$$

$$\dot{m} = \rho_2 A_2 u = 1.227\times0.21526\times136.7 = \underline{\underline{36.11 \text{ kg/s}}} \quad \boxed{\text{6 MARKS}}$$

e) Free Vortex : rv = constant.

i) Simple Radial Equilibrium $\Rightarrow$ Uniform Axial Velocity ($\partial u/\partial r = 0$)

ii) Euler Work $\Rightarrow$ Uniform Work input ($\partial T_0/\partial r = 0$)

$\boxed{\text{2 MARKS}}$

f) $\bar{r} = 0.3$ m , $v_2^{MID} = 102.5$ m/s , rv = const.

$$v_2^{TIP} = \frac{\bar{r}_2 v_2^{MID}}{r_{TIP}} = \frac{0.3\times102.5}{0.3571} = 86.1 \text{ m/s}$$

$$v_2^{HUB} = \frac{\bar{r}_2 v_2^{MID}}{r_{HUB}} = \frac{0.3\times102.5}{0.2429} = 126.6 \text{ m/s}$$

$$\alpha_2^{REL,TIP} = \tan^{-1}\left(\frac{v_2^{TIP} - r^{TIP}\Omega}{u}\right) \qquad u = 136.7 \text{ m/s}, \quad \Omega = 759.3 \text{ RAD/s}$$

$$\left.\begin{aligned}
\alpha_2^{REL,TIP} &= \tan^{-1}\left(\frac{86.1 - 0.3571\times759.3}{136.7}\right) = -53.5^\circ \\
\alpha_2^{REL,MID} &= \tan^{-1}\left(\frac{102.5 - 0.3\times759.3}{136.7}\right) = -42.5^\circ \\
\alpha_2^{REL,HUB} &= \tan^{-1}\left(\frac{126.6 - 0.2429\times759.3}{136.7}\right) = \underline{\underline{-22.9^\circ}}
\end{aligned}\right\} \boxed{\text{3 MARKS}}$$

Examiner's Comments: Generally well done except some candidates assumed isentropic flow through the rotor. Also, several candidates tried to use $\dot{m}\sqrt{c_pT_0}/P_0A$ at rotor <u>inlet</u> to calculate mass flow. (Area is at rotor <u>exit</u>).

2011 IIA Paper 3A3 (Fluid Mechanics II) Answers

Q1:

a) $\dfrac{p_s}{p_\infty} = 4.382 \quad \dfrac{p_{0s}}{p_{0\infty}} = 0.8856$

Q2:

b) $tan\,\theta = \dfrac{sin\,2\beta}{(\gamma+1) - 2\,sin^2\beta}$

c) $\beta = 67.8°$

d) $M_2 = 0.9622$ and $M_2 = 0.8829$ (both subsonic)

Q3:

b)ii) $\dfrac{V_p}{a_1} = -0.670$ iii) $p_2 = 41.87$ kPa

Q4:

a)i) $\dfrac{A_N}{A} = 0.6257$ ii) $\dfrac{F}{\dot{m}} = 776.4$ Ns/kg

b)i) $T_{02} = 861.1$ K ii) $\dfrac{F}{\dot{m}} = 930.1$ Ns/kg iii) $p_3 = 1.536$ bar

Q5:

b) $h_2 = 1.147$ m $v_2 = 3.419$ m/s

c) $x_{full} = 413.6$ m

Q6:

a) $U\left(\dfrac{u_{i+1,j} - u_{i,j}}{\Delta x}\right) = \mu\left(\dfrac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{(\Delta y)^2}\right)$

b) $\Delta x \le \dfrac{U(\Delta y)^2}{2\mu}$ and $\dfrac{\Delta x}{U} \ge 0$ (i.e. true forward difference in flow direction)

Q7:

a)i) $A = 14\times10^{-6}\ \mathrm{m}^2$ ii) $flux = -21.6\times10^{-3}$ kg/ms iii) $\dfrac{\partial\rho}{\partial t} = 1543\ \mathrm{kg/m^3s}$

c) $Y_p = 0.103$

Q8:

a) $u_1 = 136.7$ m/s $\Omega = 759.3$ rad/s

b) $v_2 = 102.5$ m/s $\alpha_2 = 36.9°$ $T_{02} = 323.2$ K

c) $P_{02} = 1.276$ bar

d) $\dot{m} = 36.11$ kg/s

f) $\alpha_2^{rel,hub} = -22.9°$ $\alpha_2^{rel,mid} = -42.5°$ $\alpha_2^{rel,tip} = -53.5°$

JPL 27May2011