Dr P R Palmer

3B4 CRIB 2011

1 a/ Several ways. Force p.u. area of the surface of the rotor is $J \times B$ (left hand rule!) ² B reverses at the poles, N-S-N-S etc, and in any sensible motor J also reverses under the poles, giving the total force. ² ∴ pole number does not affect the design. QED Proving this by volume is not necessary as say 4 & 6 pole can be compared. See below for gearboxes.

OR.

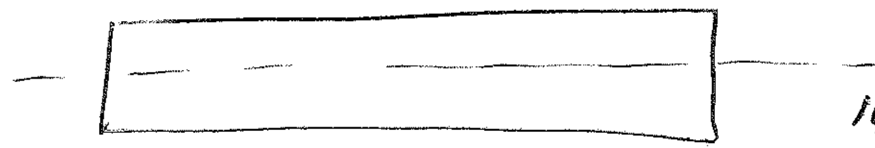
$$S = \frac{\pi}{\sqrt{2}} \cdot \pi\left(\frac{d}{2}\right)^2 l \, \frac{\omega}{p} \cdot \bar{B} \cdot \bar{J} \qquad 2$$

where ω is the frequency and p pole pairs. $\frac{\omega}{p}$ is the base speed. So for same speed $\omega \propto p$. ² Here more speed means more power hence wide use of gearboxes since smaller motor for a given power. ¹

(Above $J \times B$ is force; faster rotation all else the same means more power hence use of gearboxes. (J & B are constraints)) ¹

1b

125mm



100,000 rpm.

$$\frac{100,000}{60} \times 2\pi$$

$$S = \frac{\pi}{\sqrt{2}} \times \pi\left(\frac{d}{2}\right)^2 l \cdot \frac{\omega}{p} \; \bar{B} \; \bar{J}$$

$$= \frac{\pi}{\sqrt{2}} \times \pi\left(\frac{0.03}{2}\right)^2 \times 0.125 \times \frac{100,000}{60} \times 2\pi \cdot 0.5 \times 30,000$$

$$= 30.8 \text{ kVA} \qquad 4$$

$$P = S \times 0.8 \times 0.8 \doteq 19.7 \text{ kW}$$

$$\frac{100,00\cancel{0}}{6\cancel{0}} = 1600 \text{ Hz} \qquad 3300 \text{ Hz for 4 pole.}$$

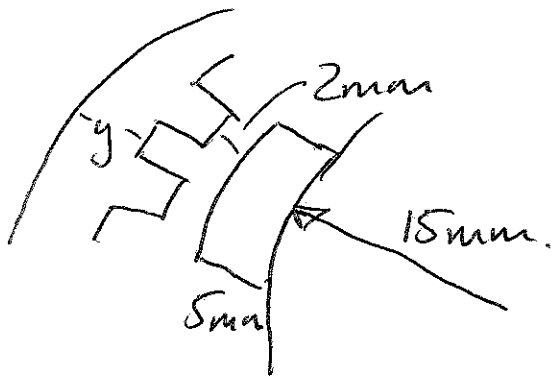
any 3

1, Long & small diameter so endwinding is small part so small losses. High frequency goes higher, still if 4 or 6 pole is used; its less of an issue 1

2, Low inertia, so can accelerate the turbo really fast ~ needs around 0.1 sec! 1

3, i Keeping magnets glued on is easier.
ii Unbalanced load on bearings is less / easier to manage.
iii Short flux path in iron parts 1 (any one)

1b cont. 2 pole 3 phase. star. ($p=1$)



$$\frac{2470}{\sqrt{3}} = 1426\,V$$

$I = 15A.$

$5\,A\,mm^{-1}$

Lets distribute by 3 ~ assume $k_d \approx 1$

$1428 = l\,\omega\,d\,N_{eff}\,B_{rms}$. $\quad B_{rms} = \frac{\pi \bar{B}}{2\sqrt{2}}$

$$1428 = 0.125 \times 2\pi.\frac{100{,}000}{60} \times 0.044 \times \frac{\pi\,0.5}{2\sqrt{2}} N_{eff}$$

$N_{eff} = 44.6 \Rightarrow 45$ turns (3×15)

$\frac{15}{5} = 3mm^2$; 15 turns $45\,mm^2$ per slot

18 slots. Assume a fill factor of 0.6

$L \quad \pi d.\frac{20}{360} = 7.7mm.$ Say 4 mm wide slot

$$\frac{45mm^2}{0.6 \times 4mm} = 18.75mm.$$

Core - $\hat{B}_c\, y = \frac{1}{2}\pi \frac{d}{2}\bar{B}$ keep losses down ~ $\hat{B}_c = 1T$

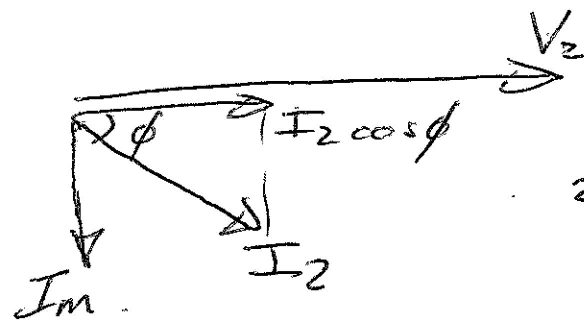
$$y = \frac{1}{2}\pi\frac{0.044}{2}.0.5 = 0.0173$$

Diameter $0.044 + 2\times0.0173 + 2\times0.0187 = 0.116$

$\overline{116\,mm}$

Iterate with distribution by 2 ~ maybe easier to make!

2 a) ~~Bily~~ phasor diagram.

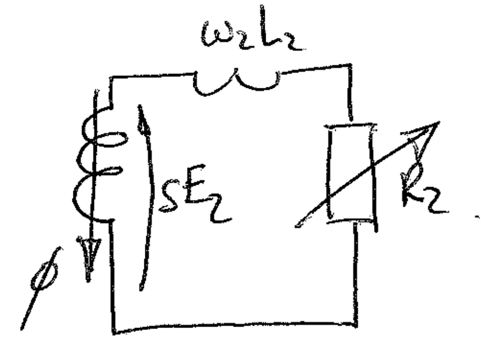


V_2 is orthogonal to the flux vector. Magnitude $\propto \hat{B}$ & ω.

2

Clearly power is $T\omega_s = V_2 I_2 \cos\phi$.

$V \propto \phi\omega \quad V_2 = k\phi \frac{\omega}{2}$

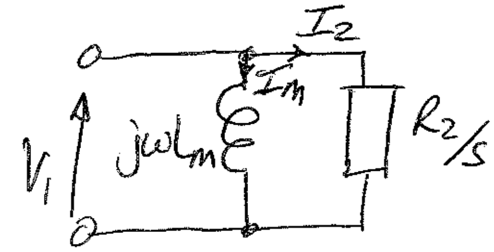


$T = \frac{V_2}{\omega_s} . I_2 \cos\phi$

$T = K\phi I_2 \cos\phi$ 2

The mechanical arrangement of the coils gives position to the voltage, current and flux, which also have magnitudes. Vector sum of the three phase windings vectors produces a single vector for each, which rotates in space.

2 a cont/ Neglect R_1, $j\omega L_1$ and $j\omega L_2$ at full flux



$$T\frac{\omega}{p} = 3 I_2^2 \frac{R_2}{s}$$

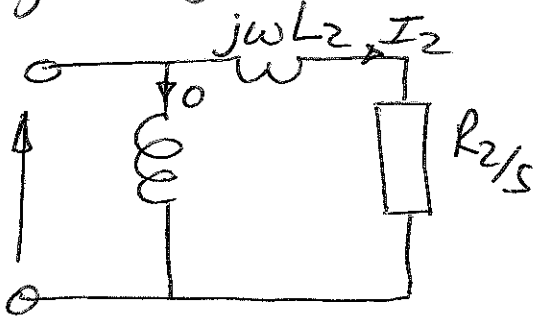
$$T = 3p\left(\frac{V_1}{\omega}\right)^2 \frac{s\omega}{R_2}$$

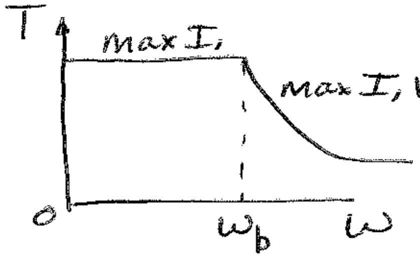
$k\phi$ as above.

In steady state $\frac{V_1}{\omega L_m}$ is constant. In a transient V_1 can rise but the current producing the flux I_m cannot change much as L_m is large. So a sharp change in V_1 and ω (and s) will produce much more I_2 and torque independently of the flux.* Treating s as a transient quantity means that the angle of V_1 with respect to the flux will be calculated correctly (Vector control)

* After a while it settles down with the right flux as $\frac{V}{\omega}$ = const

2 b/ neglecting stator terms





@ 100Hz 353V pf 0.96 $\frac{100\,kW}{3}$

$$\frac{100kW}{3}\left(\frac{1}{1-0.02}\right) = I_2^2 \, R_2/0.02$$

$$\frac{100kW}{3}\cancel{\left(\frac{1}{1-0.02}\right)} = V_1 I_2 \cos\phi$$

$$I_2 = \frac{100k}{3} \cdot \frac{1}{353} \cdot \frac{1}{0.96} = 98.36\,A$$

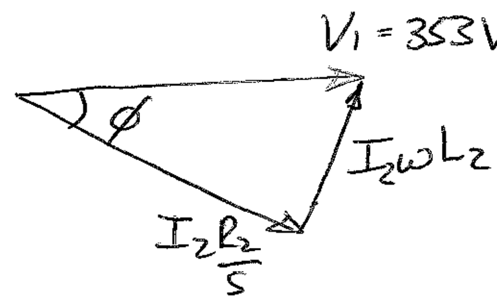
$$R_2 = 0.07$$

@ 150 Hz : $T = 3\frac{V^2}{\omega^{\cancel{2}}} \cdot \frac{s\cancel{\omega}}{R_2}$, neg. ωL_2 ; V_2 at max 353V

$\Rightarrow T \propto \frac{1}{\omega}$ "const power"

Inverter & motor current at max 98.36A.
so for same power R_2/s is const $\therefore$ $s = 0.02$ (Rated)

But check p.f.

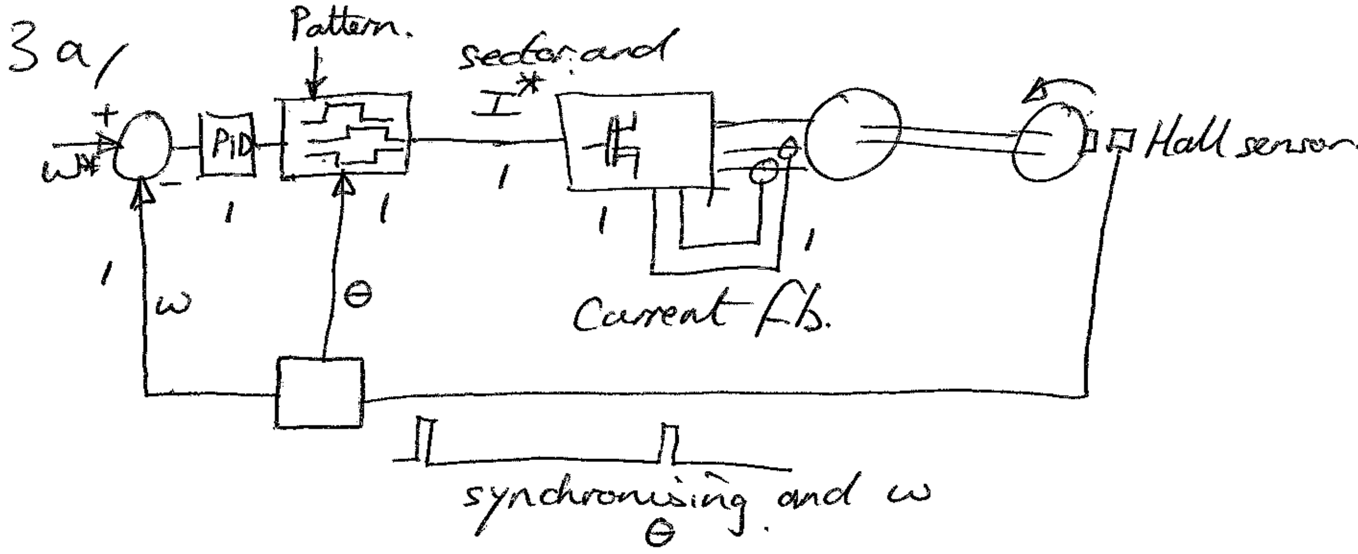


@ 100 Hz $\phi = \cos^{-1} 0.96$
$= 16.2^\circ$

@ 150Hz $\phi_2 \approx 16.2^\circ \times 1.5$
$\therefore \cos\phi_2 = 0.91$

max power $= 353 \times 3 \times 98.36 \times 0.91 = 95\,kW$
Good enough probably! so $s \approx 0.02$

3 a/

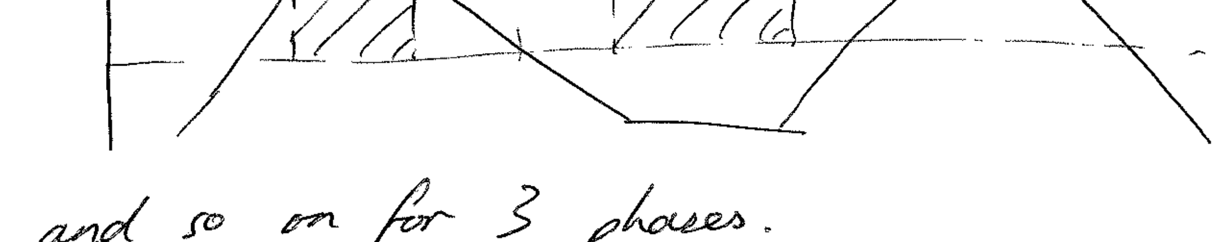


Trapezoidal means a simple pattern of switching, with current control on the inner loop to set the torque. *

Starting is tricky as there is no sychronisation. But the motor is a sychronous motor so can run open loop. So a pattern and frequency is applied, with a good magnitude of current so the rotor will move. 2

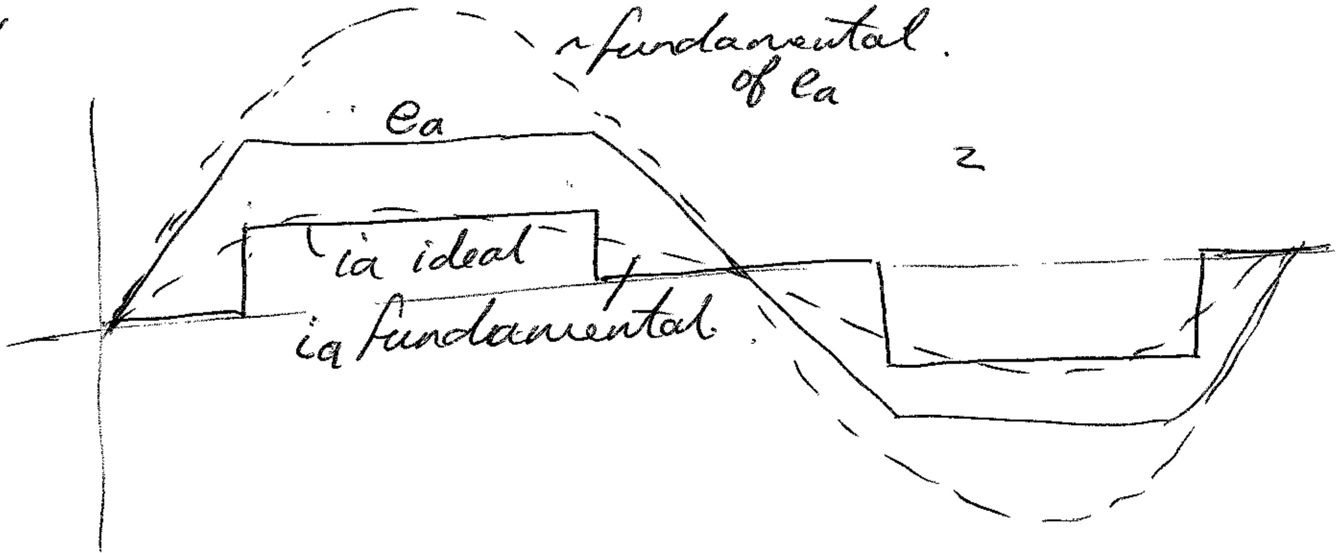
* At speed, the current is applied at the correct angles.

2

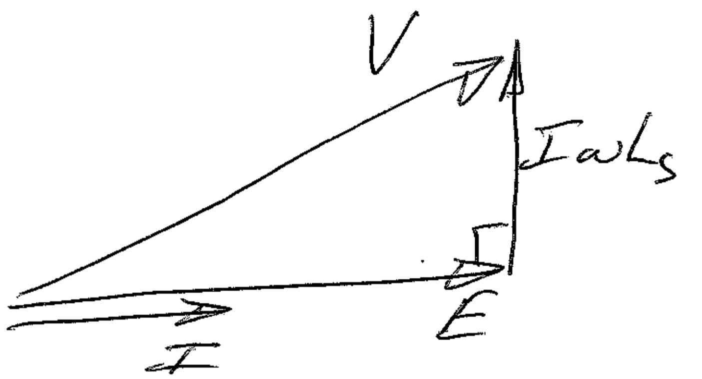


and so on for 3 phases.

3 b/



2

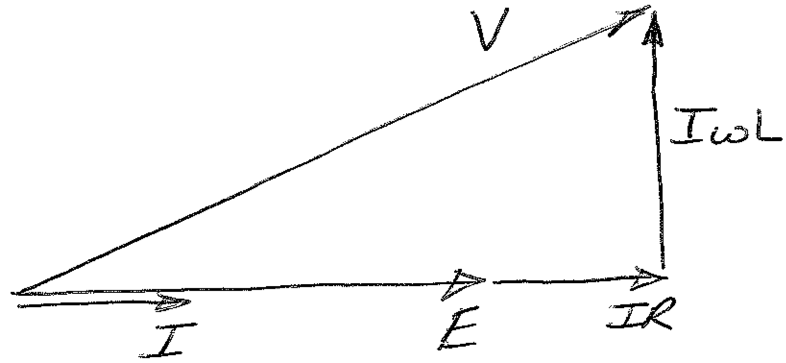


2

I in phase with E is the key element of BLDC shown as a phasor diagram for Sine BLDC or in trapezoidal/quasi square for trap. BLDC above.

2

3c/ Series resistance must be included.

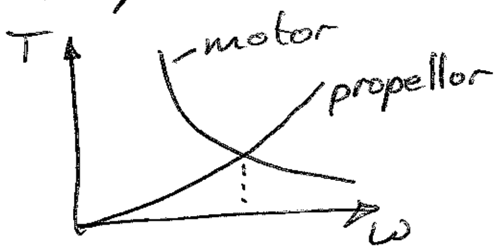


z

$E \propto \omega$

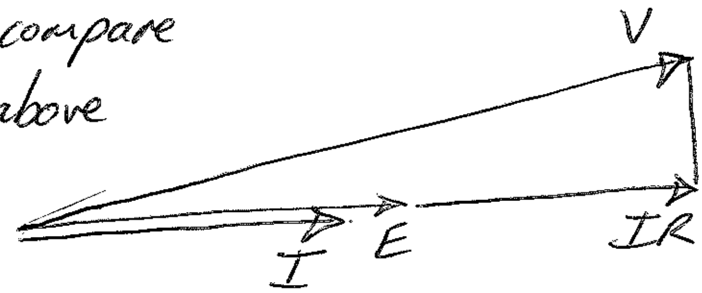
Voltage controlled so $V \propto \omega$ and angle of V adjusted w.r.t. I to keep it brushless.

<u>Hard to draw</u> as the 'new' condition depends on the load! <u>BUT</u> more torque means more I as fixed flux (PM motor), which means more voltage IR, so E drops by ω dropping, so V drops and so on until a new "load condition" is found. Fine for a propellor load!



OR/

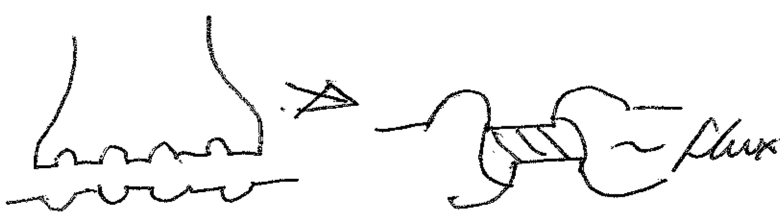
eg.
compare
above



any fair attempt!

(Recall lecture demo!)

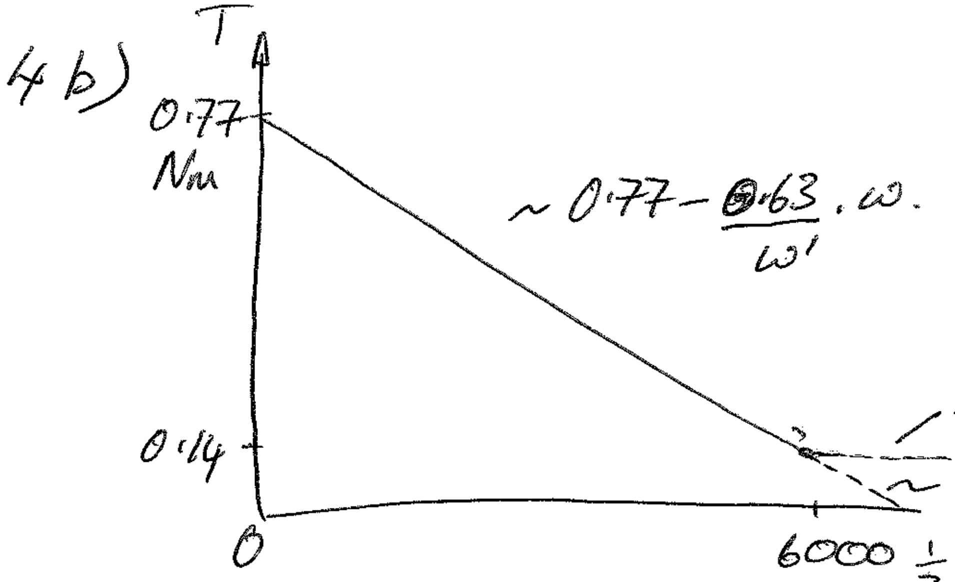
4 a/ Stepper drives are open loop, although there maybe some check on a microswitch. With the torque depending on position, it is possible to excite resonances related to the torque/current relationship with θ and the inertia $J\ddot{\theta}$. If the load does not vary, a combination of step rates and acceleration can be found which works, avoiding pullout. 4



2

By continuing a steady acceleration a <u>fixed</u> torque is required $T_e = J\ddot{\theta}$, which means the teeth are never aligned, so oscillation cannot start. 2

4 b)



$= 7333 \frac{s}{2}/s$

max speed? $\frac{0{\cdot}77}{0{\cdot}63} \times 6000$ steps/sec

0·63 Nm in 6000 ½ steps.

$\hookrightarrow \frac{6000}{400} \times 2\pi$ rads/sec $= \omega'$ (÷ 10 at the load.)

$$\hat{T}\left(1 - \frac{\omega}{\omega'}\right) = J\dot{\omega}$$

$J_L = \frac{0{\cdot}002}{10^2} = 2\times10^{-5}$

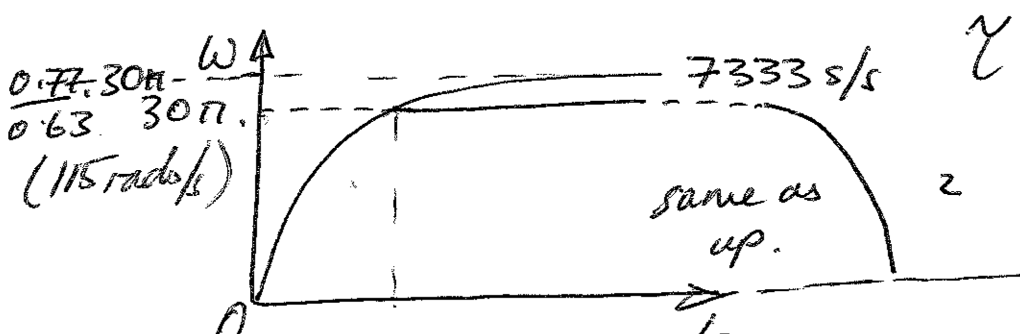
$$J\dot{\omega} + 0{\cdot}63\frac{\omega}{\omega'} = 0{\cdot}77 \quad 2$$

add to motor $J = 3\times10^{-5}$

at the motor

$$\frac{d\omega}{dt} + \frac{0{\cdot}63}{J\omega'} . \omega = \frac{0{\cdot}77}{J}$$

$\tau = \frac{\omega' J}{0{\cdot}63}$



$\tau = \frac{30\pi \times 3\times10^{-5}}{0{\cdot}63}$

2 $= 4{\cdot}49$ms.

<u>But</u> we have no data beyond 6000 $\frac{s}{2}$/s.

So max acc to 6000 ½ s/s.

$\left(1 - e^{-t/\tau}\right) = \frac{6000}{7333}$ $\quad \frac{t}{\tau} = 1{\cdot}70 \quad t = 7{\cdot}65$ms

profile as shown.

4 b cont

We need 2.5 revolutions, or 1000 $\frac{1}{2}$ steps !

$$\int_0^{t'} 7333\left(1-e^{-t/\tau}\right)dt = 500 \quad \text{(ok if less than } 7.67\,ms\text{)}$$

better, how many steps to 6000 $\frac{1}{2}$s/s. ?

$$\int_0^{\tau} 7333\left(1-e^{-t/\tau}\right)dt = 7333\left(t+\tau e^{-t/\tau}\right)\Big|_0^{\tau\times 1.7}$$

$$= 7333\left(\cancel{1.7\tau}^{\,0.7\tau} + \tau e^{-1.7} - \cancel{\tau}\right)$$

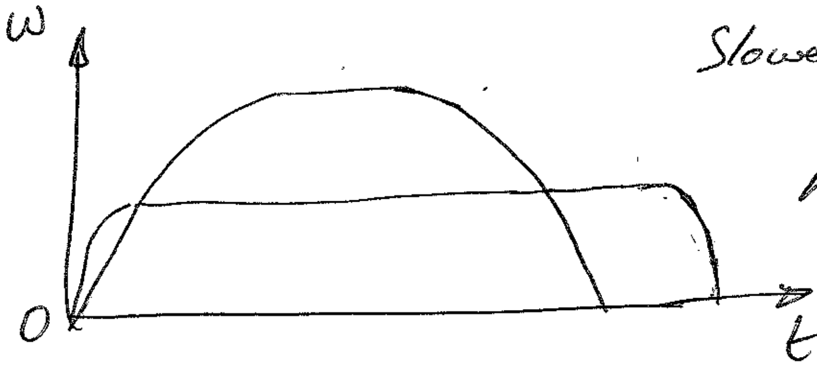
$$= 23+6 = 29$$

leaves $1000-2\times 29 = 942$ $\frac{1}{2}$ steps at 6000 $\frac{1}{2}$s/sec.

total 157 ms + 2×7.65ms.

$= \underline{172.3\,ms}$

A lower gear ratio could speed it up.



Slower to max speed, but higher max speed!