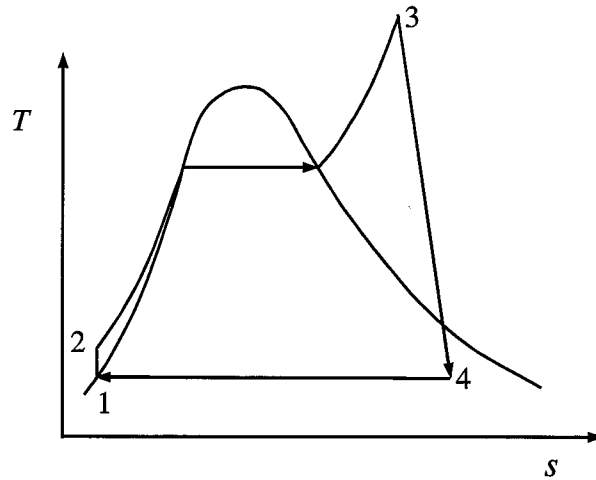


1.



$$(a) \text{ Heat input rate boiler} = \dot{m}(h_3 - h_2) = 200 \times (3423.1 - 158.6) \times 10^{-3} = 652.9 \text{ MW}$$

$$\text{Power output turbine} = \dot{m}(h_3 - h_4) = 200 \times (3423.1 - 2325.2) \times 10^{-3} = 219.6 \text{ MW}$$

$$\text{Power input feed pump} = \dot{m}(h_2 - h_1) = 200 \times (158.6 - 151.5) \times 10^{-3} = 1.4 \text{ MW}$$

$$\text{Net power output} = 219.6 - 1.4 = 218.2 \text{ MW}$$

$$\text{Thermal efficiency} = \frac{218.2}{652.9} = 0.334 \quad [15 \%$$

$$(b) \text{ Exergy supply rate in boiler} = \dot{m}[(h_3 - T_0 s_3) - (h_2 - T_0 s_2)]$$

$$= 200 \times [(3423.1 - 158.6) - 283.15 \times (6.883 - 0.521)] \times 10^{-3}$$

$$= 292.6 \text{ MW} \quad [10 \%$$

$$\text{Turbine 'lost power'} = \dot{m}T_0(s_4 - s_3) = 200 \times 283.15 \times (7.548 - 6.883) \times 10^{-3} = 37.6 \text{ MW}$$

Condenser is at constant pressure so 'lost power' is due solely to the heat transfer (with respect to the dead state because the cooling water is not included in the calculation). Thus,

$$\text{Lost power in condenser} = \dot{m}(h_4 - h_1) \left(1 - \frac{T_0}{T_{cond}}\right)$$

$$= 200 \times (2325.2 - 151.5) \left(1 - \frac{283.15}{309.31}\right) \times 10^{-3} = 36.8 \text{ MW} \quad [20 \%$$

$$\text{Total power + lost power} = 218.2 + 37.6 + 36.8 = 292.6 \text{ MW} = \text{total exergy supply rate} \quad [5 \%$$

$$\text{Rational efficiency} = \frac{\text{Net power}}{\text{Exergy supply rate}} = \frac{218.2}{292.6} = 0.746 \quad [5 \%$$

(c) The cooling water is supplied at the dead state condition so its exergy is zero. Hence,

Total exergy supply rate = 292.6 MW (same as before)

Turbine 'lost power' = 37.6 MW (same as before)

The 'lost power' in the condenser treated as a single adiabatic heat exchanger is,

$$\dot{m}_{cw}T_0(s_{cw,out} - s_{cw,in}) + \dot{m}T_0(s_1 - s_6)$$

As $c_{p,cw}$ is constant the cooling water mass flowrate is given by,

$$\dot{m}_{cw} = \frac{\dot{m}(h_4 - h_1)}{(h_{cw,out} - h_{cw,in})} = \frac{\dot{m}(h_6 - h_1)}{c_{p,cw}(T_{cw,out} - T_{cw,in})} = \frac{200 \times (2325.2 - 151.5)}{4.18 \times (30 - 10)} = 5200.2 \text{ kg/s}$$

Using $Tds = dh - vdp$ and ignoring any pressure drop in the condenser tubes (so $dp = 0$),

$$\dot{m}_{cw}T_0(s_{cw,out} - s_{cw,in}) = \dot{m}_{cw}T_0 \int_{T_{cw,in}}^{T_{cw,out}} \frac{c_{p,cw}}{T} dT = \dot{m}_{cw}c_{p,cw}T_0 \ln\left(\frac{T_{cw,out}}{T_{cw,in}}\right)$$

$$\begin{aligned} \text{Condenser 'lost power'} &= 5200.2 \times 4.18 \times 283.15 \times \ln\left(\frac{303.15}{283.15}\right) \times 10^{-3} \\ &\quad + 200 \times 283.15 \times (0.521 - 7.548) \times 10^{-3} \\ &= 420.0 - 397.9 = 22.1 \text{ MW} \end{aligned} \quad [20 \%]$$

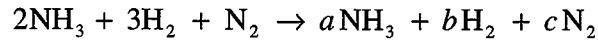
'Lost power' of cooling water at outlet = $\dot{m}_{cw}[(h_{cw,out} - h_{cw,0}) - T_0(s_{cw,out} - s_{cw,0})]$

$$\begin{aligned} &= \dot{m}_{cw}c_{p,cw} \left[(T_{cw,out} - T_{cw,in}) - T_0 \ln\left(\frac{T_{cw,out}}{T_{cw,in}}\right) \right] \\ &= 5200.2 \times 4.18 \left[(303.15 - 283.15) - 283.15 \times \ln\left(\frac{303.15}{283.15}\right) \right] \times 10^{-3} = 14.7 \text{ MW} \end{aligned} \quad [15 \%]$$

Total power + 'lost power' = 218.2 + 37.6 + 22.1 + 14.7 = 292.6 MW = Exergy supply rate [5 %]

Rational efficiency = $\frac{\text{Net power}}{\text{Exergy supply rate}} = \frac{218.2}{292.6} = 0.746$ (same as before). [5 %]

2. (a) The chemical reaction is,



Atom balance for N : $4 = a + 2c$

Atom balance for H : $12 = 3a + 2b$

Total number of moles : $n = a + b + c$

[15 %]

The equilibrium equation for reaction (9) in the databook is,

$$\left(\frac{p_{\text{N}_2}}{p_0}\right)^{-1/2} \left(\frac{p_{\text{H}_2}}{p_0}\right)^{-3/2} \left(\frac{p_{\text{NH}_3}}{p_0}\right)^{+1} = K_{p_9}$$

From the databook for reaction (9) at 600 K, $\ln(K_{p_9}) = -3.191 \rightarrow K_{p_9} = 0.04113$

The partial pressures are given by,

$$p_{\text{NH}_3} = \frac{ap}{n}, \quad p_{\text{H}_2} = \frac{bp}{n}, \quad p_{\text{N}_2} = \frac{cp}{n}$$

Hence, for $p = 10$ bar,

$$\frac{an}{c^{1/2} b^{3/2}} = 0.04113 \left(\frac{p}{p_0}\right) = 0.4113$$

[15 %]

Eliminating b and c in favour of a using the atom balances we obtain,

$$\frac{4a(8-a)}{(4-a)^{1/2} (12-3a)^{3/2}} = 0.4113 \rightarrow \frac{4a(8-a)}{3\sqrt{3}(4-a)^2} = 0.4113$$

This is a quadratic equation in a :

$$6.1372a^2 - 49.097a + 34.195 = 0$$

Thus,

$$a = \frac{49.097 \pm \sqrt{49.097^2 - 4 \times 6.1372 \times 34.195}}{2 \times 6.1372} = 7.229 \text{ or } 0.771$$

Clearly, $a = 0.771$ and hence $b = 4.844$, $c = 1.615$ and $n = 7.230$

The mole fractions are : $X_{\text{NH}_3} = \frac{a}{n} = 0.107$, $X_{\text{H}_2} = \frac{b}{n} = 0.670$, $X_{\text{N}_2} = \frac{c}{n} = 0.223$

[20 %]

From the databook, increasing T reduces K_{p_9} . The equilibrium equation then shows that a is reduced while b and c are increased. Hence X_{H_2} increases (*i.e.*, more NH_3 dissociates at higher temperature).

[10 %]

(b) (i) From $Tds = dh - vdp$ and the Maxwell relation $(\partial s/\partial p)_T = -(\partial v/\partial T)_p$ we have,

$$\left(\frac{\partial h}{\partial p}\right)_T = v + T\left(\frac{\partial s}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p \quad [10 \%]$$

(ii) Differentiating with respect to T at constant p and noting that $c_p = (\partial h/\partial T)_p$ gives,

$$\left(\frac{\partial c_p}{\partial p}\right)_T = \left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial v}{\partial T}\right)_p - T\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

From the given equation of state $v = (RT/p)(1+Bp)$,

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + RB + RT\frac{dB}{dT}$$

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = 2R\frac{dB}{dT} + RT\frac{d^2B}{dT^2}$$

Hence,

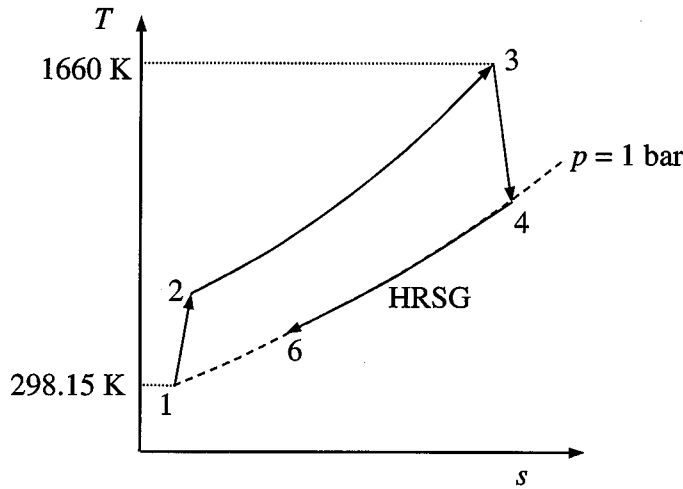
$$\left(\frac{\partial c_p}{\partial p}\right)_T = -2RT\frac{dB}{dT} - RT^2\frac{d^2B}{dT^2} = -\frac{d}{dT}\left(RT^2\frac{dB}{dT}\right)$$

Integrating with respect to p at constant T gives,

$$c_p = c_{p0} - p\frac{d}{dT}\left(RT^2\frac{dB}{dT}\right) \quad [30 \%]$$

where $c_{p0} = c_{p0}(T)$ is a function of T only.

3. (a)



[10%]

(b) (i) The formula for turbine polytropic efficiency is: $\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}\eta_{pt}}$

$$\Rightarrow r_p = \frac{p_3}{p_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{(\gamma-1)\eta_{pt}}} = \left(\frac{1660}{550+273.15}\right)^{\frac{1.33}{(0.33)0.9}} = \underline{23.13} \quad [10\%]$$

(ii) The temperature at compressor exit,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}\eta_{pc}} = 298.15 \times (23.13)^{0.4/(1.4 \times 0.85)} = \underline{857.0 \text{ K}} \quad [5\%]$$

(iii) SFEE for the adiabatic combustor ($A = \text{air-fuel mass ratio}$):

$$(A+1)h_p(T_3) - Ah_a(T_2) - h_f(T_0) = 0$$

From the definition of ΔH_{298}^0 (note that the excess air makes no difference):

$$(A+1)h_p(T_0) - Ah_a(T_0) - h_f(T_0) = \Delta H_{298}^0$$

Subtracting the second equation from the first gives,

$$(A+1)[h_p(T_3) - h_p(T_0)] - A[h_a(T_2) - h_a(T_0)] = -\Delta H_{298}^0$$

$$(A+1)c_{pp}(T_3 - T_0) - Ac_{pa}(T_2 - T_0) = -\Delta H_{298}^0$$

$$A = \left[\frac{-\Delta H_{298}^0 - c_{pp}(T_3 - T_0)}{c_{pp}(T_3 - T_0) - c_{pa}(T_2 - T_0)} \right]$$

$$A = \left[\frac{50000 - 1.15(1660 - 298.15)}{1.15(1660 - 298.15) - 1.01(857 - 298.15)} \right] = \underline{48.35} \quad [30\%]$$

(iv) Overall efficiency,

$$\eta_{ov} = \frac{\dot{W}_{net}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{(\dot{m}_f + \dot{m}_a)w_t - \dot{m}_a w_c}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{(A+1)c_{pp}(T_3 - T_4) - Ac_{pa}(T_2 - T_1)}{[-\Delta H_{298}^0]}$$

$$\Rightarrow \eta_{ov} = \frac{(48.35 + 1) \times 1.15 \times (1660 - 823.15) - 48.35 \times 1.01 \times (857 - 298.15)}{50000} = \underline{0.404}$$

[10%]

(c) Boiler efficiency,

$$\eta_b = \frac{\text{heat transferred}}{\text{max. heat transferred}} = \frac{c_{pp}(T_4 - T_6)}{c_{pp}(T_4 - T_0)} = \frac{550 - 140}{550 - 25} = 0.781$$

Overall combined cycle efficiency,

$$\eta_{cc} = \eta_{gt} + \eta_b \eta_{st} (1 - \eta_{gt}) = 0.404 + 0.781 \times 0.38 \times (1 - 0.404) = \underline{0.581}$$

Alternatively,

$$\eta_{ov} = \frac{\dot{W}_{gt} + \dot{W}_{st}}{\dot{m}_f [-\Delta H_{298}^0]} = \frac{Aw_{gt} + (A+1)c_{pp}(T_4 - T_6)\eta_{st}}{[-\Delta H_{298}^0]}$$

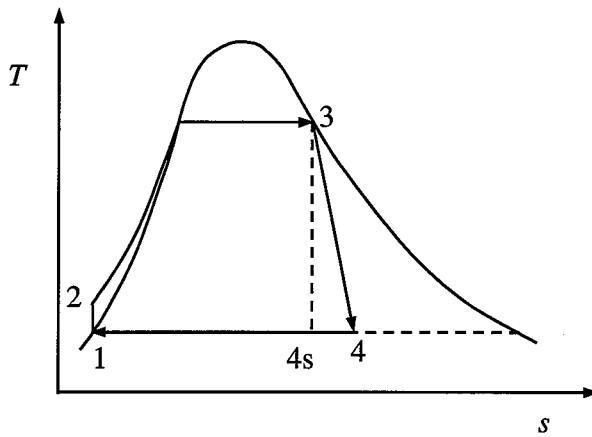
$$\Rightarrow \eta_{ov} = \frac{50000 \times 0.404 + (48.35 + 1) \times 1.15 \times (550 - 140) \times 0.38}{50000} = \underline{0.581}$$

[20%]

(d) Aero-derivatives have high pressure ratio, $r_p \sim 40$ leading to an exhaust gas temperature that is too low to drive a steam cycle. The efficiency of an aero-derivative can be increased by using steam injection. The water/steam, before being injected into the combustion chamber, is heated by the exhaust air from the turbine, thus minimising the temperature (and lost exergy) of the exhaust gas leading to higher overall efficiency.

[15%]

4.



(a) For an isentropic turbine expansion,

$$x_{4s} = \frac{s_3 - s_1}{s_{fg}} = \frac{6.070 - 0.521}{8.329 - 0.521} = 0.7107$$

Hence : $h_{4s} = h_1 + x_{4s}h_{fg} = 151.5 + 0.7107 \times (2566.6 - 151.5) = 1867.9 \text{ kJ/kg}$

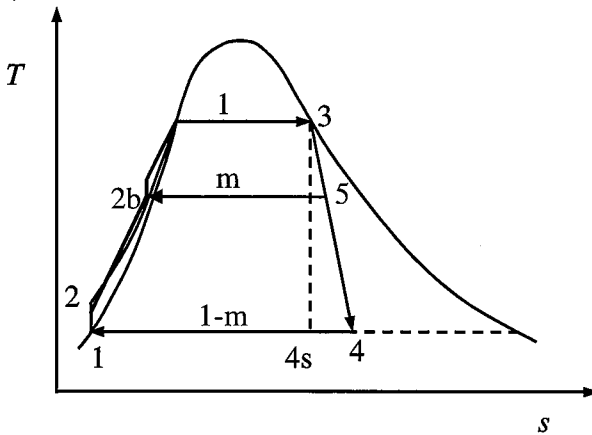
From the definition of turbine isentropic efficiency,

$$h_4 = h_3 + \eta_T(h_3 - h_{4s}) = 2800.8 - 0.86 \times (2800.8 - 1867.9) = 1998.5 \text{ kJ/kg}$$

Neglecting the feed pump work, the cycle efficiency is,

$$\eta_c = \frac{h_3 - h_4}{h_3 - h_1} = \frac{2800.8 - 1998.5}{2800.8 - 151.5} = \underline{0.303} \quad [20\%]$$

(b)



(i) SFEE for the feed heater: $mh_5 + (1 - m)h_1 = h_{2b}$

h_5 is at $p = 16 \text{ bar}$, on a straight line between points 3 and 4. [note: fairly approximate values accepted here]
From the steam chart, $h_5 = 2660 \text{ kJ/kg}$, $s_5 = 6.15 \text{ kJ/kgK}$

$$\Rightarrow m = \frac{h_{2b} - h_1}{h_5 - h_1} = \frac{858.5 - 151.5}{2660 - 151.5} = \underline{0.282} \quad [15\%]$$

(ii) Turbine work output per kg steam in boiler, $w_t = (h_3 - h_5) + (1 - m)(h_5 - h_4)$
 $\Rightarrow w_t = (2800.8 - 2660) + (1 - 0.282) \times (2660 - 1998.5) = 615.8 \text{ kJ/kg}$

$$\text{Cycle efficiency, } \eta_c = \frac{w_t}{h_3 - h_{2b}} = \frac{615.8}{2800.8 - 858.5} = 0.317$$

$$\text{Rate of solar heat input} = \text{Power out} / \text{efficiency} = \frac{11}{0.317} = 34.7 \text{ MW} \quad [15\%]$$

(iii) Mass flow rate through the boiler, $\dot{m}_{st} = \dot{W}_t / w_t = 11000 / 615.8 = 17.86 \text{ kg/s}$

Lost power in feed heater = Rate of Exergy In – Rate of Exergy Out

$$\begin{aligned} &= \dot{m}_{st} [m(h_5 - T_0 s_5) + (1 - m)(h_1 - T_0 s_1) - 1 \cdot (h_{2b} - T_0 s_{2b})] \\ \text{(considering SFEE above)} &= \dot{m}_{st} [-T_0 s_5 m - T_0 s_1 (1 - m) + T_0 s_{2b}] \\ &= \dot{m}_{st} T_0 [-s_5 m - s_1 (1 - m) + s_{2b}] \end{aligned}$$

$$= 17.86 \times (273.15 + 36.16) \times (-6.15 \times 0.282 - 0.521 \times (1 - 0.282) + 2.343)$$

$$= 1296 \text{ kW (or 1.3 MW)}$$

This is a significant amount of lost power for a 11MW plant. It is very high because the pressure (and temperature) of the bled steam is high, leading to high losses when mixed with feed water at (approximately) the condenser temperature. [25%]

(c) The lost work in the feed heater is high. However, by increasing the inlet temperature to the boiler, the feed heater raises the mean temperature of heat addition, the benefit of which is enough to outweigh the lost work in feed heater.

There are a few possible ways of increasing the efficiency further (2 required):

1. The lost work in the feed heating process could be greatly reduced by having multiple feed heaters (particularly more at lower pressures). These would suffer lower losses because the temperature differences between the feed water and bled steam would be lower. Modern power plants typically use 5-7 feed heaters, with similar temperature differences between the flows in each one.
2. The application of reheat (probably at a pressure of 10-16 bar), could lead to a marginally higher mean temperature of heat addition, but more importantly, it would reduce the wetness of the steam at the LP turbine exhaust, leading to higher overall isentropic efficiency of the turbine.
3. It may be possible to improve the current isentropic efficiency of the turbine through better design. However, this is unlikely to be possible without reducing the wetness of the steam at the turbine exhaust.

[25%]

Data for question 4 (from steam tables):

<u>Condition</u>	<u>Specific enthalpy</u> (kJ kg ⁻¹)	<u>Specific entropy</u> (kJ kg ⁻¹ K ⁻¹)	<u>Temperature</u> (°C)
Saturated vapour at 40 bar	2800.8	6.070	
Saturated liquid at 16 bar	858.5	2.343	
Saturated liquid at 0.06 bar	151.5	0.521	36.16
Saturated vapour at 0.06 bar	2566.6	8.329	36.16