

PM

3 B5 2011 CRIB

Q.1

$$a) \psi(x) = A \sin kx + B \cos kx$$

$$\psi(0) = \frac{\hbar^2 k^2}{2m} \frac{2mE}{\hbar^2} \Rightarrow B = 0$$

$$\psi(a) = 0 \Rightarrow A \sin ka = 0$$

$$ka = n\pi \quad n = 1, 2, \dots$$

$$E_n = n^2 \frac{\hbar^2 \pi^2}{2ma^2}$$

$$\psi(x) = A \sin n \frac{\pi}{a} x$$

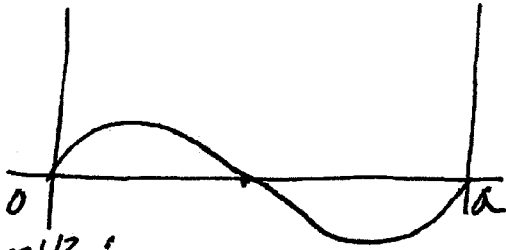
Determine A from normalization

$$\int_0^a |\psi|^2 dx = |A|^2 \int_0^a \sin^2 n \frac{\pi}{a} x dx = 1$$

$$A = \frac{2}{a}$$

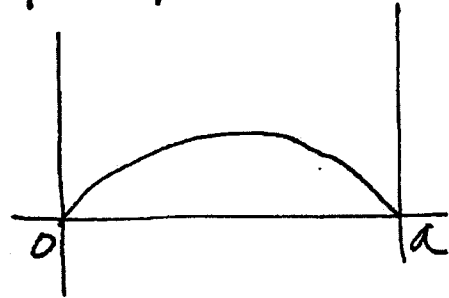
b) For $n=2$

$$\psi_2(x) = \sqrt{\frac{2}{a}} \sin \frac{2\pi}{a} x$$



For $n=1$

$$\psi_1 = \sqrt{\frac{2}{a}} \sin \frac{\pi}{a} x$$



(2)

c) For ψ_2 :

$$\int_0^{a/4} |\psi_2|^2 dx = \frac{2}{a} \int_0^{a/4} \sin^2 \frac{2\pi}{a} x dx \text{ etc.}$$

loop handed = $\frac{1}{4}$

or by inspection!

same for probability between $\frac{3a}{4}$ and a

d) For $0 < x < a$

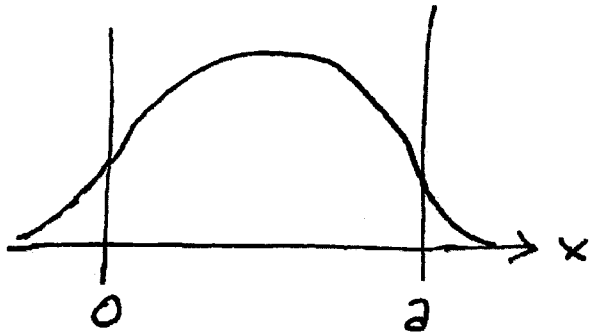
$$\psi(x) = A \sin kx + B \cos kx$$

$$\text{For } -\infty < x < 0 \quad \psi(x) = C e^{\alpha x}$$

$$\text{For } a < x < \infty \quad \psi(x) = D e^{-\alpha x}$$

where: $\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

③



In this case (finite depth of the well) one cannot assume that $\psi(0) = \psi(a) = 0$, hence the wavefunction extends outside the potential well. This is a typical quantum-mechanical effect: 'tunnelling'.

$$\begin{aligned} \text{e)} \quad \Delta E &= hf = h \frac{c}{\lambda} = \frac{3}{8} \frac{h^2}{ma^2} \\ a &= \left(\frac{3}{8} \frac{h\lambda}{cm} \right)^{1/2} = 6.74 \times 10^{-10} \text{ m} \end{aligned}$$

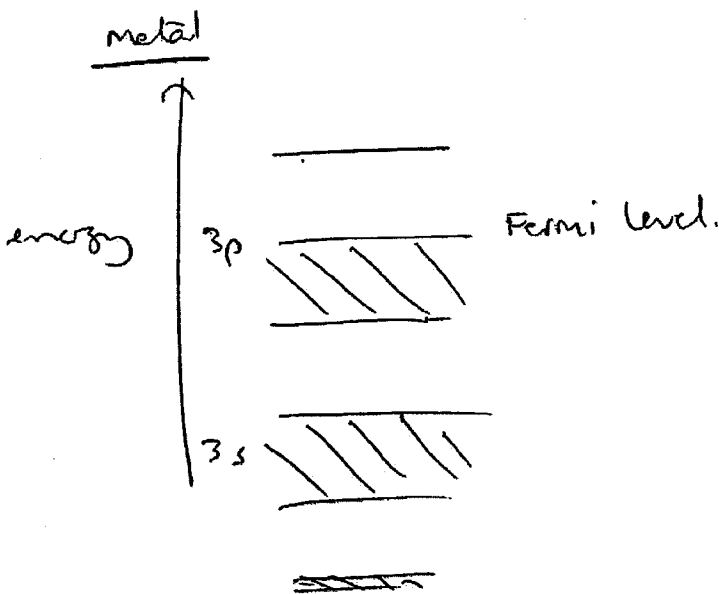
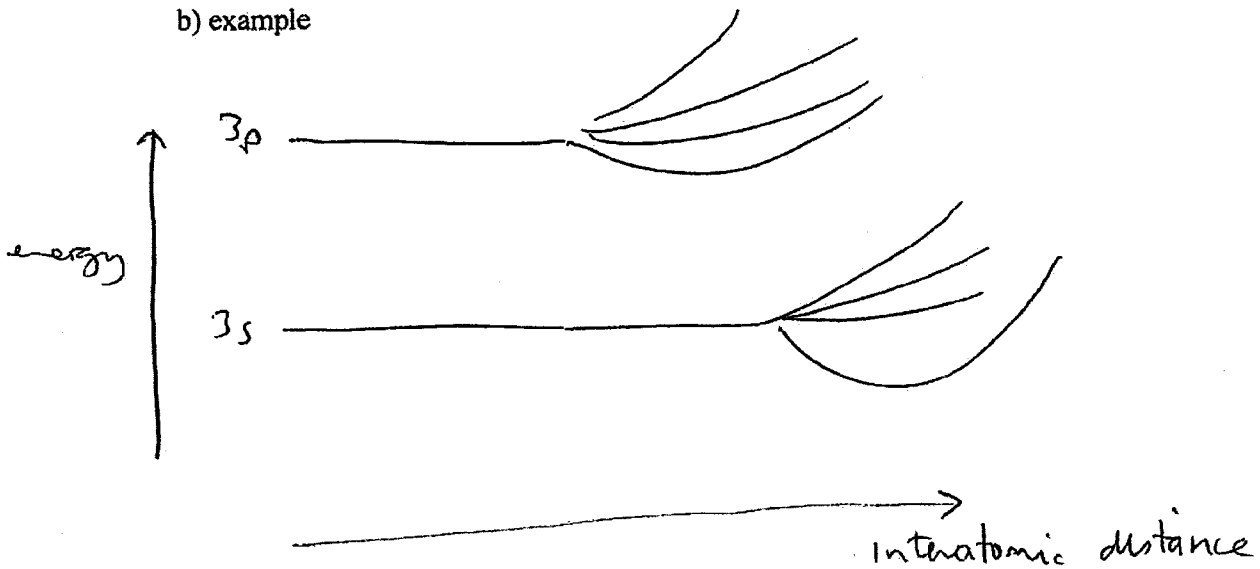
Q.2

- a) He: $1s^2$
 Ne: $1s^2 2s^2 2p^6$
 P: $1s^2 2s^2 2p^6 3s^2 3p^3$

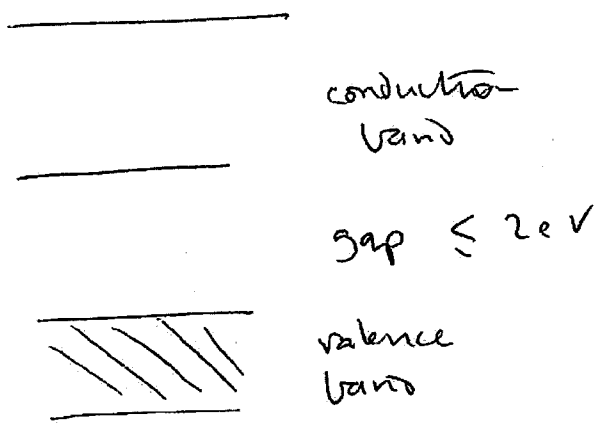
He and Ne do not form molecules because their outer shell is full, of 2 electrons in 1s shell and 8 electrons in 2s/p shell.

The only possibility for the electrons of atom 2 would be to go into the empty $2s$ shell, ^{or 3s} but this would cost too much energy.

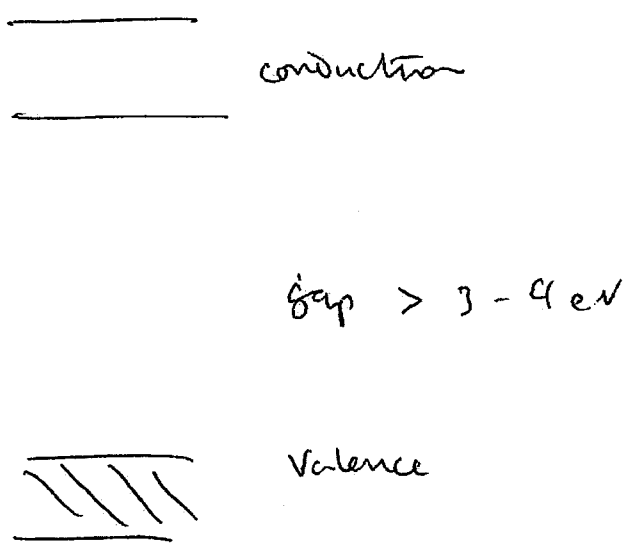
b) example



semiconductor



insulator



Pauli exclusion principle prevents an electron in the insulator valence band from changing its energy under the action of an electric field. Hence an insulator cannot conduct electricity.

The difference between a semiconductor and an insulator is whether it can be doped. A semiconductor has a smaller band gap and can be doped.

c) Na is $1s^2 2s^2 2p^6 3s^1$
 It is a metal because the last band (3s) to be filled is only half full.

Mg is $1s^2 2s^2 2p^6 3s^2$
 It is a metal, because although the 3s band is full, it overlaps in energy with the 3p.

d)

n-type Silicon

- Each Group V atom will replace a silicon atom
- This is known as *substitutional doping*
- In doing so, the Gp. V atom will use 4 of its 5 valence electrons to form covalent bonds with 4 neighbouring Si atoms
- There will now be one spare electron on the Gp. V atom
- The electron is only weakly bound
- In energy terms, it will form an energy level E_D only $\sim 0.03\text{eV}$ below E_C which will be occupied at 0°K
- Electrons may be easily excited into the CB from the energy level E_D as temperature rises
- So, the addition of each Gp. V impurity leads to an extra available conduction electron *without* the creation of a hole in the VB to produce a *n-type semiconductor*. The Gp. V impurity is called a '*donor*'-In a *n-type semiconductor* conduction is predominantly due to electrons in the conduction band which have been excited from the donor levels.

p-type Silicon

- To dope a Group IV semiconductor p-type, we add a Group III element to the material, such as Boron
- Boron $1s^2 2s^2 2p^1$ has only 3 valence electrons, but the surrounding silicon atoms provide 4 bonds
- 3 of these form covalent bonds with the 3 electrons of the Boron
- The 4th has only 1 electron, while we know it can take 2 electrons with opposite spin.
- The associated energy level E_A is located $\sim 0.030\text{ eV}$ above E_V
- At 0°K this level is empty
- At higher temperatures (300°K) an electron can jump into the empty space from other bonds; another electron can jump into the newly vacated place and so on. This can be viewed more easily in terms of *hole* rather than *electron*:
- a *hole* is bound to the Boron atom at 0°K , but can move about *at higher temperature*-
- Now there is an excess *-ve* charge $-e$ on the Boron site due to the captured electron
- The captured electron stays on the Boron site while the hole moves about
- Such Gp.III impurity is called an '*acceptor*'

To dope III-V semiconductors such as GaAs which already contains atoms from Groups III and V we use Group II elements (e.g. Zn) as acceptors and Group VI elements (e.g. S) as donors. Group IV elements can act as p-type or n-type dopants depending on their chemical preference for substituting for the Ga or the As

- C acceptor
- Si donor
- Ge amphoteric
- Sn usually a donor, but occasionally acts as an acceptor

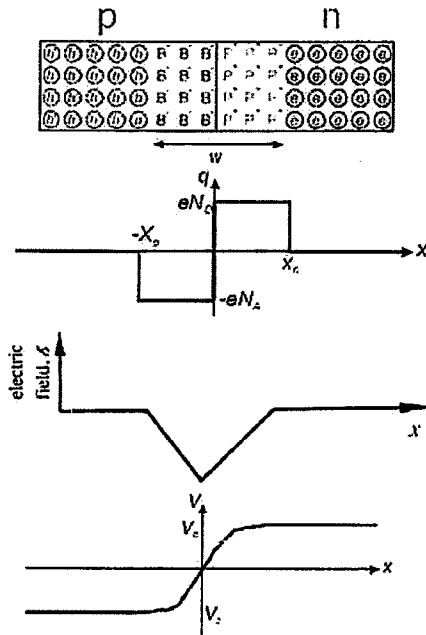
e)

From data book: $n = N_c \exp\left(\frac{E_F - E_C}{kT}\right)$
 By analogy: $p = N_v \exp\left(\frac{E_V - E_F}{kT}\right)$
 All acceptors ionized: $N_A \approx p$
 $E_F - E_V = kT \ln \frac{N_v}{N_A} = 0.174\text{ eV}$

7

Crib

Q3 (a) Space charge density distribution, electric field and electrostatic potential across unbiased junction pn junction:



(b) For the p⁺n junction the depletion region may be considered to be in the n-type region alone. The Poisson equation (Gauss law expressed in terms of voltage) states

$$\nabla^2 V = \frac{-\rho}{\epsilon_0 \epsilon_r} = \frac{-eN_D}{\epsilon_0 \epsilon_r}$$

where eN_D is the charge density due to the ionised donors. Given that V only varies in the x direction across the junction this becomes

$$\epsilon_0 \epsilon_r \frac{d^2 V}{dx^2} = -eN_D$$

Assuming that there are no electric fields outside the depletion region, i.e. the E-field is 0 at $x=w$ (whereby w is the width of the depletion region), integration gives

$$\frac{dV}{dx} = \frac{eN_D}{\epsilon_0 \epsilon_r} (w - x)$$

Integration again with the boundary condition that $V=0$ at $x=0$ gives

$$V = \frac{eN_D}{\epsilon_0 \epsilon_r} \left(wx - \frac{x^2}{2} \right)$$

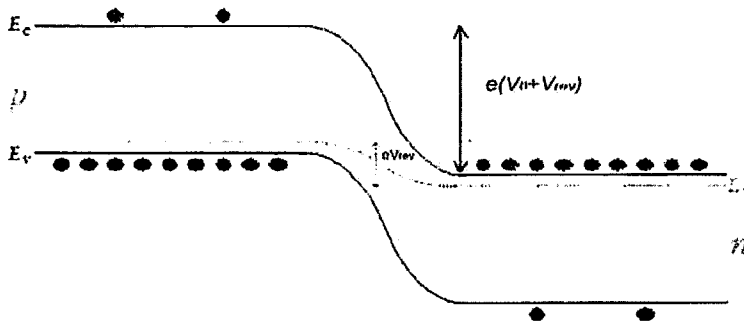
which may be evaluated at $x=w$ to give the contact potential

$$V_0 = \frac{eN_D w^2}{2\epsilon_0 \epsilon_r}$$

Rearranging gives

$$w = \left(\frac{2\epsilon_0 \epsilon_r V_0}{eN_D} \right)^{1/2}$$

(c) Band diagram for reverse biased pn junction:



The width of the depletion layer is given by

$$w = \left(\frac{2\epsilon_0 \epsilon_r (V_0 + V_{app})}{eN_D} \right)^{1/2} = \left(\frac{2\epsilon_0 12 \times (1.63V)}{e \times 10^{20} m^{-3}} \right)^{1/2} = 4.6 \mu m$$

The electric field ϵ is at its peak value at $x=0$, and

$$\epsilon = -\frac{dV}{dx} = -\frac{eN_D w}{\epsilon_0 \epsilon_r} = -\frac{e \times 10^{20} m^{-3} \times 4.6 \mu m}{\epsilon_0 \times 12} = -694 kVm^{-1}$$

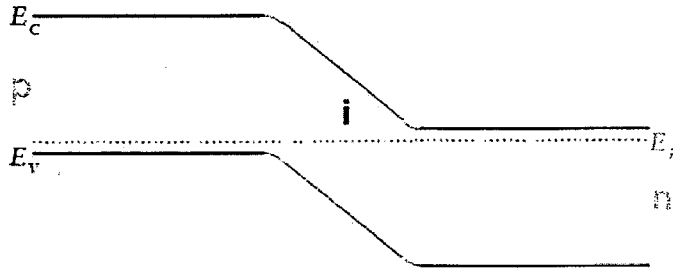
(d) The junction capacitance per unit area is given by

$$C_d = \frac{dQ}{dV} = \frac{\epsilon_0 \epsilon_r}{w}$$

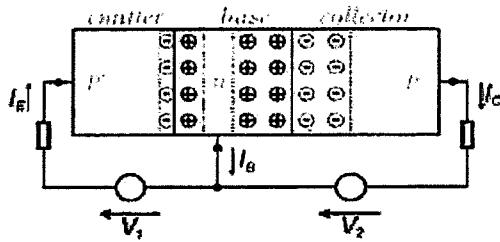
hence

$$\frac{C_{d-bias}}{C_{d-nobias}} = \frac{w_{nobias}}{w_{bias}} = \frac{2.9}{4.6} = 0.63$$

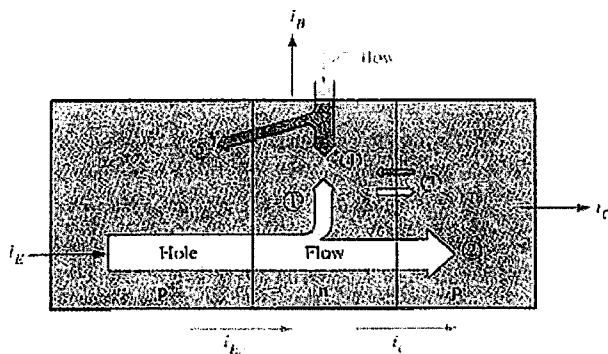
(e) Energy band diagram and electric field across unbiased p-i-n diode (constant E-field across i-Si region):



Q4 (a) A pnp BJT essentially consists of two pn junctions back to back.



In the active mode the emitter junction is under forward bias (injecting holes) and the collection junction under reverse bias (collecting injected holes). The main components of the base current are the electrons injected into the emitter (denoted 5 in Fig. below) and the electrons to compensate for those lost to recombination with injected holes in the base (4). For an uniformly doped base region there is no electric field across the base (outside the depletion regions) and the dominant transport mechanism is diffusion of holes across the base.



(b) Assuming that there are no electric fields outside the depletion regions and that the system is in a steady state, the Continuity equation for holes reduces to

$$\frac{\Delta p_n}{\tau_h} = D_h \frac{\partial^2 (\Delta p_n)}{\partial x^2}$$

which has the general solution

$$\Delta p_n(x) = C \exp\left(\frac{-x}{L_n}\right) + D \exp\left(\frac{x}{L_n}\right)$$

where C and D are constants that can be determined by applying the boundary conditions

$$\Delta p_n(0) = C + D \text{ at } x=0 \text{ and}$$

$$\Delta p_n(W_b) = 0 = C \exp\left(\frac{-W_b}{L_n}\right) + D \exp\left(\frac{W_b}{L_n}\right) \text{ at } x=W_b$$

This gives

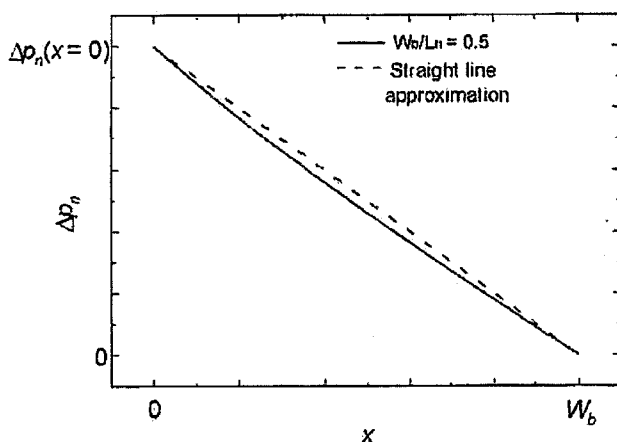
$$C = \frac{\Delta p_n(0) \exp(W_b/L_n)}{\exp(W_b/L_n) - \exp(-W_b/L_n)} \quad \text{and}$$

$$D = \frac{\Delta p_n(0) \exp(-W_b/L_n)}{\exp(W_b/L_n) - \exp(-W_b/L_n)}$$

Hence

$$\Delta p_n(x) = \Delta p_n(0) \frac{\exp((W_b - x)/L_n) - \exp((x - W_b)/L_n)}{\exp(W_b/L_n) - \exp(-W_b/L_n)}$$

(c)



For base transport factor to be close to unity recombination should be minimised, i.e. $W_b \ll L_n$

(d) Assuming that all donors are ionised, then

$$E_F = E_G - kT \ln \frac{N_C}{N_D}$$

where N_C is the effective density of states in the CB and E_G is the band gap.

From the definition of the Fermi level this means that a potential difference, V , across the base is set up with

$$V = \frac{kT}{e} \ln \frac{N_{DE}}{N_{DC}}$$

The electric field, ε , produced is uniform since the doping profile is exponential, so

$$\begin{aligned} \varepsilon &= \frac{kT}{eW_b} \ln \frac{N_{DE}}{N_{DC}} \\ &= \frac{8.62 \times 10^{-5} \times 298}{1 \times 10^{-6}} \ln \left(\frac{10^{23}}{10^{20}} \right) \text{Vm}^{-1} \\ \underline{|\varepsilon|} &= \underline{177 \text{ kVm}^{-1}} \end{aligned}$$

2011 3B5 Answers to numerical questions

Q1 a(iii): $a=6.74 \times 10^{-10} \text{m}$
c(i): 1/4; 1/4

Q2 c: $E_F - E_V = 0.174 \text{eV}$

Q3 c: $w=4.6 \text{ micron}$
 $\epsilon_{\text{max}} = -694 \text{kVm}^{-1}$
d: 0.63

Q4 d: 177kVm^{-1}