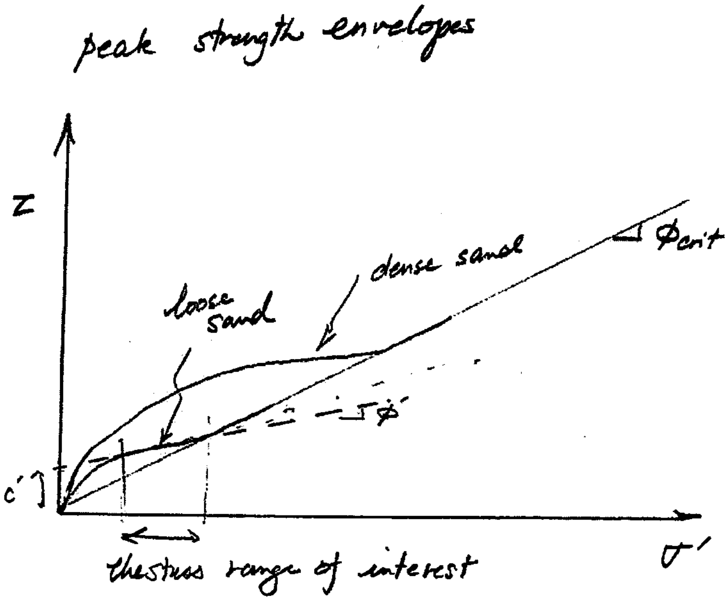
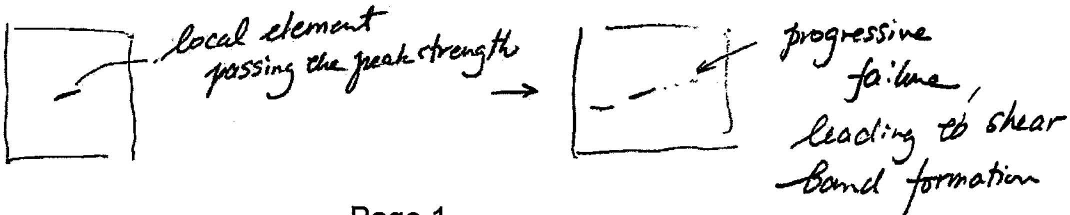
Q1

(a)



When the conventional Mohr-Coulomb model is used, it is important to make sure that the line is fitted to the actual non-linear peak strength envelope at the stress range of interest in geotechnical design

(b) Dense sand exhibits strain softening behaviors. When a particular location in the soil passes the peak and becomes weaker, the loads need to be carried by neighboring elements, which leads to progressive failure. Shear bands will develop. Shear bands have large shear strain, which will be at critical state even though the surrounding soil may not be at critical state.



Q1

(a/c) $\sigma_c = 20{,}000$ kPa for quartz sand

(i)

$$I_R = I_D \cdot I_C - 1 = \frac{e_{max} - e}{e_{max} - e_{min}} \cdot \ln(\sigma_c' / \sigma_v') - 1 = 0$$

$$\frac{e_{max} - e}{e_{max} - e_{min}} = \frac{1}{\ln(\sigma_c' / \sigma_v')}$$

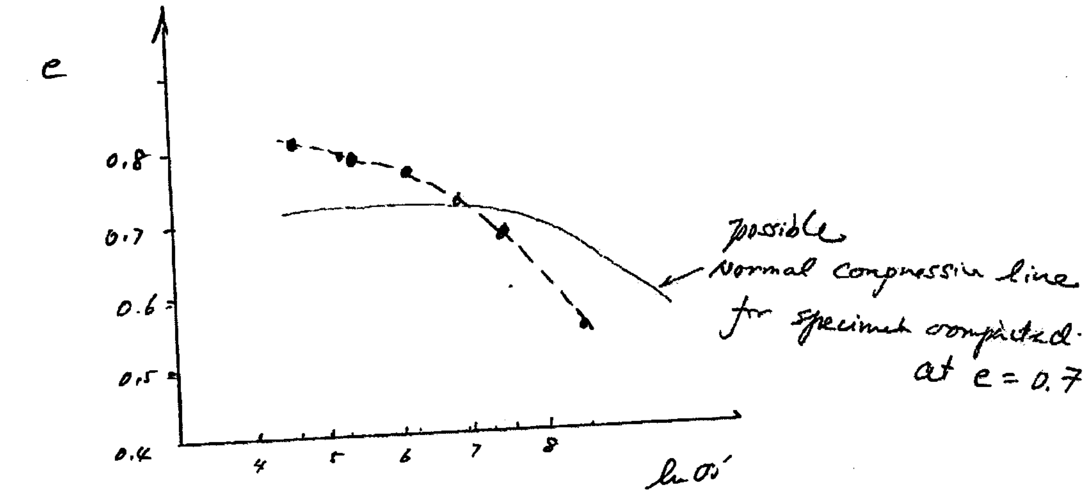
$$e = e_{max} - \frac{e_{max} - e_{min}}{\ln(\sigma_c / \sigma_v')}$$

$$= 0.90 - \frac{0.90 - 0.40}{\ln(2000 / \sigma_v')}$$

$$= 0.90 - \frac{0.50}{\ln(20000 / \sigma_v')}$$

σ_v'	100	200	500	1000	2000	5000
e	0.805	0.791	0.764	0.733	0.683	0.539
$\ln \sigma_v'$	4.61	5.30	6.21	6.90	7.60	8.52

(ii)



If the normal compression line is below the critical state line, the soil will dilate and exhibit peak strength before softening to the critical state. If the normal compression line is above the critical state line, the soil will contract during shearing and show strain hardening to the critical state strength

(ii) sub angular quartz sand

$\phi_{crit} = 36°$

$$I_D = \frac{0.90 - 0.65}{0.90 - 0.40} = 0.5$$

$$I_C = \ln(20.000/200) = 4.605$$

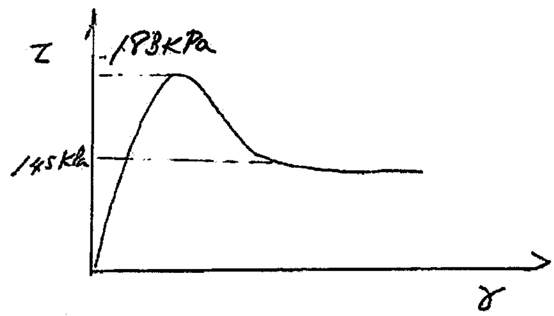
$$I_R = 4.605 \times 0.50 - 1 = 1.30$$

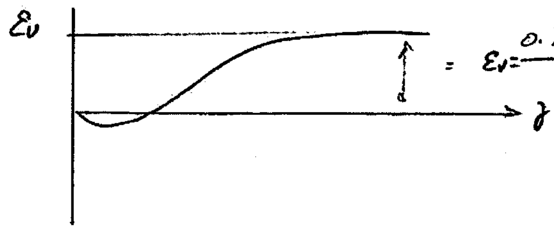
$$\phi_{max} = 5 \times I_R + \phi_{crit} = 5 \times 1.30 + 36 = 42.5°$$

peak shear strength $= 200 \cdot \tan \phi_{peak} = 183$ kPa

critical shear strength $= 200 \cdot \tan \phi_{crit} = 145$ kPa

$e_{crit} = 0.791$ from (a)





$$= \varepsilon_v = \frac{0.791 - 0.65}{1 + 0.65} = 8.5\%$$

(iii)

At critical state

$$0.80 = 0.90 - \frac{0.50}{\ln(20000/\sigma_v')}$$

$$\frac{0.50}{\ln(20000/\sigma_v')} = 0.90 - 0.80$$

$$\ln(20000/\sigma_v') = 5$$

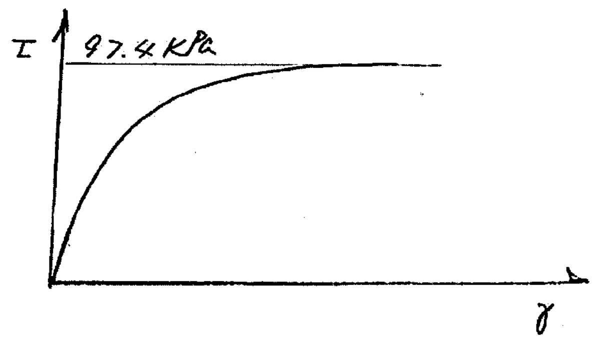
$$2000/\sigma_v' = e^{2.5}$$

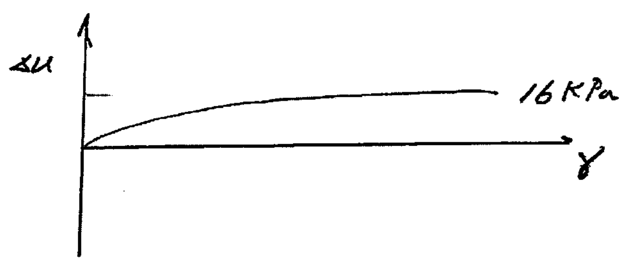
$$\sigma_v' = 134 \text{ kPa}$$

undrained shear strength = $\sigma_v' . \tan \phi_{crit} = 134 \tan 36°$

$= 97.4$ kPa

Excess pore pressure = $150 - 134 = 16$ kPa





Q2 (a) London clay

$\lambda = 0.161$

$\kappa = 0.062$

Γ at 1kPa $= 2.759$

$M_{comp} = 0.89$

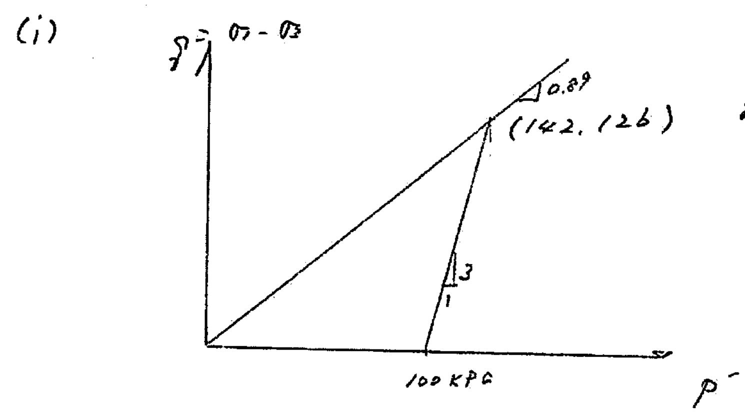
$M_{ext} = 0.69$

$N = \Gamma + \lambda - \kappa = 2.759 + 0.161 - 0.062 = 2.858$

v when normally consolidated at 100kPa

$v = 2.858 - 0.161 \ln 100 = 2.117$

(i)



$q_f = M p_f'$

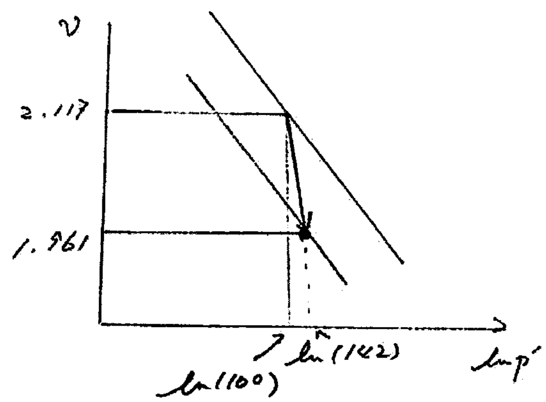
$q_f = 3(p_f' - 100)$

$0.89 p_f = 3(p_f' - 100)$

$2.11 p_f' = 300$

$p_f' = 142$ kPa

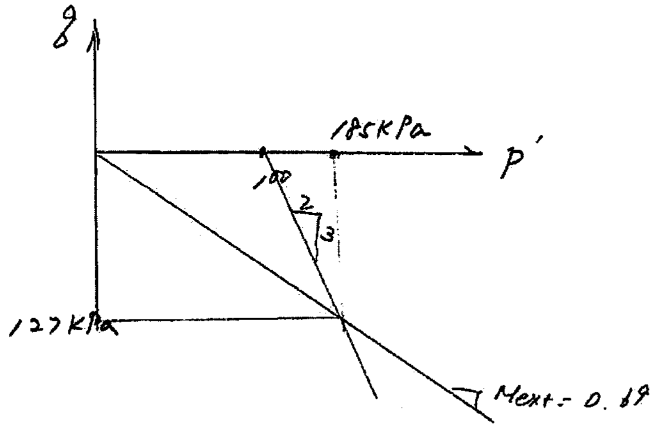
$q_f = 126$ kPa



$v_f = 2.759 - 0.161 \cdot \ln 142$

$= 1.961$

(ii)



$$q_f = M p'_f$$

$$q_f = \frac{3}{2}(p'_f - 100)$$

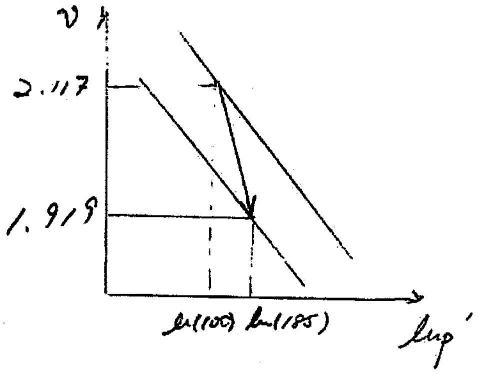
$$2q_f = 3(p'_f - 100)$$

$$2 \cdot 0.69 \cdot p'_f = 3(p'_f - 100)$$

$$1.62\, p'_f = 300$$

$$p'_f = 185\ \text{kPa}$$

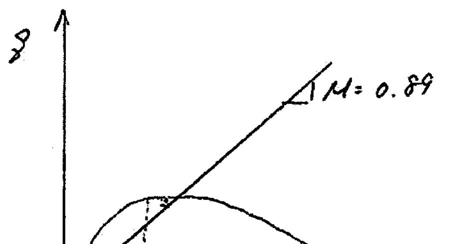
$$q_f = 0.69 \cdot 185 = 127\ \text{kPa}$$

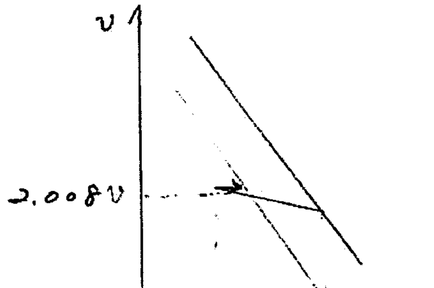


$$v = 2.759 - 0.161 \ln(185)$$

$$= 1.919$$

(iii)





v after intial consolidation

$$v = N - \lambda \ln(300) + \kappa \ln(300/100)$$

$$= 2.858 - 0.161 \ln 300 + 0.062 \ln(3)$$

$$= 2.858 - 0.918 + 0.068$$

$$= 2.008$$

Constant v in undrained Test

$$2.008 = \Gamma - \lambda \ln(p_f')$$

$$= 2.759 - 0.161 \ln p_f'$$

$$p_f' = 106 \text{ kPa}$$

$$q_f = M \cdot p_f' = 0.89 \cdot 106 = 94.34 \text{ kPa}$$

- excess pore pressure

Total pressure at failure if initial pore pressure is zero

$$P_f = 100 + \frac{94.34}{3} = 131.45$$

excess pore pressure

$$P_f - P_f' = 131.45 - 106 = \underline{25.45} \text{ kPa}$$

(ii) Cam-clay yield surface

$$q/p' = M \ln(P_c'/P')$$

$$q_y = P_y' \cdot M \ln(P_c'/P_y')$$

$$= 100 \cdot 0.89 \ln(300/100)$$

$$= \underline{97.78 \text{ kPa}} \quad \text{yield stress}$$

In undrained condition, axial strain is equal to deviator strain.

$$G = \frac{3(1-2\nu)}{2(1+\nu)} \cdot K$$

$$= \frac{3(1-2\cdot 0.2)}{2(1+0.2)} \cdot \frac{vp'}{\kappa}$$

$$= \frac{3(1-0.4)}{2(1+0.2)} \cdot \frac{2.008 \cdot 100}{0.062}$$

$$= 2.429 \text{ KPa}$$

$q_y = 3G\varepsilon_a$

$$\varepsilon_a = q_y/3G = \frac{97.78}{3.2\times 29} = \underline{1.34\,\%}$$

(iii) $2.008 = N - \lambda \ln p'$

$= 2.858 - 0.161 \ln p'$

$\underline{p' = 196\ kPa}$

Q3

(a) Idealising the tunnel construction process as a contracting cylindrical cavity under axisymmetric and undrained conditions:

constant volume (undrained) and mass continuity gives

$$2\pi r \rho = \text{constant}$$

for ρ = radial displacement at radius r

$$\therefore \rho_2 = \frac{r_1 \rho_1}{r_2}$$

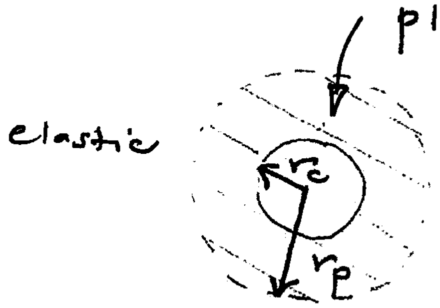
At the tunnel crown $r_1 = 5\text{m}$ and the settlement, $\rho_1 = 50\text{mm}$

Hence ρ_2 at toe of piles ($r_2 = r_1 + 5 = 10\text{m}$) given by

$$\rho_2 = \frac{5}{10} \times 50 = 25\text{mm}$$

[20%]

(b)



r_c = radius of cavity (tunnel)

r_p = radius of plastic zone

For the elastic soil and a contracting cavity

$$\sigma_r = \sigma_0 - G\frac{\delta A}{\pi r^2}$$

$$\sigma_\theta = \sigma_0 + G\frac{\delta A}{\pi r^2}$$

(δA being the reduction in cross-sectional area of the cavity / tunnel)

$$\therefore \quad \sigma_\theta - \sigma_r = 2G\frac{\delta A}{\pi r^2} \quad \text{in } \underline{\text{elastic}} \text{ zone}$$

In $\underline{\text{plastic}}$ zone $\quad \sigma_\theta - \sigma_r = 2c_u$

Hence at boundary of elastic / plastic zones ($r = r_p$)

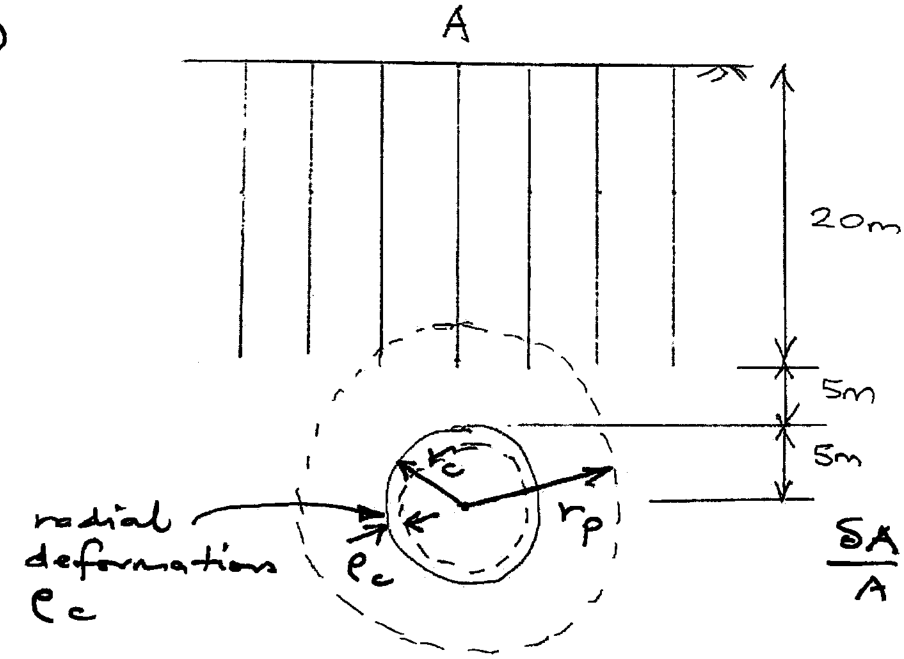
$$2G\frac{\delta A}{\pi r_p^2} = 2c_u$$

A = Current area of cavity (tunnel) $= \pi r_b^2$

$$\therefore \quad \frac{G}{c_u}\frac{\delta A}{A} = \left(\frac{r_p}{r_c}\right)^2 \Rightarrow \frac{r_p}{r_c} = \left(\frac{G}{c_u}\cdot\frac{\delta A}{A}\right)^{1/2}$$

[30%]

(c)



$$\frac{\delta A}{A} \simeq 2\frac{\rho_c}{r_c} = \frac{2\times0.05}{5} = 0.02$$

$$\frac{r_p}{r_c}^* = \left(\frac{G}{c_u}\cdot\frac{\delta A}{A}\right)^{0.5} = \left(\frac{50\times10^3}{150}\times0.02\right)^{0.5}$$

$$= 6.67^{0.5}$$

$$= 2.58 \text{ m}$$

$$\therefore \quad r_p = 2.58\times5 = 12.9 \text{ m}$$

$\therefore$ pile A extends $12.9 - 10 = \underline{2.9\text{ m}}$ into plastic zone

[25%]

(d) From Data Book

$$\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$$

$$\frac{\delta A}{A} = 0.02 \quad (\text{see part (c)})$$

$$\therefore\ \delta\sigma_c = 150\left[1 + \ln\frac{50\times10^3}{150} + \ln 0.02\right] \text{ kPa}$$

$$= 150\left[1 + 5.81 - 3.91\right] = 435 \text{ kPa}$$

$$\sigma_0 = 30 \times 20 = 600 \text{ kPa}$$

$\therefore$ radial stress (average) $\sigma_c = \sigma_0 - \delta\sigma_c$

$$= 600 - 435 = \underline{165 \text{ kPa}}$$

[25%]

Q4

(a) From the Data Book, for normally consolidated deposition

$$K_{onc} = 1 - \sin\phi_{crit} = 1 - \sin 25^\circ$$
$$= 1 - 0.42$$
$$= 0.58$$

(i) original maximum stresses :

$$\sigma_v = 30 \times 20 = 600 \text{ kPa}$$
$$u = 30 \times 10 = 300 \text{ kPa}$$
$$\therefore \sigma_v' = \sigma_v - u = 600 - 300 = 300 \text{ kPa}$$
$$\sigma_h' = K_{onc} \cdot \sigma_v' = 0.58 \times 300 = 174 \text{ kPa}$$
$$\sigma_h = \sigma_h' + u = 174 + 300 = 474 \text{ kPa}$$

$$s = \tfrac{1}{2}(\sigma_v + \sigma_h) = \tfrac{1}{2}(600 + 474) = 537 \text{ kPa}$$

A' (see (iii)) $\left\{ \begin{array}{l} s' = \tfrac{1}{2}(\sigma_v' + \sigma_h') = \tfrac{1}{2}(300 + 174) = 237 \text{ kPa} \\ t = \tfrac{1}{2}(\sigma_v - \sigma_h) = \tfrac{1}{2}(600 - 474) = 63 \text{ kPa} \end{array} \right.$

(ii) present stresses (<u>before</u> wall installed) :

$$\sigma_v = 10 \times 20 = 200 \text{ kPa}$$
$$u = 9 \times 10 = 90 \text{ kPa}$$
$$\therefore \sigma_v' = 200 - 90 = 110 \text{ kPa} \Rightarrow OCR = \frac{300}{110} = 2.73$$
$$\therefore \sigma_h' = 0.9\,\sigma_v' = 0.9 \times 110 = 99 \text{ kPa}$$
$$\sigma_h = \sigma_h' + u = 99 + 90 = 189 \text{ kPa}$$
$$s = \tfrac{1}{2}(200 + 189) = 194.5 \text{ kPa}$$

B' (see (iii)) $\left\{ \begin{array}{l} s' = \tfrac{1}{2}(110 + 99) = 104.5 \text{ kPa} \\ t = \tfrac{1}{2}(200 - 189) = 5.5 \text{ kPa} \end{array} \right.$

(iii) see graph : OA' is effective stress path during 1D normally consolidated deposition. A'B' is effective stress path during erosion of 20m of soil.

[40%]

(b) $p = \sigma_h = 120$ kPa ; $\Delta\sigma_v = 0$

$s = \frac{1}{2}(\sigma_v + \sigma_h) \Rightarrow \Delta s = \frac{1}{2}(\Delta\sigma_v + \Delta\sigma_h)$
$= \frac{1}{2}\Delta\sigma_h$

$t = \frac{1}{2}(\sigma_v - \sigma_h) \Rightarrow \Delta t = \frac{1}{2}(\Delta\sigma_v - \Delta\sigma_h)$
$= -\frac{1}{2}\Delta\sigma_h$

$\therefore \frac{\Delta t}{\Delta s} = -1$

Soil remains elastic $\Rightarrow s' =$ constant

$\sigma_v = 200$ kPa (remains unchanged)

$s = \frac{1}{2}(200 + 120) = 160$ kPa

$\therefore \Delta s = 160 - 194.5 = -34.5$ kPa

$\frac{\Delta t}{\Delta s} = -1 \Rightarrow \Delta t = 34.5$ kPa

$\therefore$ Stress state C, C' (total stresses, effective stresses)

$t = 5.5 + 34.5 = 40$ kPa

$s = 160$ kPa

$s' = 104.5$ kPa (unchanged)

$\Delta u = \Delta s = -\underline{\underline{34.5}}$ kPa

[30%]

(c) At stress state C' $t = 40$ kPa, $s' = 104.5$ kPa

σ_v remains constant $= 200$ kPa
σ_h remains constant $= 120$ kPa
$\}$ $\therefore$ t and s remain constant

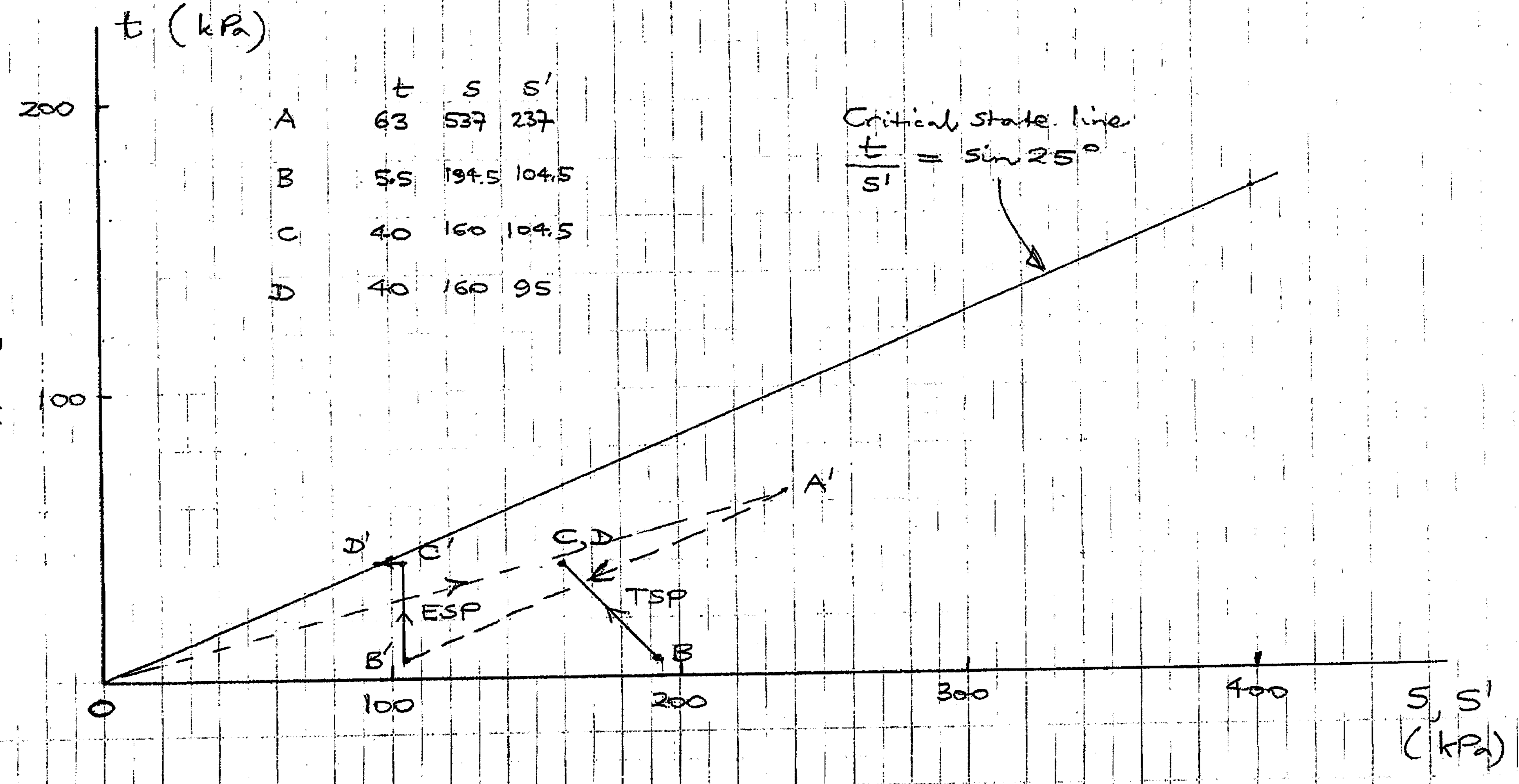
At failure, when effective stress path reaches critical state line :

$$s' = \frac{t}{\sin 25°} = \frac{40}{\sin 25°} = 95 \text{ kPa}$$

$\therefore$ pore pressure u increases by $104.5 - 95 = \underline{\underline{9.5 \text{ kPa}}}$

(i.e. only a relatively small increase in pore pressure is required to take soil element from C' to critical state line.)

[30%]


t (kPa)
200
100
0
t s s'
A 63 537 237
B 5.5 194.5 104.5
C 40 160 104.5
D 40 160 95
Critical state line
t/s' = sin 25°
A'
C, D
D'
C'
ESP
TSP
B'
B
100
200
300
400
s, s'
(kPa)

1. (a) –
   (b) –
   (c) (i) –
       (ii) –
       (iii) τ_{peak} = 183 kPa, τ_{crit} = 145 kPa, e_{crit} = 0.791
       (iv) τ_u = 98 kPa, Δu = 15 kPa

2. (a) (i) q_f = 126 kPa, ε_v = 7.4%
       (ii) q_f = 127 kPa, ε_v =9.4%
   (b) (i) q_f = 94.3 kPa, Δu = 25.5 kPa
       (ii) q_y = 97.8 kPa
       (iii) p = 196 kPa

3. (a) 25 mm
   (b) –
   (c) 2.9 m
   (d) 165 kPa

4. (a) (i) σ_v = 600 kPa, σ_h = 474 kPa, σ'_v = 300 kPa, σ'_h = 174 kPa
       (ii) OCR = 2.73, σ_v = 200 kPa, σ_h = 189 kPa, σ'_v = 110 kPa, σ'_h = 99 kPa
       (iii) –
   (b) -34.5 kPa
   (c) 9.5 kPa