3A6 Exam 2011 Crib

1)

(a) (i) Considering an element of length dx from x to $x+dx$ along the rod, the energy balance is:

$$\underbrace{-kA\frac{dT}{dx}+kA\left(\frac{dT}{dx}+\frac{d^2T}{dx^2}\,dx\right)}_{\text{net heat conduction}}+\underbrace{\dot{Q}A\,dx}_{\text{volumetric heat release}}\underbrace{-hP(T-T_\infty)\,dx}_{\text{convection}}=0$$

where $A=\pi R^2$ and $P=2\pi R$.

Dividing by $kA\,dx$ and cancelling out terms, we have:

$$\frac{d^2T}{dx^2}+\frac{\dot{Q}}{k}-\frac{hP}{kA}(T-T_\infty)=0$$

$$\frac{d^2T}{dx^2}+\frac{\dot{Q}}{k}-\frac{2h}{kR}(T-T_\infty)=0$$

(ii) We consider the ratio between the two last terms in the equation:

$$N_q=\frac{\dot{Q}R}{2h(T-T_\infty)}$$

T is a variable of the same order as T_1 and T_2, so either serves as a representative magnitude. The non-dimensional quantity of interest is therefore:

$$N_q=\frac{\dot{Q}R}{2h(T_1-T_\infty)}$$

If $N_q>>1$ then the heat losses are negligible. If $N_q<<1$ then the volumetric heat release is negligible.

(iii) For negligible convection losses, the term $\frac{2h}{kR}(T-T_\infty)<<\frac{\dot{Q}}{k}$, so we have:

$$\frac{d^2T}{dx^2}+\frac{\dot{Q}}{k}=0$$

which integrates to:

$$T(x)=-\frac{\dot{Q}}{2k}x^2+Ax+B$$

The boundary conditions are $T(0)=T_1$ and $T(L)=T_2$, so that:

$$B=T_1\qquad T(L)=T_1-\frac{\dot{Q}}{2k}L^2+AL=T_2\quad\longrightarrow A=\left[(T_2-T_1)+\frac{\dot{Q}}{2k}L^2\right]/L$$

$$T(x) = T_1 + \frac{\dot{Q}L^2}{2k}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - (T_1 - T_2)\frac{x}{L}$$

(b) (i) We perform an energy balance similar to the one in (a), but now for the radial element inside the rod, in which there is only conduction (in two directions, x and r, and volumetric heat generation:

$$\underbrace{-k\,2\pi r\,dr\frac{\partial T}{\partial x} + k\,2\pi r\,dr\left(\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}\,dx\right)}_{\text{net heat conduction in } x} +$$

$$\underbrace{-k\,2\pi r\,dx\frac{\partial T}{\partial r} + k\,2\pi(r+dr)\,dx\left(\frac{\partial T}{\partial r} + \frac{d^2 T}{dr^2}\,dr\right)}_{\text{net heat conduction in } r} + \underbrace{\dot{Q}\,2\pi r\,dr\,dx}_{\text{volumetric heat release}} = 0$$

We expand the brackets and neglect higher order terms to obtain:

$$k\,2\pi r\,dr\frac{\partial^2 T}{\partial x^2}\,dx + k\,2\pi\,dx\underbrace{\left(\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r^2}\right)}_{\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}dr + \dot{Q}\,2\pi r\,dr\,dx = 0$$

We divide by $2\pi k\,dr\,dx$ to get the desired equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{Q}}{k} = 0$$

(ii) Assuming that there is no conduction along x, and that the surface temperature is T_s, we can integrate the equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{Q}}{k} = 0$$

to yield:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{\dot{Q}}{k}$$

$$\left(r\frac{\partial T}{\partial r}\right) = -\frac{\dot{Q}r^2}{2k} + K_1$$

$$T = -\frac{\dot{Q}r^2}{4k} + K_1 \ln r + K_2$$

K_1 must be zero so the solution does not blow up at $r = 0$. We use $T(R) = T_s$ to obtain:

$$T(R) = -\frac{\dot{Q}R^2}{4k} + K_2 = T_s$$
$$K_2 = T_s + \frac{\dot{Q}R^2}{4k}$$
$$T(r) = T_s + \frac{\dot{Q}R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

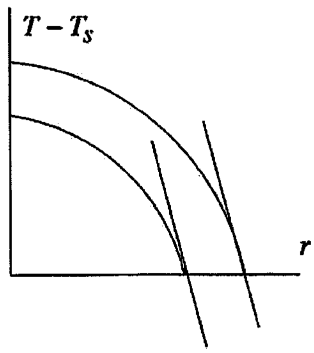
(iii) The heat flux at the surface on the inside of the rod must match the heat loss:

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{\dot{Q}R}{2k}$$

But the gradient is fixed. In order to match the conditions, T_s will rise or fall so that in either case:

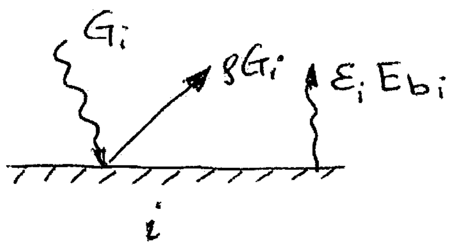
$$\frac{\dot{Q}R}{2k} = h(T_s - T_\infty) \qquad \text{or}$$
$$= \sigma\varepsilon(T_s^4 - T_\infty^4)$$

Note that the maximum (inner) temperature will rise or fall accordingly as well.



2)

(a)



Radiosity $J_i = \rho G_i + \varepsilon_i E_{bi}$ — (1)

Net heat transfer leaving the surface $Q_i = A_i (J_i - G_i)$ — (2)

Kirchoffs law:

$\rho_i + \alpha_i + \cancel{\tau_i}^{\,0} = 1$, $\alpha_i = \varepsilon_i$

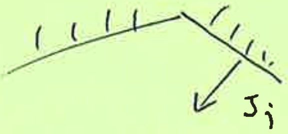
$\Rightarrow \boxed{\rho_i = 1 - \varepsilon_i}$ — (3)

from (1) & (3) $G_i = \dfrac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}$ — (4)

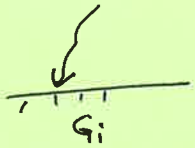
using (2) $\dfrac{Q_i}{A_i} = \dfrac{\varepsilon_i (E_{bi} - J_i)}{(1 - \varepsilon_i)}$

$\Rightarrow \boxed{Q_i = \dfrac{\varepsilon_i A_i (E_{bi} - J_i)}{(1 - \varepsilon_i)}}$

(b)



$$G_i = \sum_{j=1}^{N} \frac{J_j A_j F_{ji}}{A_i} \qquad (1)$$



$$A_i F_{ij} = A_j F_{ji} \qquad (2)$$

$$J_i = G_i \rho_i + \varepsilon_i E_{bi} \qquad (3)$$

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{\rho_i} \qquad (4)$$

(4) + (1) :

$$\frac{J_i - \varepsilon_i E_{bi}}{\rho_i} = \sum_{j=1}^{N} \frac{J_j A_j F_{ji}}{A_i} =$$

$$= J_i \frac{A_i F_{ii}}{A_i} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{J_j \overbrace{A_j F_{ji}}^{A_i F_{ij}}}{A_i}$$

$$= J_i F_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} J_j F_{ij} \qquad (5)$$

SOLVING FOR J_i :

$$J_i \left(\frac{1}{\rho_i} - F_{ii} \right) = \sum_{\substack{j=1 \\ i \neq j}}^{N} J_j F_{ij} + \frac{\varepsilon_i}{\rho_i} E_{bi}$$

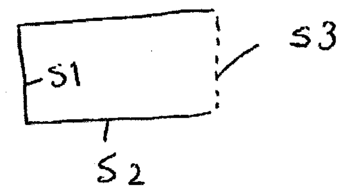
$$J_i = \frac{\rho_i}{1 - \rho_i F_{ii}} \left\{ \frac{\varepsilon_i}{\rho_i} E_{bi} + \sum_{\substack{j=1 \\ i \neq j}}^{N} J_j F_{ij} \right\}.$$

$$\left[J_i = \frac{1}{1 - (1-\varepsilon_i) F_{ii}} \left\{ \varepsilon_i E_{bi} + (1-\varepsilon_i) \sum_{\substack{j=1 \\ i \neq j}}^{N} J_j F_{ij} \right\}. \right]$$

QED

(C)

(i)



$A_1 = \frac{\pi D^2}{4} = 7.854 \times 10^{-5}\ m^2$

$A_2 = \pi D L = 6.283 \times 10^{-4}\ m^2$

$\epsilon = 0.9$

<u>Define:</u> $F_{13} = F_{31} = f, \quad \frac{A_1}{A_2} = \alpha$

$$\cancel{F_{11}}^{\,0} + F_{12} + F_{13} = 1 \quad \Rightarrow \quad \boxed{F_{12} = (1-f)} \quad — \ (5)$$

$F_{22} + F_{21} + F_{23} = 1$; also $F_{21} = F_{23}$

$$F_{22} = 1 - 2F_{21} ;$$

but $A_1 F_{12} = A_2 F_{21} \Rightarrow F_{21} = \alpha F_{12}$

from (5) $F_{21} = \alpha(1-f)$

$$\Rightarrow \boxed{F_{22} = 1 - 2\alpha(1-f)}$$

By symmetry: $F_{13} = F_{31}$

$F_{21} = F_{23} = \alpha(1-f)$

$F_{32} = F_{12}$

Given $f = 0.6$; $\alpha = \frac{A_1}{A_2} = \frac{\pi D^2}{4}\frac{1}{\pi D L} = \frac{1}{4}\overset{0.5}{\frac{D}{L}} = \frac{1}{8}$.

$\Rightarrow$

$F_{11} = 0$	$F_{12} = 0.4$	$F_{13} = 0.6$
$F_{21} = 0.05$	$F_{22} = 0.9$	$F_{23} = 0.05$
$F_{31} = 0.6$	$F_{32} = 0.4$	$F_{33} = 0$

Black body emission:

$$E_{b_1} = \sigma T_1^4 = \sigma\, 1000^4 = 5.67\times10^4 \ \text{W/m}^2$$

$$E_{b_2} = \sigma T_2^4 = \sigma\, 1000^4 = 5.67\times10^4 \ \text{W/m}^2$$

$$E_{b_3} = \sigma T_3^4 = \sigma\, 300^4 = 4.593\times10^2 \ \text{W/m}^2$$

(ii)

$$J_1 = \frac{1}{1-F_{11}(1-\epsilon)}\left[\epsilon E_{b_1} + (1-\epsilon)(J_2 F_{12} + J_3 F_{13})\right]$$

$$J_2 = \frac{1}{1-F_{22}(1-\epsilon)}\left[\epsilon E_{b_2} + (1-\epsilon)(J_1 F_{21} + J_3 F_{23})\right]$$

$$J_3 = \frac{1}{1-F_{33}(1-\epsilon)}\left[\epsilon E_{b_3} + (1-\epsilon)(J_2 F_{32} + J_1 F_{31})\right]$$

Substituting the values for F_{ij} & $\epsilon = 0.9$

$$\boxed{\begin{aligned} J_1 &= 0.9\, E_{b_1} + 0.04\, J_2 + 0.06\, J_3 \\ J_2 &= 0.9891\, E_{b_2} + 5.495\times10^{-3} J_1 + 5.495\times10^{-3} J_3 \\ J_3 &= 0.9\, E_{b_3} + 0.04\, J_2 + 0.06\, J_1 \end{aligned}}$$

(iii) Total radiative heat loss is

$$Q_t = Q_1 + Q_2$$

using part (a) $Q_1 = \frac{\varepsilon A_1 (E_{b_1} - J_1)}{(1-\epsilon)}$

$$Q_1 = \frac{0.9 \times 7.854 \times 10^{-5}}{0.1} (E_{b_1} - J_1)$$

$$\boxed{Q_1 = 7.069 \times 10^{-4} (E_{b_1} - J_1)}$$

$$Q_2 = \frac{\varepsilon A_2 (E_{b_2} - J_2)}{(1-\epsilon)} = \frac{0.9 \times 6.283 \times 10^{-4}}{0.1} (E_{b_2} - J_2)$$

$$\boxed{Q_2 = 5.655 \times 10^{-3} (E_{b_2} - J_2)}$$

$$Q_t = -7.069 \times 10^{-4} J_1 - 5.655 \times 10^{-3} J_2 + 6.362 \times 10^{-3} E_{b_1}$$

$$\boxed{Q_t = 360.72 - 7.069 \times 10^{-4} J_1 - 5.655 \times 10^{-3} J_2}$$

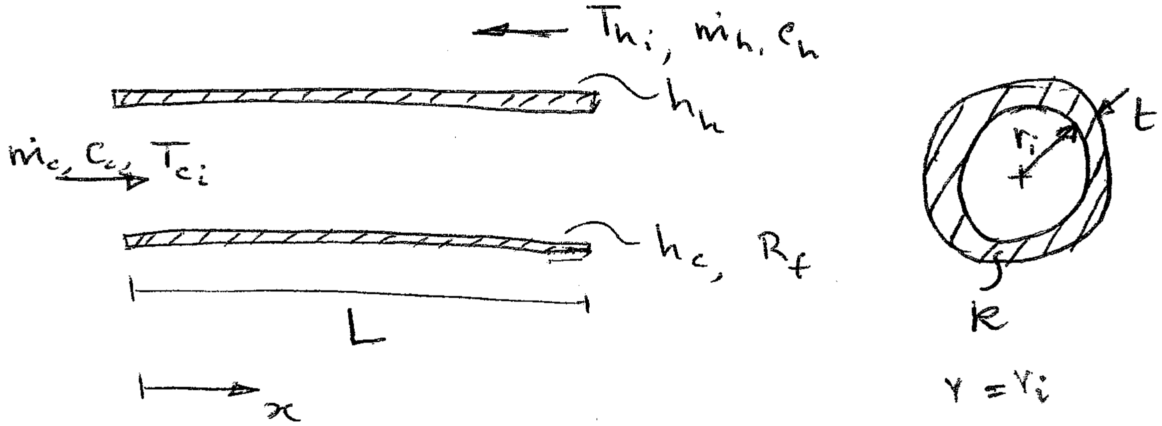
$$= \varepsilon^* A_3 E_{b_1} \qquad A_3 = A_1$$

$$\Rightarrow \boxed{\varepsilon^* = 81 - 9 \frac{J_1}{E_{b_1}} - 72 \frac{J_2}{E_{b_1}}}$$

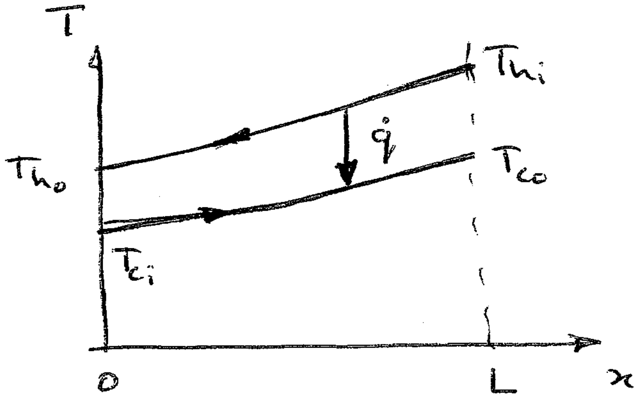
$E_{b_1} = 5.67 \times 10^4$ W/m^2

$$\Rightarrow \boxed{\varepsilon^* = 81 - 1.587 \times 10^{-4} J_1 - 1.269 \times 10^{-3} J_2}$$

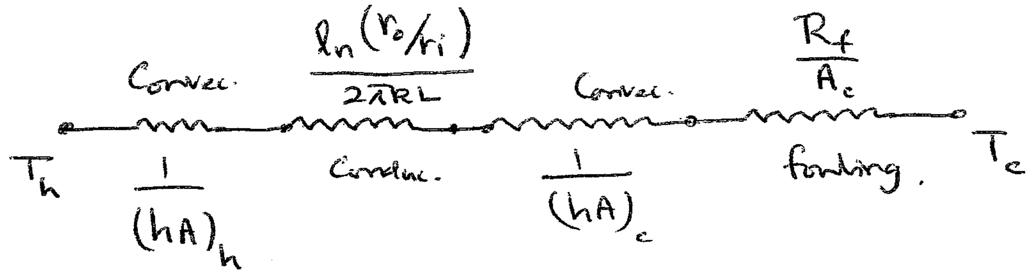
3)



(a)



(b)



Total resistance

$$\frac{1}{UA} = \hat{R}_t = \frac{1}{(hA)_h} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{(hA)_c} + \frac{R_f}{A_c}$$

$A = A_c$ (U - based on the inside area)

$$\frac{1}{U} = \left(A_c \hat{R}_t\right) = R_t = \frac{A_c}{h_h A_h} + \frac{A_c \ln\left(\frac{r_o}{r_i}\right)}{2\pi kL} + \frac{1}{h_c} + R_f$$

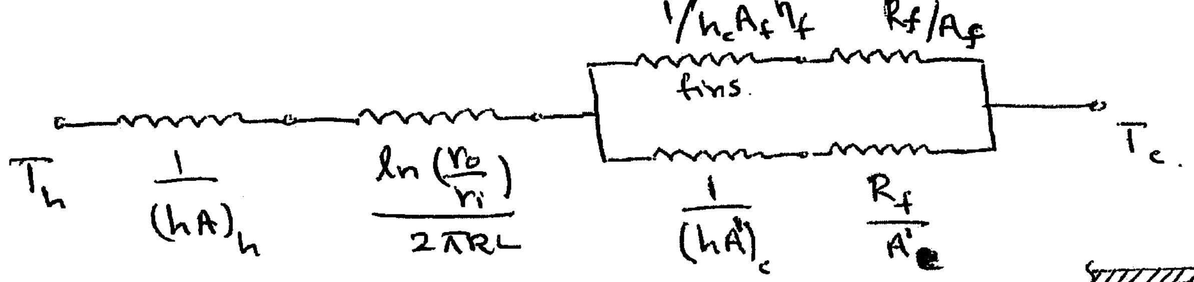
$A_c = 2\pi r_i L$, $\qquad A_h = 2\pi(r_i+t)L$

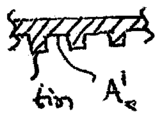
$$\frac{r_o}{r_i} = \left(1+\frac{t}{r_i}\right)$$

$$\Rightarrow \boxed{R_t = \frac{r_i}{(r_i+t)h_h} + \frac{r_i}{k}\ln\left(1+\frac{t}{r_i}\right) + \frac{1}{h_c} + R_f}$$

$$\Rightarrow \quad \underline{\underline{U = R_t^{-1}}}$$

(c) When fins are used on the cold side:





Comment:

The convection via the prime surface area (A') and the enhanced convection via the fins would act in parallel. Thus these two resistances are to be included as above. The fouling resistance will act in series with each of these two resistances.

$A_f = n\pi d l$

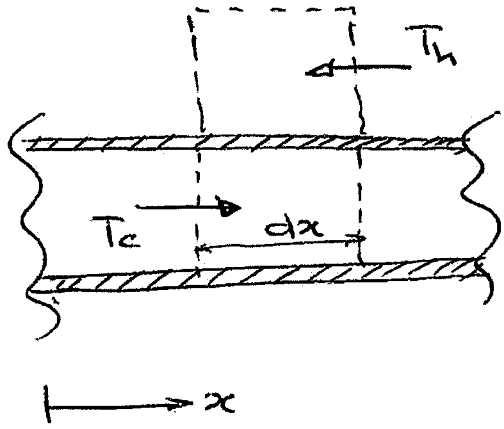
n - number of fins
d - diameter of the fin
l - length of the fin

η_f - fin efficiency

A'_c - prime surface area on the cold side
$(A - A_f) = (2\pi r_i L - A_f)$

(11)

(d)



$$\dot{Q} = (T_{hi} - T_{ho})\dot{m}_h c_h$$
$$= C_h (T_{hi} - T_{ho})$$

also

$$\dot{Q} = \dot{m}_c c_c (T_{co} - T_{ci})$$
$$= C_c (T_{co} - T_{ci})$$

Hot side:

$$-\dot{m}_h c_h \, dT_h = -UP\,dx\,(T_h - T_c)$$

$$\Rightarrow \quad \frac{dT_h}{dx} = \left(\frac{UP}{C_h}\right)(T_h - T_c) \quad \text{— (1)}$$

Cold side:

$$\dot{m} c_c \, dT_c = UP\,dx\,(T_h - T_c)$$

$$\Rightarrow \quad \frac{dT_c}{dx} = \left(\frac{UP}{C_c}\right)(T_h - T_c) \quad \text{— (2)}$$

Subtract (2) from (1)

$$\frac{d}{dx}(T_h - T_c) = UP(T_h - T_c)\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\Rightarrow \quad \boxed{\frac{d\,\Delta T}{dx} = UP\,\Delta T\left(\frac{1}{C_h} - \frac{1}{C_c}\right)}$$

Heat capacities are matched

$$C_h = C_c = C$$

$$\Rightarrow \quad \frac{d\Delta T}{dx} = 0 \quad \Rightarrow \quad \boxed{\Delta T = \text{Const.}}$$

as required.

from (1)

$$\frac{dT_h}{dx} = \left(\frac{UP}{c}\right)\Delta T$$

$$\int_0^x dx \quad \Rightarrow \quad \boxed{T_h(x) = T_{h_o} + \left(\frac{UP}{c}\right)\Delta T\, x} \quad \text{— (3)}$$

since U is Const.

Ily from (2)

$$T_c(x) = T_{c_i} + \left(\frac{UP}{c}\right)\Delta T\, x$$

$$\Rightarrow \quad \boxed{T_h \;\&\; T_c \text{ vary linearly with } x.}$$

(e) from (3)

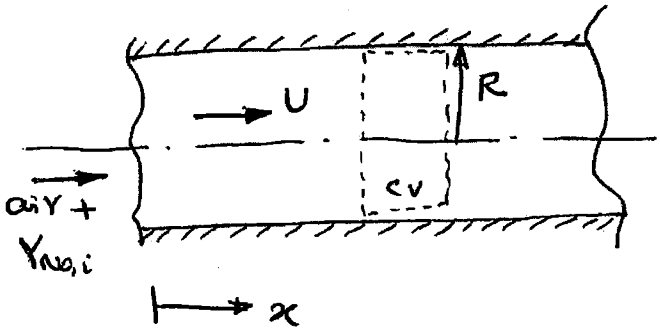
$$\frac{(T_{h_i} - T_{h_o})\,c}{(T_{h_i} - T_{c_o})\,UP} = \boxed{L = \frac{(T_{c_o} - T_{c_i})\,c}{(T_{h_i} - T_{c_o})\,UP}} \quad (4)$$

from (4) for given T_{ci}, T_{hi}, T_{co}, L is reduced as

(1) c decreases, (2) U is increased

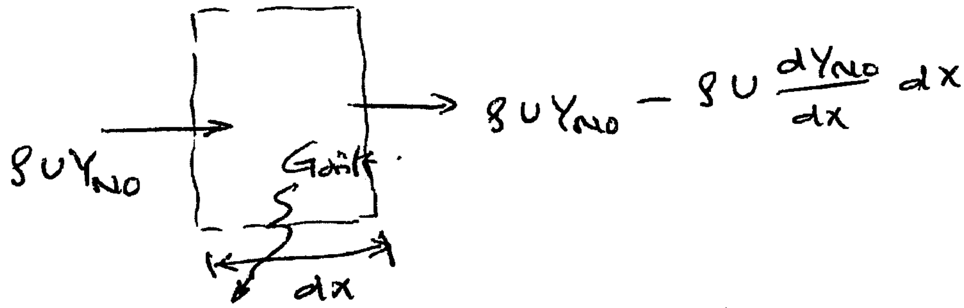
(3) P is increased.

(4)



$$h_m = \frac{2.18\,D}{R} \quad m/s$$

(a) For the control volume shown:



mass balance for NO.

$$\underset{\text{Steady}}{\cancel{\text{rate of change}}}^{\,0} = \text{in} - \text{out} + \cancel{\text{Source}}^{\,0} - \cancel{\text{sink}}^{\,0}.$$

(Since the chemical reaction occurs on the surface)

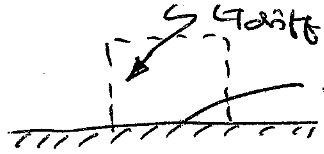
$$A_c \rho U Y_{NO} - A_c \rho U Y_{NO} + A_c \rho U \frac{dY_{NO}}{dx} dx - G_{diff} P\, dx = 0$$

$$\Rightarrow \rho U A_c \frac{dY_{NO}}{dx} = G_{diff} P = \rho h_m (Y_{NO,s} - Y_{NO}) P$$

for Convective mass transfer.

$$\Rightarrow \boxed{\frac{dY_{NO}}{dx} = \frac{h_m}{U} \frac{P}{A_c} (Y_{NO,s} - Y_{NO})}$$ as required.

(b) on the surface:



$\dot{\Omega}_{NO} = -K\rho Y_{NO,s}$

Since the NO diffusing towards the surface is consumed by the surface reaction

diffusive flux = reactive flux

$$\Rightarrow \rho h_m \left(Y_{NO,s} - Y_{NO}\right) = -K\rho Y_{NO,s}$$

Rearranging:

$$\boxed{\frac{Y_{NO,s}}{Y_{NO}} = \left(\frac{h_m}{h_m + K}\right)}$$

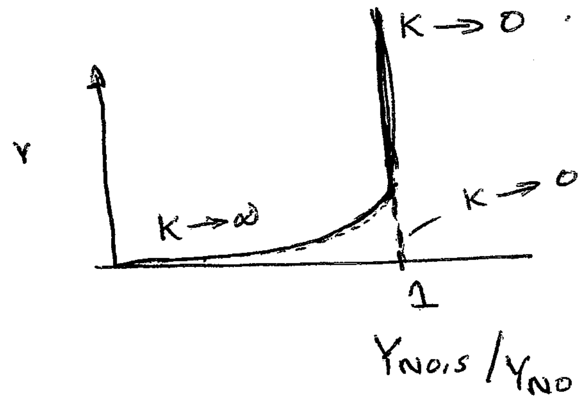
if $K \to 0$, kinetic limit (slow reaction)

$$Y_{NO,s} \to Y_{NO}$$

if $K \to \infty$, diffusion limit (very fast reaction)

$$\frac{Y_{NO,s}}{Y_{NO}} = \left(\frac{h_m/K}{h_m/K + 1}\right) \Rightarrow Y_{NO,s} \to 0$$

$\Rightarrow$



(c) diffusion limited $\Rightarrow$ $Y_{NO,s} = 0$

$$\Rightarrow \quad \frac{dY_{NO}}{dx} = -\left(\frac{h_m P}{UA}\right) Y_{NO}$$

integrate from 0 to x

$$\Rightarrow \quad Y_{NO}(x) = Y_{NO,i} \exp\left[-\frac{h_m P}{UA} x\right]$$

$$\frac{P}{A} = \frac{2\pi R}{\pi R^2} = \frac{2}{R}$$

$$\therefore \quad \boxed{Y_{NO}(x) = Y_{NO,i} \exp\left[-\frac{2h_m}{UR} x\right]}$$

(d) $$\frac{Y_{NO,L}}{Y_{NO,i}} = 0.01 = \exp\left[-\frac{2h_m L}{UR}\right]$$

$$\frac{4.36 \mathcal{D} L}{UR^2} = -\ln(0.01) = 4.605$$

$$\Rightarrow \quad L = 1.056 \frac{UR^2}{\mathcal{D}} = 1.056 \frac{\dot{V}}{\pi \mathcal{D}}$$

$\dot{V}$ = Volumetric flow rate = 2 lpm = 3.333×10^{-5} m^3/s

$\mathcal{D} = 1.12 \times 10^{-5}$ m^2/s

$$\Rightarrow \quad \boxed{L = 1.0 \text{ m.}}$$

IIA Paper 3A5 2011

Thermodynamics and Power Generation - Numerical answers

1.
(a) 218.2 MW, 33.4%
(b) 292.6 MW, Turbine lost power = 37.6 MW
Condenser lost power = 36.8 MW
74.6%
(c) Condenser lost power = 22.1 MW
Cooling water lost power = 14.7 MW
Turbine lost power = 37.6 MW
74.6%

2.
(a) (i) 0.107, 0.670, 0.223

3.
(b) (i)23.13 (ii) 857 K (iii) 48.35 (iv) 40.4%
(c) 58.1%

4.
(a) 30.3%
(b) (i) 0.282 (ii) 31.7%, 34.7 MW (iii) 1.30 MW

Numerical Answers

3A6 – Heat and Mass Transfer, 2011

2) c)(i)	$F_{11} = 0$	$F_{12} = 0.4$	$F_{13} = 0.6$
	$F_{21} = 0.05$	$F_{22} = 0.9$	$F_{23} = 0.05$
	$F_{31} = 0.6$	$F_{32} = 0.4$	$F_{33} = 0$

$E_{b1} = E_{b2} = 5.67\text{E}04$ W/m^2, Eb3 = 4.593E02 W/m^2

4) d) 1 m

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1/June/2011