2011 Part IIA Module 3C8 Machine Design

1. a) If there is no slip then the velocities of the two discs at the contact point are equal.

$$V_1 = \Omega r_c \quad , \quad V_2 = \omega r_s$$

where $r_s = r_d \sin\alpha$ and ω is angular vel of the intermediate disc. If $V_1 = V_2$ then

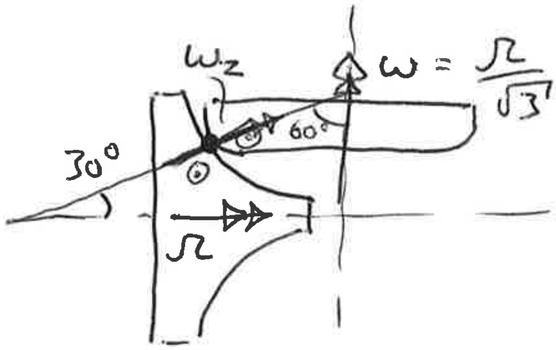
$$\Omega r_c = \omega r_d \sin\alpha$$

$$\Omega r_d (1 - \cos\alpha) = \omega r_d \sin\alpha .$$

$\left.\begin{array}{l} \cos\alpha = \frac{1}{2} \\ \sin\alpha = \frac{\sqrt{3}}{2} \end{array}\right\}$ so $\quad \Omega \cancel{r_d}\left(1 - \frac{1}{2}\right) = \omega \cancel{r_d} \frac{\sqrt{3}}{2}.$

$$\underline{\underline{\omega = \frac{\Omega}{\sqrt{3}}}}$$

Spin velocity ω_z



$$\omega_z = \Omega \cos 30^\circ - \omega \cos 60^\circ$$

$$= \Omega \frac{\sqrt{3}}{2} - \frac{\Omega}{\sqrt{3}} \cdot \frac{1}{2}$$

$$= \frac{\Omega}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{\Omega}{\sqrt{3}}.$$

$$\underline{\underline{\omega_z = \frac{\Omega}{\sqrt{3}}}}$$

b) $$\frac{P_{lost}}{P_{transmitted}} = \frac{6F\Delta v + 6M\omega_z}{3F\bar{V}}$$

where $$\bar{V} = \Omega r_c = \Omega r_d (1-\cos\alpha) = \frac{\Omega r_d}{2}$$

$$\omega_z = \frac{\Omega}{\sqrt{3}} \quad \text{(from part (b))}$$

$$\left.\begin{aligned} M &= \nu N a I_M \\ F &= \nu N I_F \\ \Delta v &= e\,\omega_z \end{aligned}\right\} \text{data sheet.}$$

thus $$\frac{P_{lost}}{P_{trans.}} = \frac{(6\cancel{\nu N} I_F e\omega_z + 6\cancel{\nu N} a I_M \omega_z)}{3\cancel{\nu N} I_F \Omega r_d/2}$$

$$= \frac{4(I_F e\omega_z + I_M a\omega_z)}{I_F \Omega r_d}$$

$$= \frac{4}{\sqrt{3}}\left(\frac{I_F e\Omega + I_M a\Omega}{I_F \Omega r_d}\right)$$

$$= \frac{4}{\sqrt{3}}\left(\frac{e}{r_d} + \frac{a}{r_d}\frac{I_M}{I_c}\right)$$

c) i) let $I_F = 0.75$

from data sheet diagram, $\frac{e}{a} = 0.74$, $I_M = 0.33$

thus
$$\frac{P_{lost}}{P_{transmitted}} = \frac{4}{\sqrt{3}}\left(\frac{0.74a}{r_d} + \frac{a}{r_d}\frac{0.33}{0.75}\right)$$

$$= 2.73\frac{a}{r_d}$$

ii) clearly, minimise loss by making r_d large and contact size 'a' small.

But large r_d means large contact radius R and thus large 'a' for given contact force. However contact forces will be reduced as r_d increases, for a given input torque T.

From data sheet for circular contacts, $a = \left[\frac{3}{4}\frac{PR}{E^*}\right]^{\frac{1}{3}}$

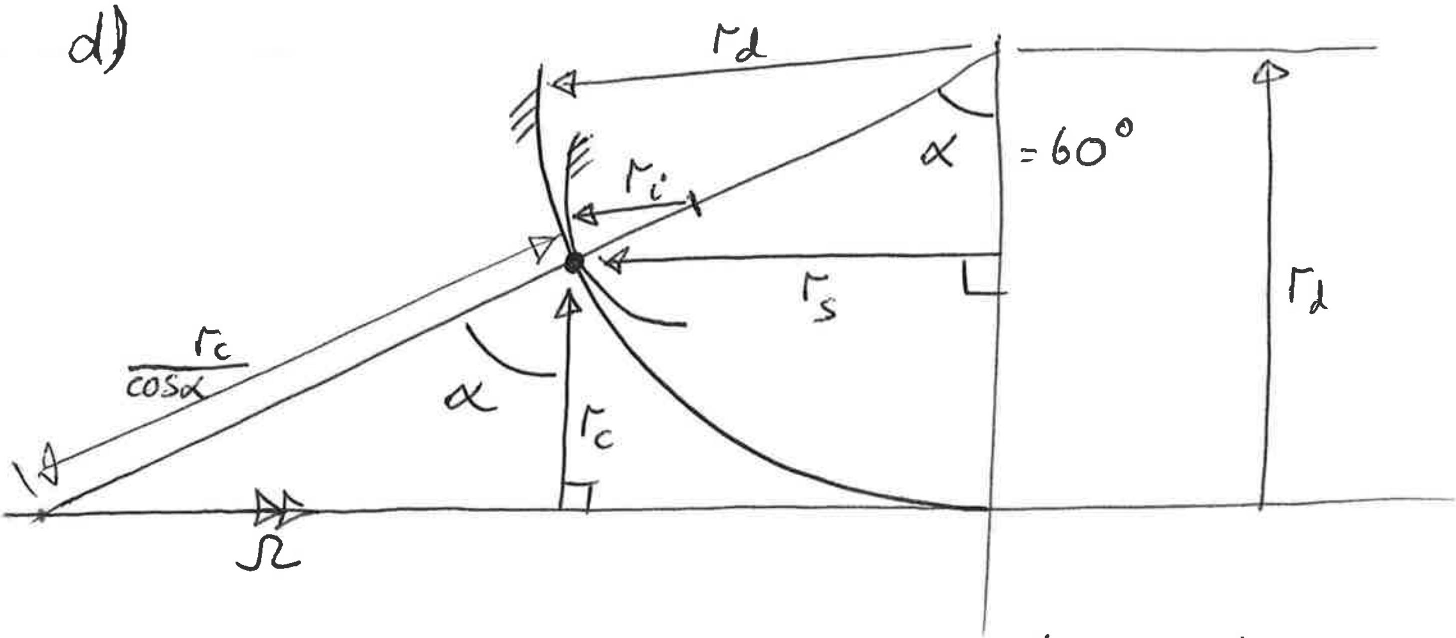
Contact force $P \propto \frac{T}{r_d}$ and $r_d \propto R$

so
$$\frac{a}{r_d} \propto \frac{\left[\frac{T}{R}\frac{R}{E^*}\right]^{\frac{1}{3}}}{R}$$

$$\frac{P_{lost}}{P_{trans}} \propto \frac{a}{r_d} \propto \frac{[T/E^*]^{\frac{1}{3}}}{R}$$

so for given input torque, minimise fraction of power lost by maximising r_d and E^*.

d)



$$r_c = r_d - r_d\cos\alpha = r_d(1-\cos\alpha)$$

In plane of diagram the reduced radius R_1 is given by $\frac{1}{R_1} = \frac{1}{r_i} - \frac{1}{r_d}$

Perpendicular to the plane of the diagram and to the contact plane the reduced radius is given by

$$\frac{1}{R_2} = \frac{1}{r_d} + \frac{1}{r_c/\cos\alpha}$$

$$= \frac{1}{r_d} + \frac{1}{r_d\left(\frac{1-\cos\alpha}{\cos\alpha}\right)}$$

$$= \frac{1}{r_d} + \frac{1}{r_d\left(\frac{1}{\cos\alpha}-1\right)} \qquad \alpha = 60° \quad \cos\alpha = \frac{1}{2}$$

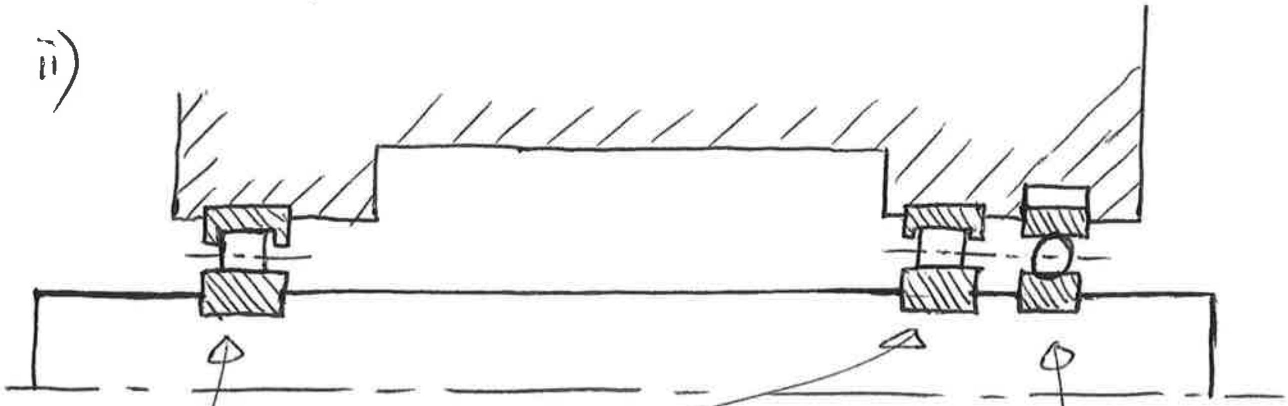
$$\therefore \frac{1}{R_2} = \frac{2}{r_d}$$

For circular contact $R_1 = R_2$, $\frac{1}{r_i} - \frac{1}{r_d} = \frac{2}{r_d}$

$$\underline{\underline{r_i = r_d/3}}$$

2. a) i) The risk is that differential thermal expansion of the shaft and housing will result in large axial forces on the bearings, leading to reduced life. Care must also be taken to ensure good alignment of the bearing housings and shaft to prevent tilting moments on the bearings.

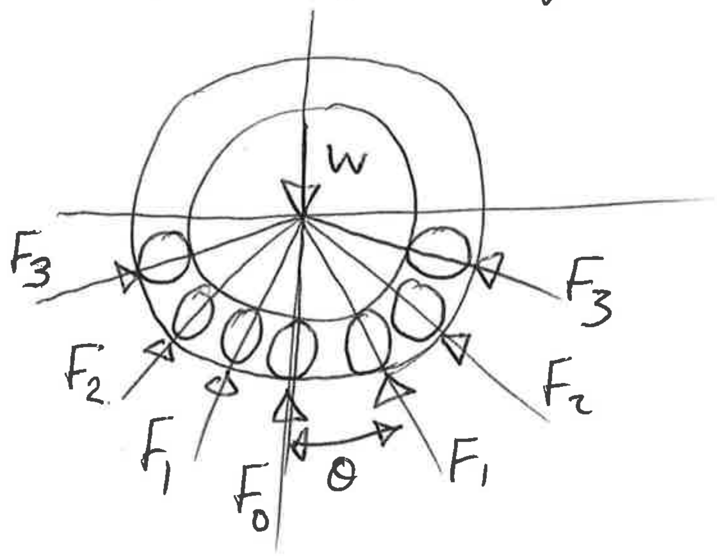
ii)



Inner and outer tracks of the roller bearings are positively located in radial and axial directions. Roller bearing cannot take axial load.

Inner and outer tracks of ball bearing are positively located in axial direction, but clearance in radial direction, to prevent radial load.

b) 14 rollers, so 7 carry the radial load.
Angular separation of rollers is $\frac{2\pi}{14} = \frac{\pi}{7}$ rad.



assume F varies with $\cos n\theta$

so $W = F_0 + 2F_1 \cos\theta + 2F_2 \cos 2\theta + 2F_3 \cos 3\theta$

$F_1 = F_0 \cos\theta$

$F_2 = F_0 \cos 2\theta$

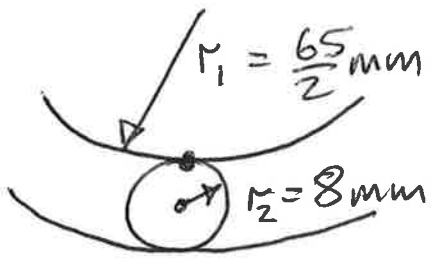
$F_3 = F_0 \cos 3\theta$.

$\therefore W = F_0 \left(1 + 2\cos^2\theta + 2\cos^2(2\theta) + 2\cos^2(3\theta)\right)$

$\theta = \frac{\pi}{7}$

$\therefore$ $\underline{\underline{W = 3.5 F_0}}$

Hertz contact:



$\frac{1}{R} = \frac{2}{65} + \frac{1}{8}$

$R = 6.42$ mm

(contact with inner track is critical)

line contact: $p_0 = \left\{ \frac{P' E^*}{\pi R} \right\}^{\frac{1}{2}}$

where $p_0 = 4\,GPa$, $R = 6.42 \cdot 10^{-3}\,m$
$E^* = 115\,GPa$

$$\therefore \quad P' = \frac{p_0^2 \pi R}{E^*} = 2.8\,MN/m.$$

$P = P'w$ $\quad$ w is length of roller, 19mm

$P = 2.8 \cdot 10^6 \cdot 0.019$

$F_0 = P = 53.3\,kN.$

hence bearing load $\underline{\underline{W = 3.5 F_0 = 186.6\,kN}}$

ii) the data sheet static load rating is 112kN.
The discrepancy may be due to a slight barelling of the roller to avoid stress concentrations at the ends of the roller. The bearing manufacturer may make a different assumption regarding the load sharing of the rollers.

c) $$L_{10} = \left(\frac{C}{P}\right)^{\frac{10}{3}}$$ where P is now radial load on bearing, C is dynamic load rating, 110kN.

$$0.1 = \left(\frac{110{,}000}{P}\right)^{\frac{10}{3}}$$ (L_{10} is number of revs in units of 10^6)

$\underline{\underline{P = 220\,kN.}}$

c) continued..

Generally it is advisabe to limit the load on the bearing to the static load rating ($P \leq C_0$) to prevent permanent deformation of the tracks. (Brinelling)

However it is possible to allow $P > C_0$ if the bearing is rotating, because the permanent deformation will occur smoothly around the bearing tracks.

3 a) tabular method

	ω_s		ω_c	ω_o
	A	BD	C	$\overline{E}$
let $BD = m$, $C = 0$	$-m\frac{B}{A}$	$+m$	0	$-m\frac{D}{E}$
add n	n	n	n	n
	$n - m\frac{B}{A}$	$m + n$	n	$n - m\frac{D}{E}$

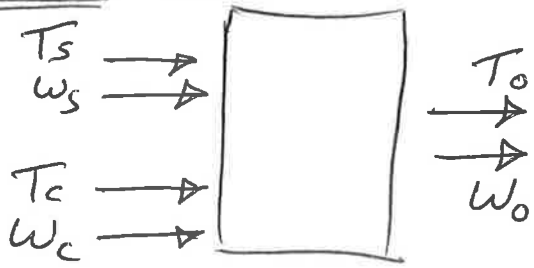
from table
$$\omega_s = n - m\frac{B}{A}$$
$$\omega_c = n$$
$$\omega_o = n - m\frac{D}{E}$$

eliminate n $\therefore$
$$\omega_s = \omega_c - m\frac{B}{A}$$
$$\omega_o = \omega_c - m\frac{D}{E}$$

eliminate m :
$$\omega_s = \omega_c - (\omega_c - \omega_o)\frac{BE}{AD}$$

$\frac{AD}{BE} = \alpha$ $\therefore$
$$\underline{\underline{\omega_o = \alpha\omega_s + \omega_c(1-\alpha)}}$$

b) i) $\omega_c = 0 \quad \therefore \quad \underline{\underline{\omega_o = \alpha \omega_s}}$



+ve power is input

power conserved $\quad P_s + P_c + P_o = 0$

$$T_s\omega_s + T_c\omega_c + T_o\omega_o = 0$$

$\omega_c = 0 \quad \therefore \quad T_s\omega_s + T_o\omega_o = 0$

$$T_s\omega_s + T_o\alpha\omega_s = 0$$

$$\omega_s(T_s + \alpha T_o) = 0$$

$$\underline{\underline{T_o = -\frac{T_s}{\alpha}}}$$

$P_c = 0 \therefore P_s + P_o = 0$

$$\underline{\underline{P_s = -P_o}}$$

ii) $\omega_s = 2\omega_c \quad \therefore \quad \omega_o = \alpha 2\omega_c + (1-\alpha)\omega_c$

$$= (2\alpha + 1 - \alpha)\omega_c$$

$$= (\alpha + 1)\omega_c$$

$$\underline{\underline{\omega_o = \frac{(\alpha+1)}{2}\omega_s}}$$

torque ratio is independent of speeds, so

$$\underline{\underline{T_o = -\frac{T_s}{\alpha}}} \quad (\text{as for } \omega_c = 0)$$

power $\quad \dfrac{P_s}{P_o} = \dfrac{T_s\omega_s}{T_o\omega_o} = -\alpha \cdot \dfrac{2}{\alpha+1} = \underline{\underline{-\dfrac{2\alpha}{\alpha+1}}}$

c) now $(P_S + P_C)0.95 + P_0 = 0.$

i) $\omega_c = 0$, so $\omega_0 = \alpha\omega_s$

$$0.95\, T_s\omega_s + T_0\omega_0 = 0$$

$$T_0 = -0.95\, T_s \frac{\omega_s}{\omega_0}$$

$$T_0 = -\frac{0.95\, T_s}{\alpha}$$

ii) $\omega_s = 2\omega_c$, so $\omega_0 = \frac{(\alpha+1)\,\omega_s}{2}$

$$T_s\omega_s\, 0.95 + T_c\omega_c\, 0.95 + T_0\omega_0 = 0$$

$$T_s\cancel{\omega_s}\, 0.95 + T_c\frac{\cancel{\omega_s}}{2}\, 0.95 + T_0(\alpha+1)\frac{\cancel{\omega_s}}{2} = 0$$

but $T_s + T_c + T_0 = 0$

$$T_c = -(T_s + T_0)$$

$$0.95\, T_s - \frac{0.95}{2}(T_s + T_0) + T_0\frac{(\alpha+1)}{2} = 0$$

$$\frac{T_s\, 0.95}{2} = T_0\left(\frac{0.95}{2} - \frac{\alpha}{2} - \frac{1}{2}\right)$$

$$T_0 = -\frac{T_s\, 0.95}{0.05 + \alpha}$$

4a) power $= T.\omega$

$$10{,}000 = T.\ 2000 . \frac{2\pi}{60}.$$

pinion torque $\underline{T = 47.75\,Nm}$

so force acting along pressure line is P where

pinion teeth N_p.

$$P . \frac{m N_p}{2} . \cos 20 = T$$

$$P = T . \frac{2}{m N_p \cos 20}$$

$$P' = \frac{P}{w} \quad \text{where} \quad w = m N_p$$

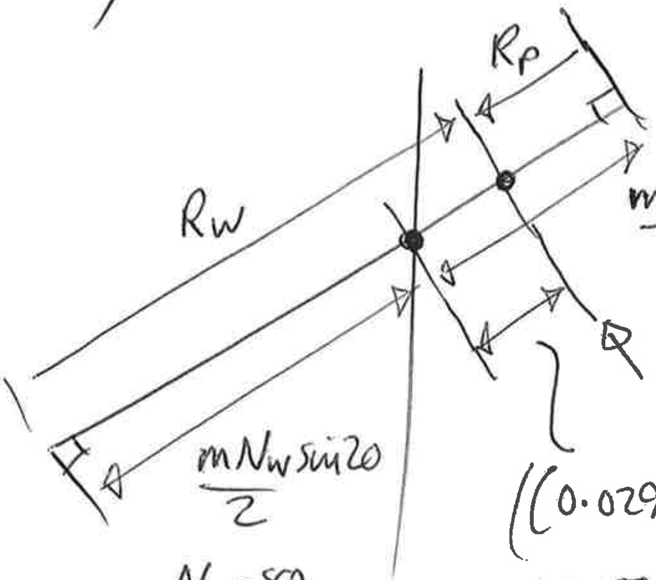
(square pinion)

$$P' = \frac{T}{m^2 N_p^2} . \frac{2}{\cos 20}$$

$$= \frac{10.10^3}{2000} . \frac{60}{2\pi} . \frac{2}{m^2 20^2 \cos 20}$$

$$\underline{\underline{p' = \frac{0.254}{m^2}}}$$

b) consider contact stress for double contact.



$N_p = 20.$

$N_W = 80$

$\left((0.02924 N_W^2 + N_W + 1)^{\frac{1}{2}} - 0.1710 N_W \right) m$

$N_W = 80 \quad \therefore \quad = 2.69m$

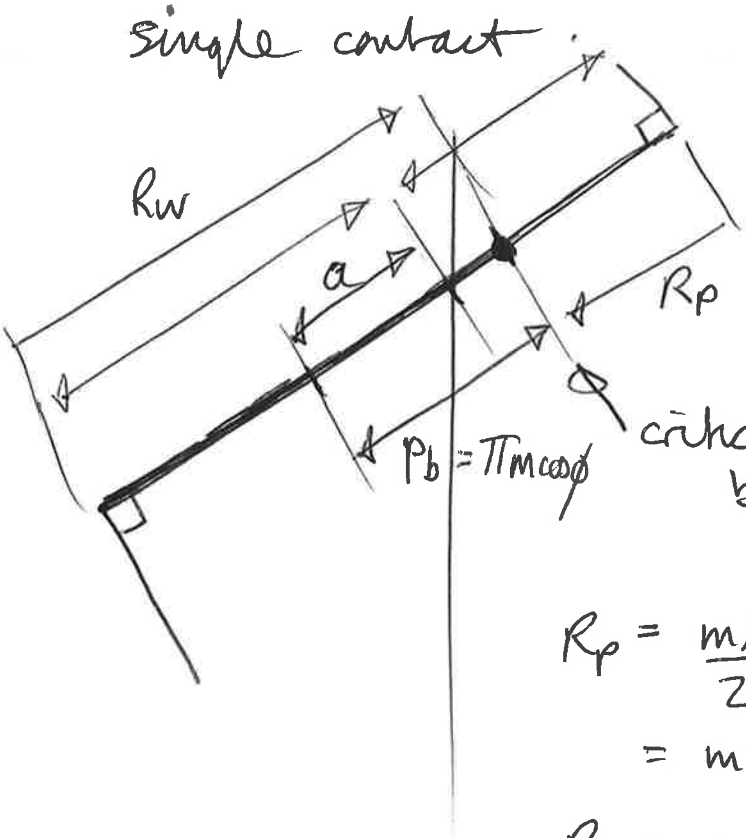
$R_p = \frac{m N_p}{2} \sin 20 - 2.69m = m\, 0.73$

$R_W = \frac{m N_W}{2} \sin 20 + 2.69m = m\, 16.57$

$\therefore \quad \frac{1}{R} = \frac{1}{m}\left(\frac{1}{0.73} + \frac{1}{16.37} \right)$

$\underline{\underline{R = m \cdot 0.7}}$

single contact.



$$a = \left((0.02924 N_p^2 + N_p + 1)^{\frac{1}{2}} - 0.171 N_p \right) m$$
$$= 2.30m$$

critical point, one base pitch back from the addendum.

$$R_p = \frac{mN_p}{2} \sin 20 + 2.3m - \pi m \cos 20$$
$$= m\, 2.76$$

$$R_W = \frac{m(N_p + N_W)}{2} \sin 20 - R_p$$
$$= m\, 14.34$$

$$\therefore \frac{1}{R} = \frac{1}{m}\left(\frac{1}{14.34} + \frac{1}{2.76} \right)$$
$$\underline{\underline{R = 2.31m}}$$

line contacts $\quad p_0 \propto \left(\dfrac{P'}{R} \right)^{\frac{1}{2}}$

single: $\quad p_0 \propto \left(\dfrac{P'}{2.31m} \right)^{\frac{1}{2}}$

double: $\quad p_0 \propto \left(\dfrac{P'}{2} \dfrac{1}{0.7m} \right)^{\frac{1}{2}}$

hence double contact is critical

contact stress $p_0 = \left\{ \dfrac{P' E^*}{2\pi R} \right\}^{\frac{1}{2}}$

note $\dfrac{P'}{2}$ for double contact

$$= \left\{ \frac{0.254}{m^2} \, \frac{1}{2\pi} \cdot \frac{E^*}{0.7m} \right\}^{\frac{1}{2}}$$

let $p_0 = \sigma_s = 1500 . 10^6$

$E^* = 115 . 10^9$

$$\sigma_s^{\,2} = \frac{0.254}{2\pi \, 0.7} \cdot \frac{115 . 10^9}{m^3} = (1500 . 10^6)^2$$

$\therefore \; \underline{\underline{m = 1.43 \text{ mm}}}$

check bending stress (single tooth)

$J = 0.35$ (data sheet)

$P'_T = P' \cos\phi$

$= \dfrac{0.25}{m^2} \cos 20$

$$\sigma_b = \frac{P'_T}{J m}$$

$$400 . 10^6 = \frac{0.25}{m^2} \cos 20 . \frac{1}{0.35m}$$

$$m^3 = \frac{0.25 \cos 20}{0.35 . 400 . 10^6}$$

$\underline{\underline{m = 1.18 \text{ mm}}}$

Hence contact stress is limiting,

so choose $\underline{\underline{m = 1.5 \text{ mm}}}$ (next preferred size up)