1. a) $\varepsilon_{v\,ult} = \frac{\Delta\sigma_v}{E_o} = \frac{100}{10000} = 1\%$

$\therefore \rho_{ult} = \varepsilon_{v\,ult} \times H = 1\% \times 10\text{m} = \underline{100\text{ mm}}$

b) For $R_v = 0.9$, the Fourier solution tabulated on p. 6 of the Data Book gives:

$T_v \doteq 0.84$

$\therefore t_{90} = 0.84 \frac{d^2}{c_v} = \frac{0.84 \times 5^2}{1} = \underline{21 \text{ years}}$

c) After 25 months the first tank has

$T_v = \frac{c_v t}{d^2} = 1 \times \frac{25}{12} \times \frac{1}{25} = \frac{1}{12}$

This is the transition between Phases 1 and 2 in parabolic isochrone theory. After a further 25 months $T_v^{(1)} = \frac{1}{6}$, $T_v^{(2)} = \frac{1}{12}$, so the first tank is in Phase 2 and the second is at the transition.

In Phase 1, $R_v = \sqrt{\frac{4}{3} T_v} = \frac{1}{3}$

For second tank $\rho_{ult} = 100\text{mm} \times \frac{65}{100}$
$= 65\text{mm}$

So $\frac{d\rho^{(2)}}{dt} = \rho_{ult} \frac{dR_v}{dt} = 65 \times \sqrt{\frac{4}{3}\frac{c_v}{d^2}} \cdot \frac{1}{2} t^{-1/2}$

$\therefore \frac{d\rho^{(2)}}{dt} = 75 t^{-1/2} = \underline{5.2 \text{ mm/y}}$

In Phase 2, $R_v = \left[1 - \frac{2}{3}\exp\left(\frac{1}{4} - 3T_v\right)\right]$

1. c) cont.

$$\text{So } \frac{d\rho^{①}}{dt} = 100.\left(-\frac{2}{3}\right).\left(-\frac{3c_v}{d^2}\right)\exp\left(\frac{1}{4}-3T_v\right)$$

$$\therefore \frac{d\rho^{①}}{dt} = \frac{200}{25}\exp(-0.25) = \underline{6.2 \text{ mm/y}}$$

d) After 25 months of load the second tank is settling slower than the first. But immediately after loading the rate of settlement of the first tank must have been less than the second ($t^{-1/2} \to \infty$). So between 0 and 25 months the two rates of settlement will be equal — a turning point in differential settlement. This satisfies:

$$7.5t^{-1/2} = 8\exp\left(\frac{1}{4}-\frac{1}{4}-\frac{3t}{25}\right)$$

$$\therefore \frac{e^{0.12t}}{\sqrt{t}} = 1.07$$

Try $t = 1$ year $\to 1.13$: nearly

$t = 1.1 \to 1.09$: close

$t = 1.15 \to 1.07$ ✓ $\underline{@ \text{ 13.8 months}}$

At this time : $\rho^{①} = 100\left[1 - \frac{2}{3}\exp(-0.12\times1.15)\right]$

$\therefore \rho^{①} = 42$ mm

And $\rho^{②} = 65\sqrt{\frac{4}{3}\times\frac{1.15}{25}} = 16$ mm

So $\Delta\rho^{①} = 42 - 33 = 9$ mm

$\Delta\rho^{②} = 16$ mm

Differential settlement ~ 7 mm

1 d) cont.

Whereas at $t = \infty$

$$\rho^{①} = 100 \text{ mm} \qquad \Delta\rho^{①} = 67 \text{ mm}$$
$$\rho^{②} = 65 \text{ mm} \qquad \Delta\rho^{②} = 65 \text{ mm}$$

Differential settlement ~ − 2 mm

So the maximum differential settlement is ~7mm at ~ 14 months

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NB Although the question was popular, and had an average mark of 13.3/20, some candidates gave settlements instead of settlement rates as the answer to (c) thereby losing 3 marks. Not a single candidate correctly identified the differential settlement of the pipe, forgetting that it was only fixed in place after the first tank had already settled 33mm. So the marks could have been even higher

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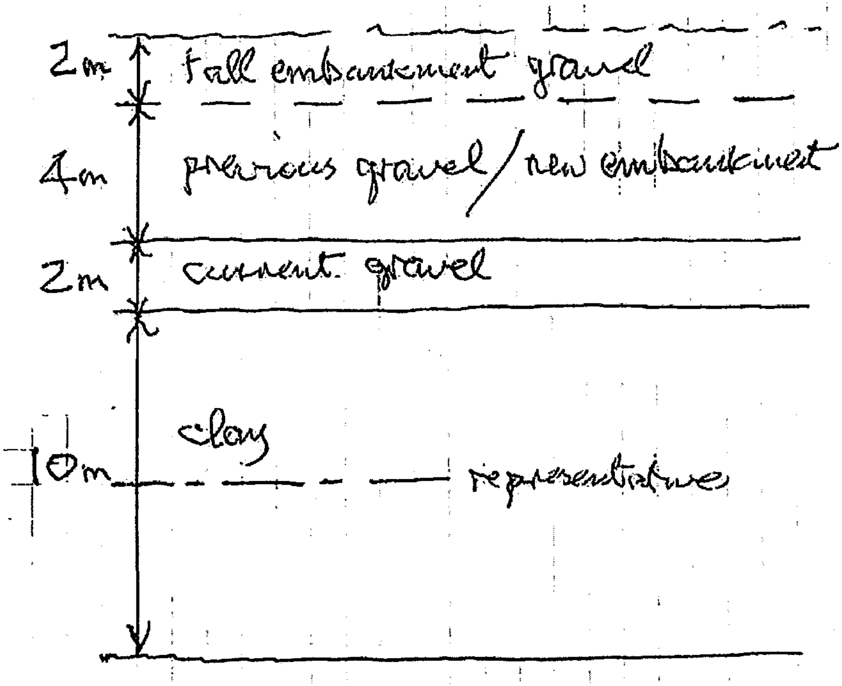
2.



a) Dry density of gravel: $\gamma = \dfrac{2.65 \times 9.81}{1.6}$

$= 16.2 \text{ kN/m}^3$

Find unit weight of clay by iteration. Assume initially $\gamma = 16 \text{ kN/m}^3$. Then at the centre of the clay, involving normally consolidated London clay under 6 m (total) of gravel:

$$\sigma'_v = 6 \times 16.2 + 5 \times (16 - 9.8) = 128 \text{ kPa}$$

$$v = \Gamma + \lambda - \kappa - \lambda \ln \sigma'_v$$

$$= 2.759 + 0.161 - 0.062 - 0.161 \ln 128$$

so $v = 2.077$

i.e. $e = 1.077$ so a better estimate of the submerged (buoyant) unit weight of the clay is

$$\gamma' = \frac{(G_s - 1)}{v}\gamma_w = \frac{1.75 \times 9.81}{2.077} = 8.3 \text{ kN/m}^3$$

2 a) cont.

Then an improved calculation for σ'_v at the centre of the clay layer, under 6 m of gravel, is:

$$\sigma'_v = 6 \times 16.2 + 5 \times 8.3 = 139 \text{ kPa}$$

giving $v = 2.067$

After erosion of 4 m of gravel, to get the current situation, vertical stress will reduce by about $4 \times 16.2 = 65$ kPa. By the present day this will have become effective, and the clay will have swollen along a κ-line. So:

$$\sigma'_v = 139 - 65 = \underline{74 \text{ kPa}}$$

$$v = 2.067 + 0.062 \ln(139/74)$$

or $v = 2.067 + 0.039 = \underline{2.106}$

$\therefore$ $e = 1.106$ and $\underline{w = 1.10/2.75 = 40\%}$

b) A 4 m embankment returns the clay to its pre-erosion state, $v = 2.067$.

So settlement $\rho = \dfrac{\Delta v}{v_0} \times 10\text{ m} = \dfrac{0.039 \times 10}{2.106}$

$\therefore$ $\underline{\rho = 0.185 \text{ m}}$

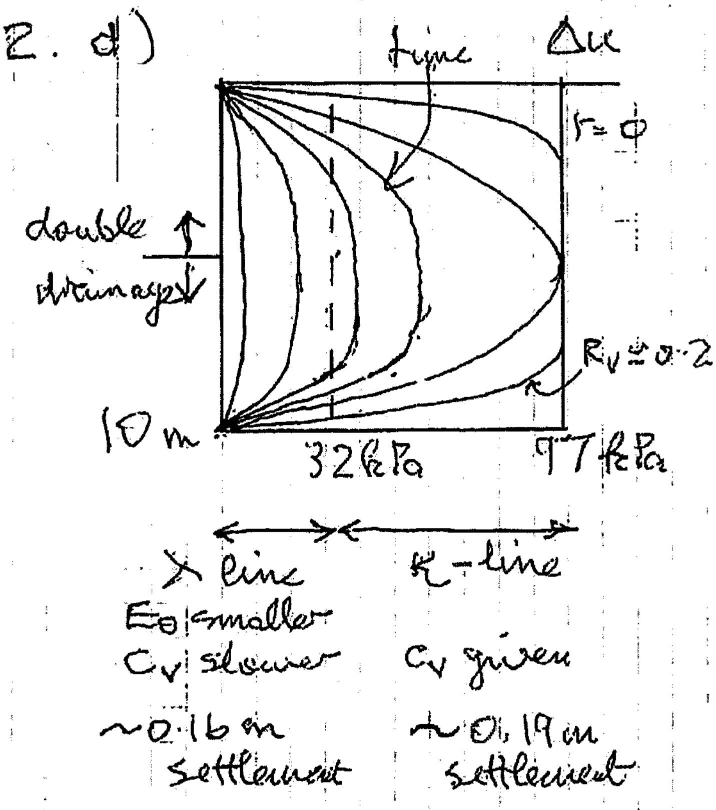
c) An extra 2 m of gravel causes extra, normal consolidation on the λ-line as σ'_v goes from 139 kPa to 171 kPa.

$$v = 2.067 - 0.161 \ln \frac{171}{139} = 2.034$$

Then $\Delta\rho = \dfrac{0.033}{2.106} \times 10 = 0.16 \text{ m}$

So $\underline{\rho = 0.19 + 0.16 = 0.35 \text{ m}}$

2. d)



$$E_o = \frac{\sigma_v' \, V}{\lambda \text{ or } \kappa}$$

$$C_v = \frac{E_o k}{\gamma_w}$$

$$\frac{\lambda}{\kappa} = 2.6$$

e) If C_v on κ-line is $1 m^2/y$

C_v on λ-line is $1/2.6 \simeq 0.38 m^2/y$

Using Fourier solutions from the Data Book:

at $R_v = 0.2$, $T_v \simeq 0.03$

at $R_v = 0.9$, $T_v \simeq 0.84$

For the 4 m embankment we know $C_v = 1 m^2/y$

so $t_{20} = 0.03 \times \frac{5^2}{1} = 0.75 y \simeq 9$ months

$t_{90} = 0.84 \times \frac{5^2}{1} = 21 y$

For the 6 m embankment, some of the settlement will be on the λ-line with $C_v = 0.38 m^2/y$. But at $R_v = 0.2$ most of the clay remains on the κ-line: see isochrone in (a). So 20% of 0.35 m of settlement, i.e. 0.070 m, will occur in 9 months.

2. e) cont.

At $R_v = 0.9$ there will have been 90% of 0.35 m or 0.315 m of settlement. Assume that almost all the 0.19 m of κ-settlement has occurred at that time. That would leave 0.125 m of the 0.16 m of λ-settlement which must also have occurred, or $R_v = 0.78$ for that component. From Fourier, $T_v = 0.52$ so

$$t_{90 \text{ overall}} \simeq 0.52 \times \frac{25}{0.38} \simeq 34 \text{ years}$$

This is rather approximate since the uppermost and lowermost layers of clay will have drained from $\kappa \rightarrow \lambda$ early in the process

---

Candidates' average mark was 12.6/20 which is good. Needless marks were lost by failing to track the stress history properly. The subtlety of iterating to find the density of the clay was lost on some, together with a couple of marks. The variation in C_v from κ to λ was identified only by a few, and no-one established a sound methodology for dealing with it in (e).

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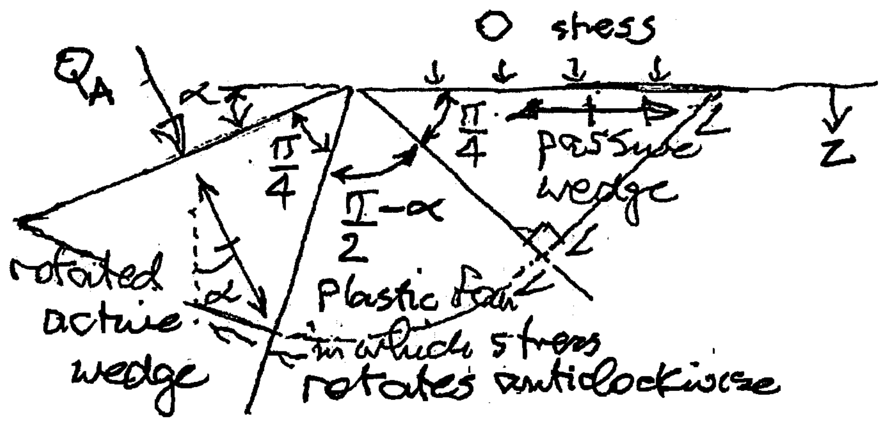
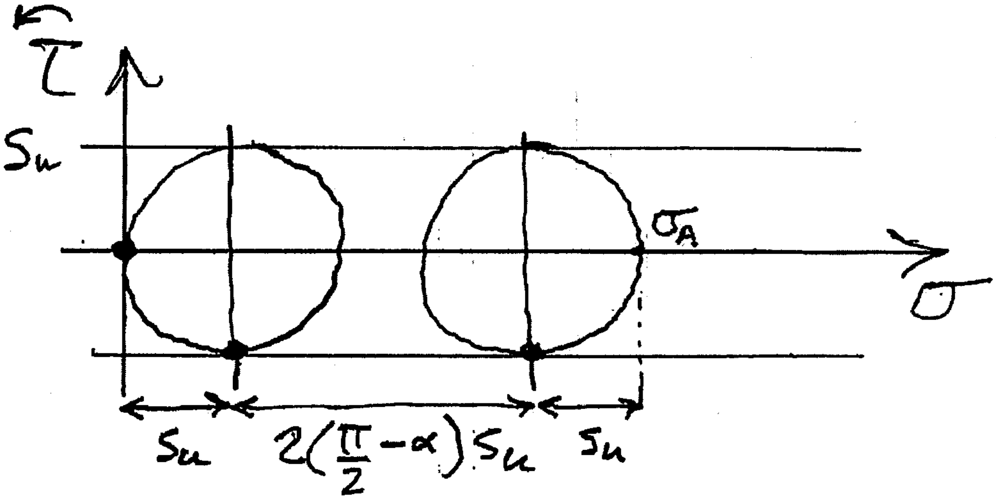
3 a) Scheme A



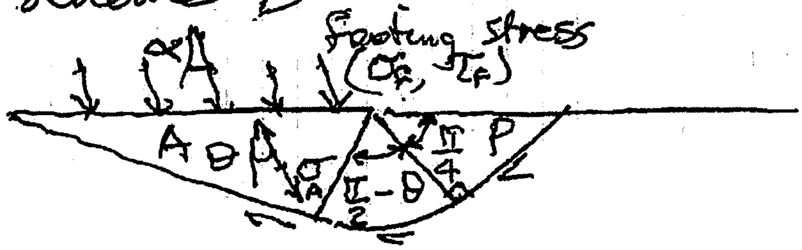


$$\frac{Q_A}{A} = \sigma_A = S_u\left[2+\pi-2\alpha\right]$$

For $\alpha = 20° = 0.349$ radians

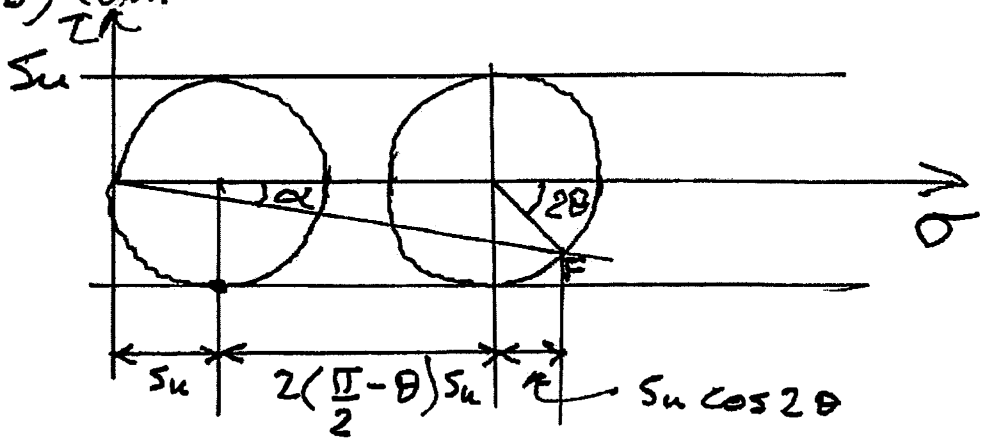
$\sigma_A = \underline{4.444\, S_u}$

b) Scheme B



For the footing: $\tan\alpha = \dfrac{\tau_F}{\sigma_F}$

3 b) cont.



From this construction $\alpha \neq \theta$.

Indeed: $\tau_F = \sigma_F \tan\alpha = S_u \sin 2\theta$

And : $\sigma_F = S_u\left[1 + \pi - 2\theta + \cos 2\theta\right]$

$$\therefore \tan\alpha = \frac{\sin 2\theta}{\left[1+\pi-2\theta+\cos 2\theta\right]}$$

If $\alpha = 20^\circ$, $\tan\alpha = 0.364$

$$\therefore \frac{\sin 2\theta}{\left[1+\pi-2\theta+\cos 2\theta\right]} = 0.364$$

θ is quite small, so begin with $\cos 2\theta \simeq 0.9$?
and $\sin 2\theta \simeq 2\theta$, so first approx.

$2\theta = 1.835 - 0.728\theta$

$\theta \simeq 1.443$ radians $= 82.7^\circ$?

No, $\sin 2\theta$ needs to be bigger!

$\theta = 1$ radian? $= 57.3^\circ$: $0.527 \neq 0.364$

$\theta = 0.8$ radian $= 45.8^\circ$: $0.398 \neq 0.364$

$\theta = 0.75$ radian $= 43.0^\circ$: $0.368 \neq 0.364$

$\theta = 0.743$ radian $= 42.6^\circ$: 0.364 ✓

3b) cont.

So $\sigma_f = 2.74\ s_u$, $\tau_f = 0.996\ s_u$ } resultant $2.92\ s_u$

$\alpha_f = 20°$ ✓

c) Self weight stresses of γz can be superimposed throughout without altering the solution in any other way. If γ is chosen as γ_{soil}, the foundation will then be supported by additional stresses $\gamma_{soil} z$ whereas it requires contact stresses $\gamma_{concrete} z$ to balance its self-weight. But $\gamma_{concrete} \simeq \gamma_{soil}$. Therefore, self weight of concrete and soil roughly balance each other and the solution stands.

Scheme A generates $4.44\ s_u$ per unit area of contact. Scheme B generates $2.92\ s_u$ per unit area. The plan area of Scheme A would therefore be:

$$\frac{2.92}{4.44} \cos 25° \simeq 0.62 \times \text{that of Scheme B}.$$

There is more concrete to place, and a more troublesome slope to cast it on, but Scheme A is likely to be more efficient overall.

3. Feedback: The candidates' average mark was 12.2, but only 26 out of 46 candidates attempted it, and 2 of those wrote almost nothing so they seemed to have run out of time. So their typical performance was good. Marks were lost by candidates generally failing to use their correct algebra to compute numerical values of bearing capacity for an inclination of 20°.

4. a) Gabions

$G_s = 2.7 \quad e = 0.6$

$\gamma_{dry} = \gamma_w \dfrac{2.7}{1.6} = 16.5 \text{ kN/m}^3$

$\gamma_{sat} = \gamma_w \dfrac{(2.7+0.6)}{1.6} = 20.2 \text{ kN/m}^3$

$\gamma' = \gamma_{sat} - \gamma_w = 10.4 \text{ kN/m}^3$

Sand (quartz)

$G_s = 2.65 \quad e = 0.7$

$\gamma_{dry} = 15.3 \text{ kN/m}^3$

$\gamma_{sat} = 19.3 \text{ kN/m}^3$

$\gamma' = 9.5 \text{ kN/m}^3$

Archimedes principle is to allocate buoyancy to each m^3 of wet soil lying below the water table, regarding the soil as displacing water.

b) Having applied Archimedes we remove the water from further consideration, so we now have "heavy sand" above the water table and "light sand" below. Heavy sand creates more destabilizing lateral earth pressures, so the critical case is with the water level taken at its lowest credible elevation.

With water at −2m BGL,

$W' = B\left[2 \times 16.5 + 3 \times 10.4\right] = \underline{64.2\,B \text{ kN/m}}$

c) For the sand, $K_a = (1 - \sin 32^\circ)/(1 + \sin 32^\circ)$

So $K_a = 0.307$ and $K_p = 1/K_a = 3.25$

4c) Then for P_A':

z	σ_v'	σ_h'	$\delta P_A'$
0	0	0	
			$\leftarrow 9.4$ @ 3.67 m
2 m	30.6 30.6 + 0	9.4 9.4	
			$\leftarrow 28.2$ @ 1.5 m $\leftarrow 13.1$ @ 1 m
5 m	30.6 + 28.5	9.4 + 8.75	

0 − 2 m: $\delta P_A' = \frac{9.4}{2} \times 2 = 9.4$ kN/m

2 − 5 m: $\delta P_{A,1}' = 9.4 \times 3 = 28.2$ kN/m

$\delta P_{A,2}' = \frac{8.75}{2} \times 3 = 13.1$ kN/m

So $P_A' = 9.4 + 28.2 + 13.1 = \underline{50.7 \text{ kN/m}}$ $\underline{@\ 1.77\text{ m}}$

For P_p': $P_p' = \frac{1}{2} \times 3.25 \times 9.5 \times 0.5^2$

so $\underline{P_p' = 3.86 \text{ kN} \ @\ 0.17\text{ m}}$

d) $F_h' = P_A' - P_p' = 46.8$ kN/m

$F_v' = W' = 64.28$ kN/m

And for moment equilibrium about the centre of the base:

$F_v' e = 9.4 \times 3.67 + 28.2 \times 1.5 + 13.1 \times 1 - 3.9 \times 0.17$

So $\underline{e = \frac{1.39}{B} \text{ m}}$

Using Meyerhof, the effective base width

$$B' = B - 2e = B - \frac{2.78}{B}$$

So the vertical bearing stress

$$\sigma_v' = \frac{F_v'}{B'} = \frac{64.2B}{\left(B - \frac{2.78}{B}\right)} = \frac{64.2}{\left(1 - \frac{2.78}{B^2}\right)}$$

The bearing capacity

$$\sigma_f' = q_0' N_q + \frac{1}{2} B' N_\gamma \gamma'$$

where, for $\phi = 32°$, $N_q = 23$ and $N_\gamma = 28$

So $\sigma_f' = 109 + \left(B - \frac{2.78}{B}\right) 133$

If we believe we have incorporated enough safety in our assumptions, we can equate σ_v' to σ_f'.

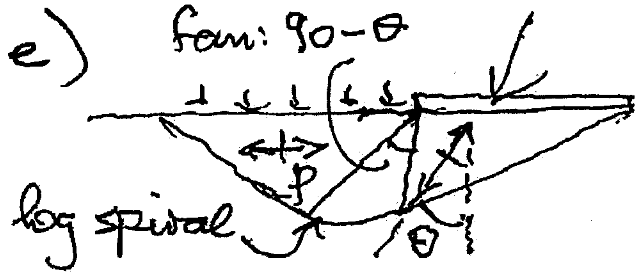
$$\frac{64.2}{\left(1 - \frac{2.78}{B^2}\right)} = 109 + 133\, B\left(1 - \frac{2.78}{B^2}\right)$$

Try $1 - \frac{2.78}{B^2} = \frac{1}{3}$ so that $B = 2.04\,m$

193 c.f. 199 OK ?

So take $\underline{B = 2\,m}$

e) fan: $90-\theta$



N_q solution: impose rotation θ in the σ_1' direction beneath the footing. Find N_q,

N_γ reduces $\sim \times 0.5$?

4. Feedback: Answered by only 26/46 candidates with an average success of 9.6/20 which is poor. Many candidates were running out of time. There was one perfect solution. Some candidates did very badly because they could not (or did not) calculate submerged density properly, or otherwise failed to calculate correctly the vertical effective stresses. Some used Ka and Kp on water pressures as well as effective stresses. Some failed to capitalise on the fact that with hydrostatic conditions, the water forces cancel out (so long as γ' is used for the sand and rocks under water). Only a few had sufficient time, energy and knowledge to calculate a bearing stress using Meyerhof, and then to equate it to the bearing capacity.