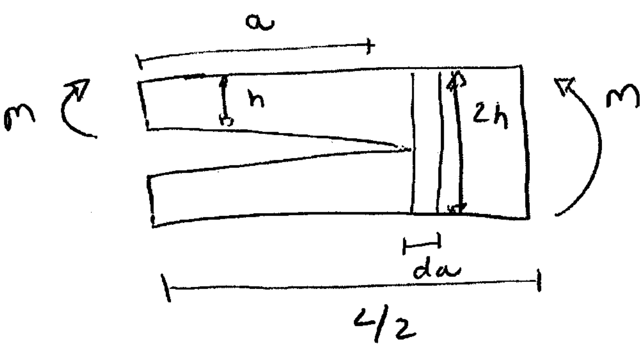
1(a) G is the rate of change of potential energy with crack length $G = -\frac{\partial U}{\partial a}$. Fracture occurs when $G = G_c$

K is a measure of the ~~mg~~ magnitude of the stress singularity in the vicinity of a sharp crack in ~~a~~ a ~~nom~~ nominally elastic material. Fracture occurs when $K = K_c$.

(b) ~~Consider a section of a beam of depth b~~



For 1/2 of ~~...~~ specimen (one crack tip): $U = \frac{6M^2}{EB}\left[\frac{(\frac{L}{2} - a)}{(2h)^3} + \frac{a}{h^3}\right]$

when crack grows da

$$\Rightarrow \quad -dU = \frac{6M^2}{BE(2h)^3}\left(\frac{L}{2}-a\right) - \left[\frac{6M^2}{BE(2h)^3}\left(\frac{L}{2}-a-da\right) + \frac{6M^2}{BEh^3}\,da\right]$$

$$-dU = \frac{21}{4}\,\frac{M^2}{BEh^3}\cdot da$$

1. Cont'd.

$$G = -\frac{1}{B}\frac{\partial u}{\partial a}$$

$$G = \frac{21}{4}\frac{m^2}{Eh^3B^2}$$

(c)

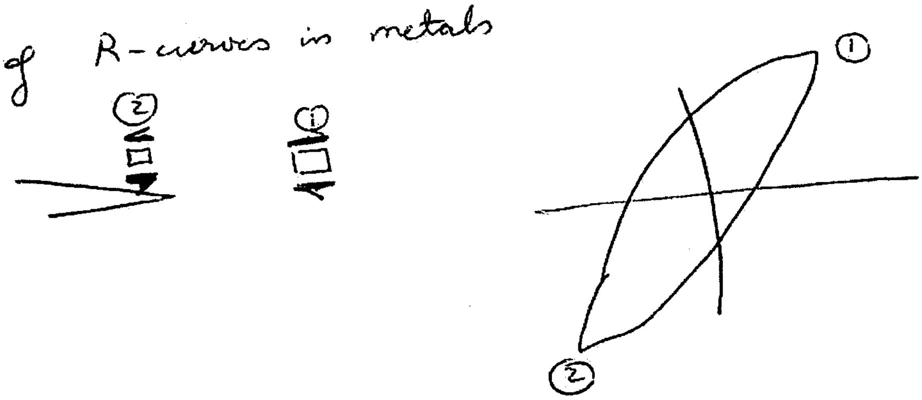
$$G_c = \frac{21}{4}\frac{65^2}{(25\times10^{-3})^2\times70\times10^9\times(5\times10^{-3})^3}$$

$$G_c = 4056 \ J/m^2$$

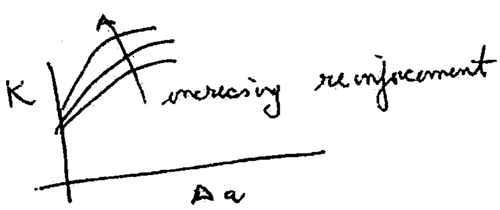
(d) Additional losses arise in the plastic zone surrounding the crack tip process zone.

2 (a) In SSY, the plastic zone size $r_p \approx \frac{K^2}{\pi \sigma_Y^2}$ is much less than the leading dimensions like plate size, crack length etc. In SSY, the non-linear zone is completely embedded within an outer K-field which determines the state in the non-linear zone & thus K is an adequate parameter to co-relate fracture.

(b) Hysteresis in the stress-strain curve is the origin of R-curves in metals



The R-curve effect increases with fibre re-inforcement due to more crack ~~bridd~~ bridging in the wake of the crack



2.c (i) $K_I = \dfrac{2P}{\sqrt{2\pi L}}$ & $\sigma_{yy} = \dfrac{K_I}{\sqrt{2\pi r}}$ @ $\theta = 0°$

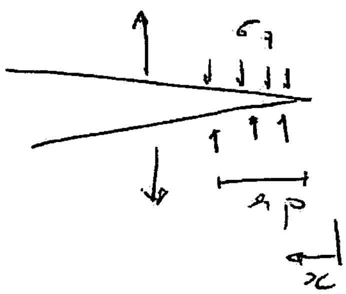
By Irwin (plane stress), yielding occurs when

$$\sigma_{yy} = \sigma_Y$$

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2$$

$$r_p = \frac{L}{\pi^2}\left(\frac{P}{\sigma_Y L}\right)^2$$

(ii)



$K_I^{(P)}$ at fictitious crack tip $= \dfrac{2P}{\sqrt{2\pi(L+r_p)}}$

$$K_I^{(\sigma_Y)} = -\int_0^{r_p} \frac{2\sigma_Y\,dx}{\sqrt{2\pi x}} = -\frac{4\sigma_Y r_p^{\frac{1}{2}}}{\sqrt{2\pi}}$$

$$K_I = K_I^{(P)} + K_I^{(\sigma_Y)} = 0$$

ie $r_p = \dfrac{L}{2}\left[\left\{1+\left(\dfrac{P}{L\sigma_Y}\right)^2\right\}^{\frac{1}{2}} - 1\right]$

Engineering Tripos Part IIA, 2010/11

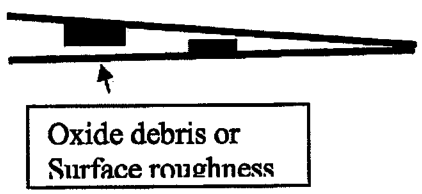
Crib for Paper 3C9: Engineering Fracture Mechanics of Materials and Structures

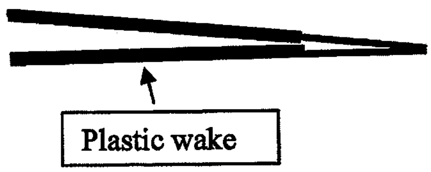
A3. (a) Fatigue crack closure is the phenomenon whereby the tip of a fatigue crack closes under a tensile remote load. Consequently, the cyclic crack opening (and thereby the crack growth rate) are reduced. Possible sources of fatigue crack closure are:

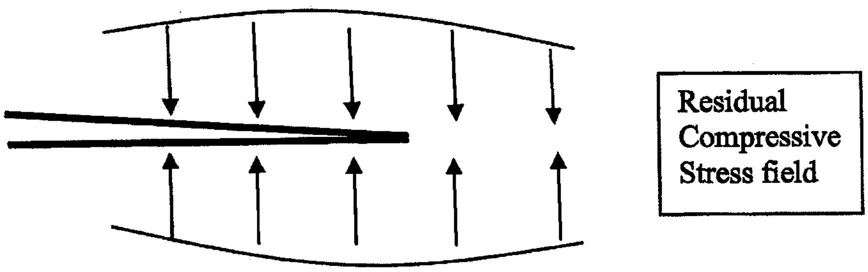
(i) Near threshold, oxide or fracture surface roughness leads to wedging of the crack, thereby preventing the crack from closing.

(ii) Within the Paris regime, plasticity-induced crack closure occurs. As the crack grows through the plastic zone at its tip, it leaves a residual wake of stretched material. Typically, for thin sheets in plane stress, $K_{op} \approx 0.5K_{\max}$, whereas for thick plates in plane strain, $K_{op} \approx 0.2K_{\max}$.

(iii) When a crack grows through a residual stress field, the K due to crack face loading by the residual stress field is added to the K

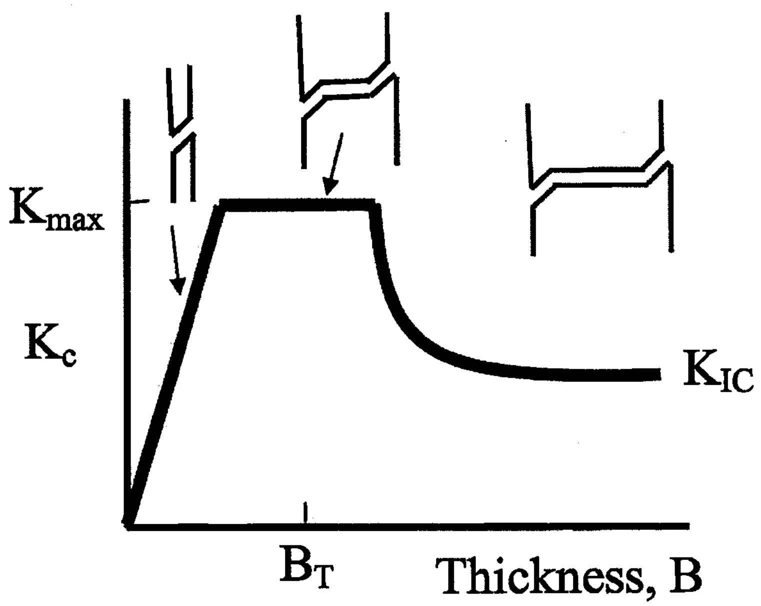






(b) Contact fatigue cracks develop under loading by the wheel, particularly when the train is braking or accelerating. Note that the elastic contact stress field beneath a circular roller in plane strain involved only compressive stresses, and so it is difficult to predict that mode I opening cracks develop under the elastic loading. The mechanism can involve a pumping mechanism in to the cracks and also shear delamination by the growth of voids from inclusions. Grinding of the surface of the tracks polishes the surface and removes persistent shear bands and any nucleated cracks.

(c) The fracture mechanism in a sheet depends upon thickness B: very thin sheet has a low toughness, but as the sheet thickness increases there is a plateau in toughness. At large values of sheet thickness the toughness drops to the plane strain value K_{IC}.

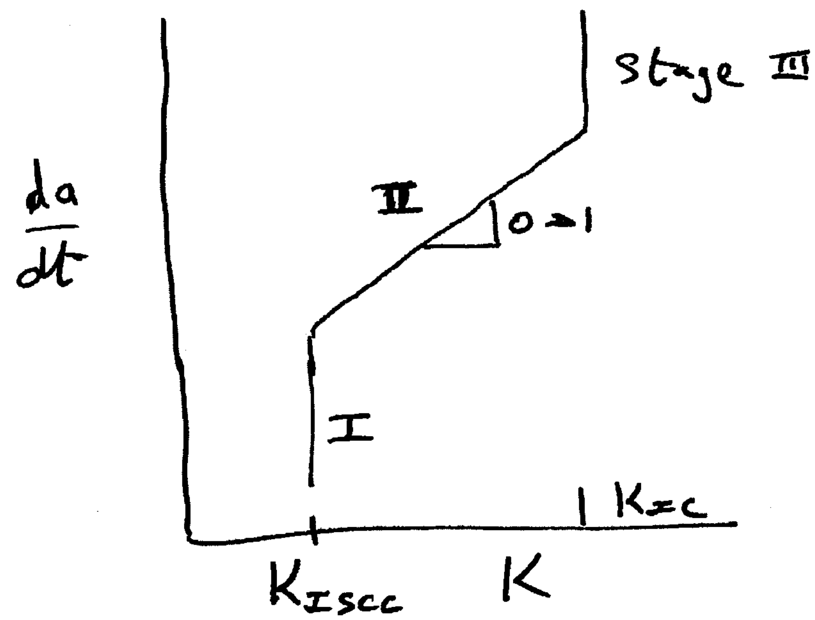


The plane strain asymptote of $K_C = K_{IC}$ is attained when the thickness B attains the value $B \geq 2.5(K_{IC} / \sigma_Y)^2$. In thick specimens that exceed this criterion, shear lips form at the edges of the specimen of width about equal to $r_p = \frac{1}{\pi}(K_{IC} / \sigma_Y)^2$. There is high plastic constraint directly ahead of the crack tip such that the tensile stress $=3\,\sigma_Y$. This promotes both cleavage and microvoid coalescence.

For thinner specimens, the stress state ahead of the crack tip is closer to that of plane stress, and the shear lips occupy a substantial fraction of the fracture surfaces. When $B < \frac{1}{\pi}(K_{IC} / \sigma_Y)^2$ the shear lips exist across the full thickness of the specimen and failure is by plastic shear-off without the need for microvoid coalescence and cleavage.

By laminating sheets of thickness equal to the transition value of B_T each sheet is able to thin at the crack tip and the peak value of fracture toughness K_{max} is attained.

4. (a)



Stages I & II

Consider the film rupture model

A stable passive film forms along the crack flanks whereas active corrosion occurs at the crack tip. This differing corrosion rate can be due to

- a differing pH at the crack tip from the crack flanks.
- the active plasticity at the crack tip changes the chemical potential, ~~for~~ driving corrosion.

4. (a) At a sufficiently low K value (8)
the degree of plasticity at the
crack tip is small and a protective
oxide layer forms at the crack tip.
This arrests crack advance and a
threshold K is attained, termed K_{ISCC}.

Stage III

At high K (near $K = K_{IC}$) a faster crack
advance mechanism intervenes: static
mechanisms such as microvoid coalescence or
cleavage.

Stress corrosion cracking (SCC) is the nucleation and growth of cracks under a combination of stress and a corrosive nonmetallic fluid. This also leads to slow crack growth below K_{Ic} - giving slow crack growth. SCC also applies to non-metallic materials (glass by moisture, and some polymers below their glass transition temperature). Normally ductile metals can fail in a brittle manner due to SCC. In metals the interaction of stress and environment comes about because the localised crack tip plasticity changes the electrochemical behaviour of the metal, making it much more susceptible to corrosion. SCC is sensitive to environment chemistry, temperature, strain-rate. SCC growth rates in general are slower than in LME - partly because SC involves diffusion *away* from the crack tip, whereas LME requires transport *to* the crack tip.

Examples of SCC: most major alloy systems (Al, steels, Mg, Ti) in aqueous Cl solutions; Cu, Fe alloys in ammonia.

4. (b)

$$\frac{da}{dt} = 6\times10^{-6}\,K$$
$$= 6\times10^{-6}\times 1.13\times\sigma\sqrt{\pi a}$$
$$= 1.20\times10^{-5}\times\sigma\sqrt{a}$$

Integrating, to failure:

$$\int_{a_i}^{a_f}\frac{da}{\sqrt{a}} = 1.20\times10^{-5}\,\sigma\, t_f$$

$$\left[2\sqrt{a}\right]_{a_i}^{a_f} = 1.20\times10^{-5}\,\sigma\, t_f$$

$$2\left(\sqrt{a_f}-\sqrt{a_i}\right) = 1.20\times10^{-5}\,\sigma\, t_f$$

The final crack size a_f is given by:

$$a_f = \left(\frac{K_c}{1.13\times\sigma\sqrt{\pi}}\right)^2 = \left(\frac{60}{1.13\times 540\sqrt{\pi}}\right)^2 = 3.08\times10^{-3}\ m$$

The initial flaw size a_i which grows to this length in 1 hour at 540 MPa is given by:

$$2\left(\sqrt{3.08\times10^{-3}}-\sqrt{a_i}\right) = 1.20\times10^{-5}\times 540\times 1$$

$$\Rightarrow a_i = 2.73\times10^{-3}\ m$$

The proof test must cause cracks greater than this crack length to propagate, for the vessel to be safe. Hence:

$$\therefore \sigma_p = \frac{K_c}{1.13\sqrt{\pi a_i}} = \frac{60}{1.13\sqrt{\pi\times 2.73\times10^{-3}}} = 573\ MPa$$

This is comfortably below the yield stress of 1800 MPa, so that is OK. Should also check that the conditions also satisfy LEFM, and plane-strain (as this is the fracture toughness value used). We must assume that dimensions of the vessel (wall thickness) are sufficiently large, but can check that the initial crack length is sufficiently large compared to the plastic zone size. The usual LEFM condition is:

$$a_i \ge 2.5\left(\frac{K}{\sigma_y}\right)^2.$$

At fracture, $K = K_{Ic}$, hence $2.5\left(\frac{K}{\sigma_y}\right)^2 = 2.5\left(\frac{60}{1800}\right)^2 = 2.7\,mm.$

The initial crack length of 2.73 mm is therefore just long enough for LEFM to be valid at fracture (at the proof stress). All cracks larger than this would see the working stress, and hence K < K_{Ic}. The criterion above will thus automatically be satisfied.