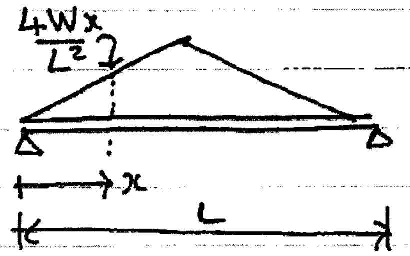
1a)

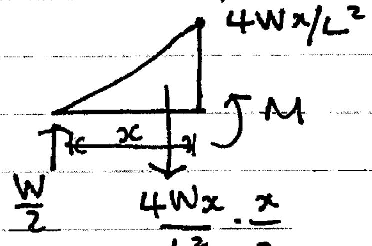


TOTAL LOAD $= W$

$\therefore$ INTENSITY AT $x = \frac{2W}{L} \cdot \frac{x}{L} \cdot \frac{1}{2} = \frac{4Wx}{L^2}$

(CHECK AT $\frac{L}{2} = 2W/L$ ✓)

FREE BODY DIAGRAM:



$\circlearrowright$ ABOUT x

$$M = \frac{Wx}{2} - \frac{2Wx^3}{L^2}$$

BUT $M = EI\left(-\frac{d^2v}{dx^2}\right)$ WHERE v = VERTICAL DEFLECTION

$$\therefore EI\frac{d^2v}{dx^2} = \frac{2Wx^3}{3L^2} - \frac{Wx}{2}$$

$$EI\frac{dv}{dx} = \frac{2Wx^4}{12L^2} - \frac{Wx^2}{4} + C$$

AT $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$

$$\therefore C = \frac{WL^2}{16} - \frac{W\cancel{L^4}^{2}}{96\cancel{L^2}}$$

$$= \frac{5WL^2}{96}$$

$$\therefore EI\frac{dv}{dx} = \frac{Wx^4}{6L^2} - \frac{Wx^2}{4} + \frac{5WL^2}{96}$$

$$\Rightarrow v = \frac{1}{EI}\left(\frac{Wx^5}{30L^2} - \frac{Wx^3}{12} + \frac{5WL^2x}{96}\right)$$

$\therefore$ AT $x = \frac{L}{2}$

$$v = \frac{1}{EI}\left(\frac{W\cancel{L^5}^{3}}{960\cancel{L^2}} - \frac{WL^3}{96} + \frac{5WL^3}{192}\right)$$

$$= \frac{WL^3}{EI}\left(\frac{1 - 10 + 25}{960}\right)$$

$$= \frac{WL^3}{60}$$

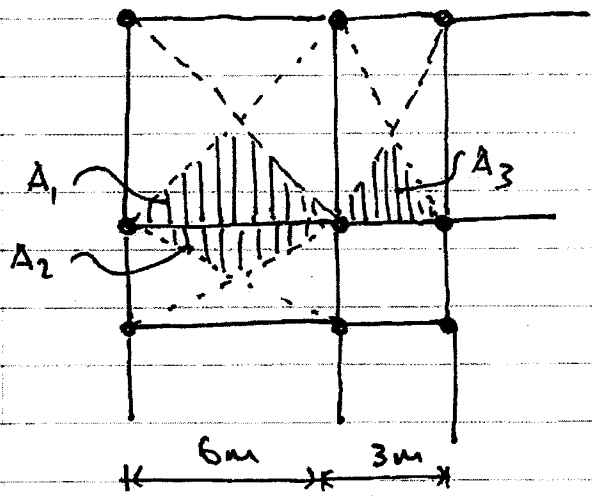
b) i) LOADS : $D.L. = 0{\cdot}3 \times (2400 \times 9{\cdot}81) \times 10^{-3} = 7{\cdot}1\ kN/m^2$

$L.L. = 5\ kN/m^2$

TOTAL SUPERIMPOSED LOAD (SLS) $= 12{\cdot}1\ kN/m^2$

DESIGN (FACTORED) TOTAL (ULS). $= 17{\cdot}9\ kN/m^2$

LOAD PATH (TWO-WAY SPANNING SLABS):

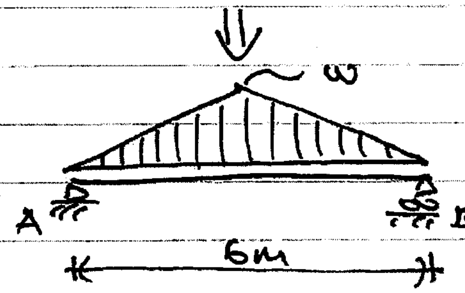


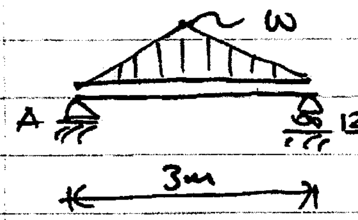
$A_1 = (6\times6)/4 = 9m^2$] 6m BEAM

$A_2 = 6/2 \times 1{\cdot}5 = 4{\cdot}5m^2$] 6m BEAM

$A_3 = 4{\cdot}5m^2$] 3m BEAM.

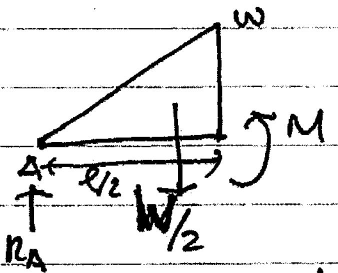
$\therefore$ TOTAL CONTRIBUTORY AREA ONTO 6m BEAM $= 9 + 4{\cdot}5 = 13{\cdot}5m^2$





	3m SPAN	6m SPAN
w (SLS)	$12{\cdot}1 \times 3 = 36{\cdot}3\ kN/m$	$12{\cdot}1 \times (3+1{\cdot}5) = 54{\cdot}5\ kN/m$
w (ULS)	$17{\cdot}9 \times 3 = 53{\cdot}7\ kN/m$	$17{\cdot}9 \times 4{\cdot}5 = 80{\cdot}6\ kN/m$
W (SLS)	$12{\cdot}1 \times 3 \times 1{\cdot}5 = 54{\cdot}5\ kN$	$12{\cdot}1 \times 4{\cdot}5 \times 3 = 163{\cdot}6\ kN$
W (ULS)	$17{\cdot}9 \times 3 \times 1{\cdot}5 = 80{\cdot}6\ kN$	$17{\cdot}9 \times 4{\cdot}5 \times 3 = 241{\cdot}7\ kN$
$R_A = R_B$ (ULS)	$80{\cdot}6/2 = 40{\cdot}3\ kN$	$241{\cdot}7/2 = 120{\cdot}9\ kN$
$\therefore$ M (ULS)	$40{\cdot}3\,(1{\cdot}5 - 0{\cdot}5) = 40{\cdot}3\ kNm$	$120{\cdot}9\,(3-1) = 241{\cdot}8\ kNm$

MAX. BM @ MID SPAN:



$$M = R_A \cdot \frac{l}{2} - \frac{W}{2} \cdot \frac{l}{6}$$

	3m SPAN	6m SPAN
$Z_{REQUIRED} = M/355$	$113.5\ cm^3$	$681\ cm^3$
MAX. ALLOWABLE $\delta = l/200$	8.3 mm	16.7 mm
$\therefore I_{REQUIRED} = WL^3/60E\delta$ (SLS)	$1407\ cm^4$	$16794\ cm^4$

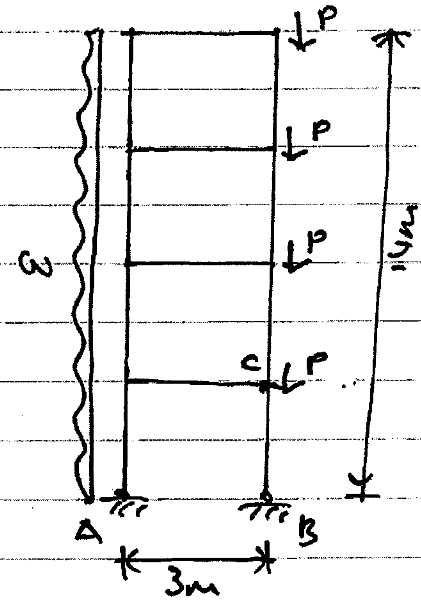
$\therefore$ 3m SPAN PROVIDE 203x102x23 UB $\left[I = 2105\,cm^4,\ Z_p = 234\,cm^3 \right]$ *

6m SPAN PROVIDE 406x178x54 UB $\left[I = 18720\,cm^4,\ Z_p = 1055\,cm^3 \right]$ *

(* DOMINATES)

b ii) LARGER BEAM SIZES REQUIRED DUE TO LATERAL TORSIONAL BUCKLING. THIS WOULD AFFECT BOTH THE 3m AND 6m SPANS ~~AS BOTH ARE DOMINATED BY~~ BUT THE 3m SPAN WOULD BE LESS AFFECTED DUE TO LARGE SPARE CAPACITY IN Z_p.

b iii)



LOADS ON COLUMN BC:

$w = \frac{1}{2}\left((2+1.5) \times (6+3+6)\right) = 26.25\,kN/m$

$P = 2(40.3) + 2(120.9) = 322.4\,kN$

$\therefore$ AXIAL FORCE IN BC DUE TO WIND LOAD ($\therefore$ ↻ ABOUT B) $= 26.25 \times 7/3$

$= 61.25\,kN$

$\therefore$ TOTAL AXIAL FORCE $= 61.25 + (322.4 \times 4)$

$= 1351\,kN$

PLASTIC SQUASHING AREA $= 1351\text{ kN}/355 = 38.1\text{ cm}^2$

ASSUME $\chi \simeq 0.75$

$\therefore$ AREA REQUIRED $= 50.8\text{ cm}^2$

TRY $203 \times 203 \times 46$ UC

$L = 3.5\text{m}$; $r_y = 5.13\text{cm}$; $\lambda_o = \pi\sqrt{\dfrac{210 \times 10^3}{355}} = 76.4$

$\lambda = L/r_y = 68 \Rightarrow \bar{\lambda} = \dfrac{\lambda}{\lambda_o} = 0.89$

$r/y = 53/(203.6/2) = 0.52 \rightarrow$ CURVE B

$\Rightarrow \chi \simeq 0.7$

$\therefore P = 58.7\text{cm}^2 \times 0.7 \times 355$

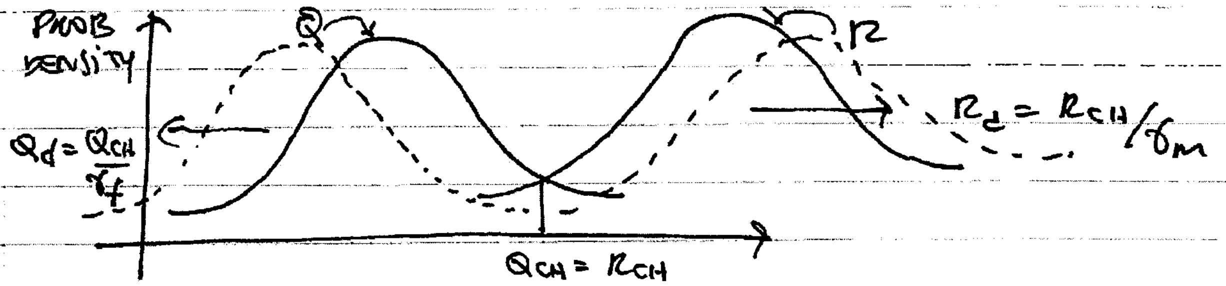
$= 1459\text{kN} > 1351\text{kN}$

$\therefore$ PROVIDE $203 \times 203 \times 46$ UC

2 (a) LIMIT STATE → LIMIT OF ACCEPTABILITY

SLS → EXCESSIVE DEFORMATIONS, VIBRATION ETC.

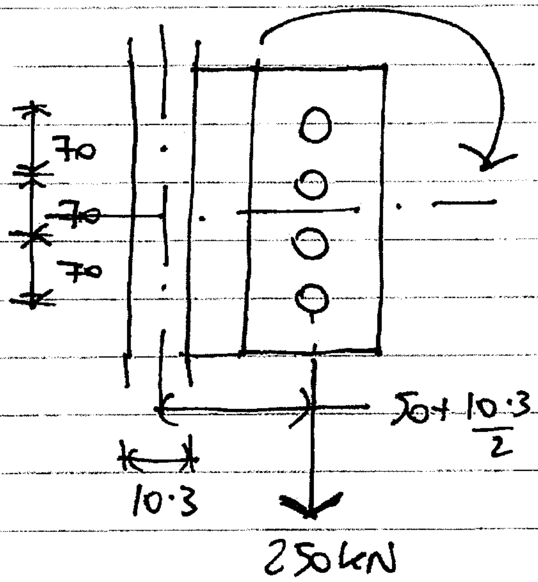
ULS → FAILURE / COLLAPSE



Q → LOADS ; Q_{CH} → CHARACTERISTIC LOAD

R → RESISTANCES ; R_{CH} → " RESISTANCE

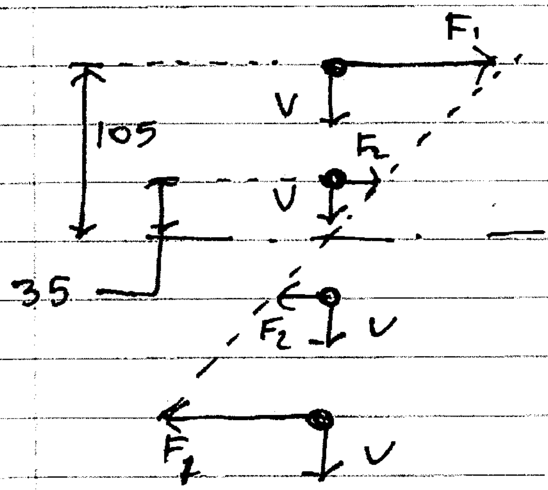
(b) i) BEAM / CLEAT BOLTS.



$$M = 250 \times \left(50 + \frac{10.3}{2}\right) = 13.8\,\text{kNm}$$

VERTICAL SHEAR FORCE / BOLT

$$= \frac{250}{4} = 62.5\,\text{kN}$$



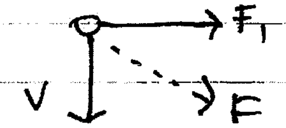
$$F_1 = \frac{13.8 \times 10^3 \times 105}{\left[2(105)^2 + 2(35)^2\right]}$$

$$= 59.1\,\text{kN}$$

$$F_2 = \frac{13.8 \times 10^3 \times 35}{\left[2(105)^2 + 2(35)^2\right]}$$

$$= 19.7\,\text{kN}$$

∴ HEAVIEST LOADED BOLT



$$F = \sqrt{59.1^2 + 62.1^2}$$

$$= 85.72 \text{ kN}$$

∴ SHEAR STRESS IN BOLTS (DOUBLE SHEAR)

$$= \frac{85.72 \times 10^3}{\underbrace{2}_{\text{SHEAR PLANES}} \times \underbrace{4}_{\text{BOLTS}} \times \pi r^2}$$

$$= 34 \text{ N/mm}^2 < (0.6 \times 460)$$

∴ SHEAR STRENGTH IS SUFFICIENT.

~~CHECK WEB THICKNESS:~~

$$\cancel{\text{MIN } t = \frac{85.72 \times 10^3}{355 \times d}}$$

$$\cancel{\therefore d = \frac{85.72 \times 10^3}{355 \times}}$$

WEB BEARING CAPACITY $= 355 \times 20 \times 7.4\text{mm}$

$= 52.4 \text{ kN} < 85.72 \text{ kN}$

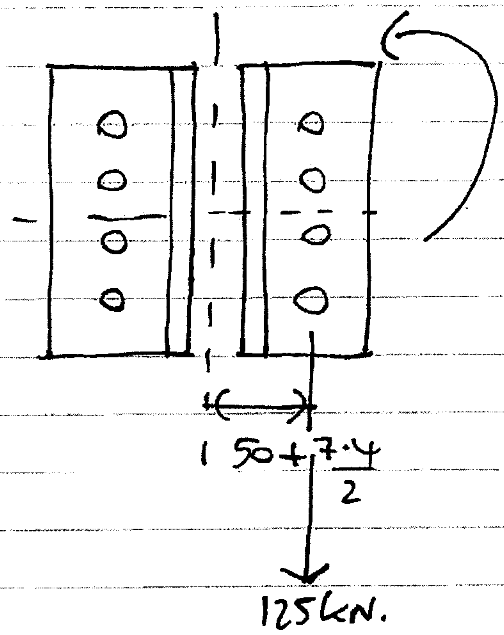
∴ WEB REQUIRES STRENGTHENING:

$$t = \frac{(85.72 - 52.4) \times 10^3}{355 \times d}$$

$$= 4.7\text{mm}$$

∴ PROVIDE 5mm THICK PLATE

CLEAT / COLUMN. BOLTS.

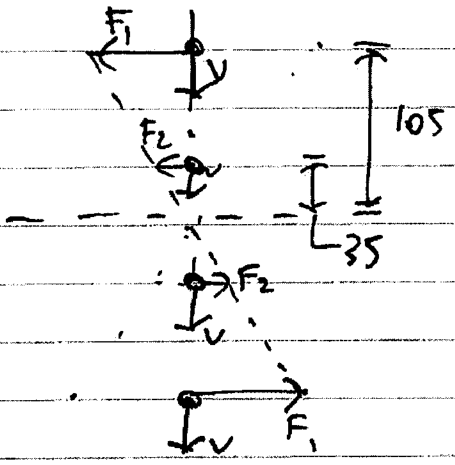


$$M = 125\left(50 + \frac{7.4}{2}\right)$$

$$= 6.7\ \text{kNm}$$

VERTICAL SHEAR FORCE / BOLT

$$= 125/4 = 31.25\ \text{kN}$$



$$F_1 = \frac{6.7 \times 10^3 \times 105}{\left[2(105^2) + 2(35)^2\right]} = 28.7\ \text{kN}$$

$$F_2 = 9.6\ \text{kN}$$

HEAVIEST LOADED BOLT $= \sqrt{31.25^2 + 28.7^2}$

$= 42.4\ \text{kN}$

SHEAR STRESS IN BOLTS (SINGLE SHEAR) $= \dfrac{42.4 \times 10^3}{4\pi r^2}$

$= 33.73 < (0.6 \times 460)$ ∴ OK.

WEB CAPACITY $= 355 \times 20 \times 10.3$

$= 73.1\ \text{kN} > 42.4\ \text{kN}$ ∴ OK.

b ii) MAX. SHEAR FORCE IN CLEAT = 125 kN

$t_{shear} = 125 \times 10^3 / 355 \times 0.6 \times 280 = \underline{2.1\,mm}$

$t_{bending} = 12My / \sigma d^3 \qquad \left(\sigma = \frac{My}{I} \; ; \; \underbrace{I = tb^3/12}_{\text{ignores holes – revisit if critical!}} \right).$

$= \dfrac{12 \times 13.8 \times 10^6 \times 140}{355 \times 280^3}$

$= \underline{2.9\,mm}$

$t_{bearing} = \left(\dfrac{85.72}{2}\right) \times 10^3 \Big/ 355 \times 20$

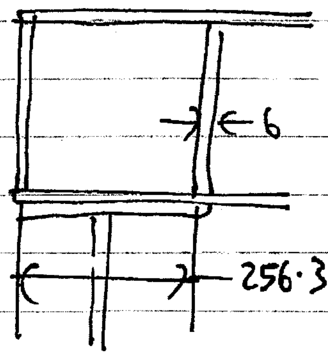
$= \underline{\underline{6.0\,mm}}.$

~~c) SHEAR FORCE BASED ON~~

c) SHEAR CAPACITY BASED ON COMP. STRENGTH OF WEB + STIFFENERS

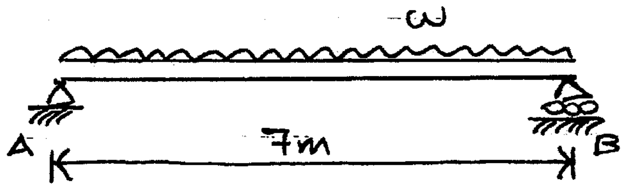
$= \left[\underbrace{(256.3 \times 10.3)}_{\text{WEB}} + \underbrace{((171.5 - 7.4) \times 6 \times 2)}_{\text{STIFFENERS.}} \right] \times 355$

$= \underline{\underline{1636\,kN}}$



d) c,d PROVIDES A MUCH LARGER SHEAR CAPACITY, BUT INVOLVES WELDING (HENCE COST) AND IT IS NOT SUITABLE FOR MULTI-STOREY FRAMES AS COLUMN WOULD BE INTERRUPTED AT EVERY FLOOR i.e. SLOWER CONSTRUCTION + CUMULATIVE CONSTRUCTION TOLERANCES IN VERTICAL LOAD PATH.

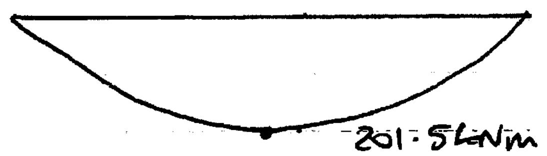
3 a)



ULS L.L. $= 18 \times 1.6 \qquad = 28.8\ \text{kN/m}$

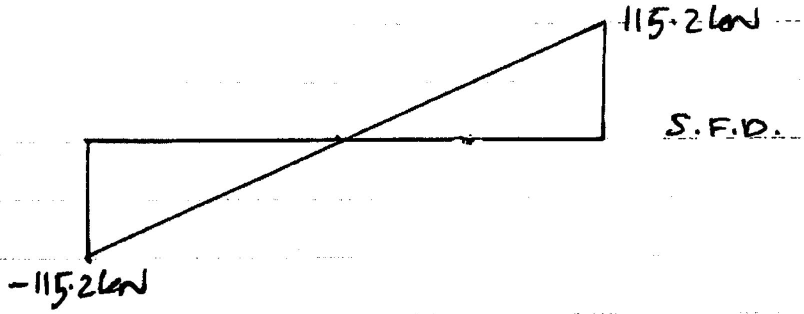
ULS D.L. $= 23.5 \times 0.5 \times 0.25 \times 1.4 \qquad = 4.1\ \text{kN/m}$

ULS TOTAL $= w \qquad = 32.9\ \text{kN/m}$



B.M.D. $\quad M_{MAX} = 32.9 \times 7^2/8$

$= \underline{201.5\ \text{kNm}}$



S.F.D. $\quad R_A = R_B = 32.9 \times 7/2$

$= \underline{115.2\ \text{kN}}$

b) CONCRETE CRUSHING $\quad M_c = 0.225 f_{cu} b d^2/\gamma_c \quad$ WHERE $d = 500 - (40+12$…

$= 0.225 \times 40 \times 250 \times 447.5^2/1.5$

$= 300.4\ \text{kNm} > M_{MAX}\ (201.5\ \text{kNm})$

$\therefore$ NO COMPRESSION STEEL REQUIRED.

$$M_u = \frac{A_s f_y}{\gamma_s} d\left(1 - 0.5\frac{x}{d}\right) = \frac{A_s f_y}{\gamma_s}\left(d - 0.5x\right) \qquad -(1)$$

WHERE $x = f_y A_s / 0.6 f_{cu} b \cdot \left(\gamma_c/\gamma_s\right)$

$= 460 A_s / 0.6 \times 40 \times 250 \cdot \left(\frac{1.5}{1.15}\right)$

$\therefore x = 0.1 A_s$

$\therefore$ SUB INTO (1):

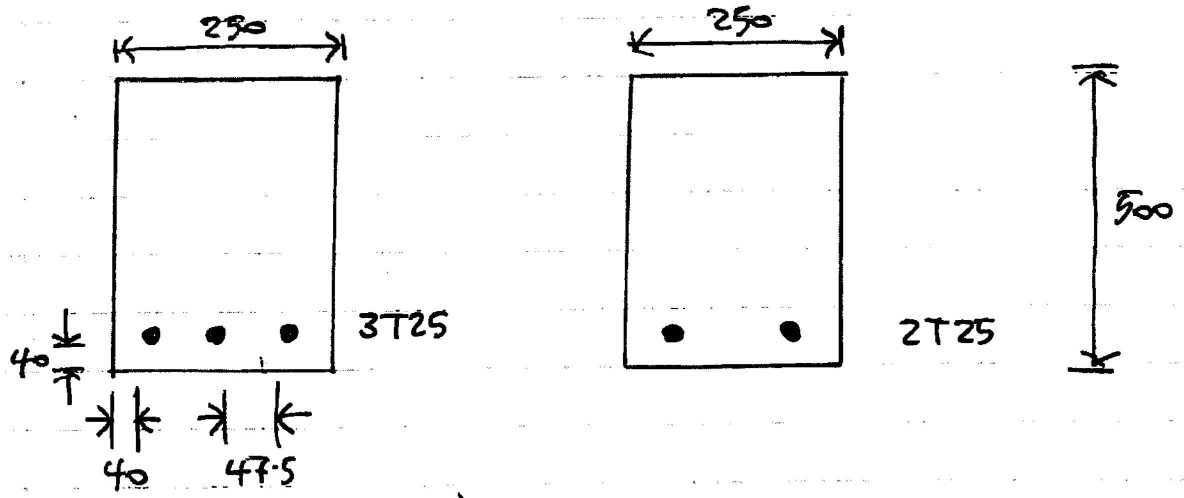
$$201.5 \times 10^6 = \frac{A_s \cdot 460}{1.15}\left(447.5 - \frac{0.1 A_s}{2}\right)$$

$$20 A_s^2 - 179 \times 10^3 A_s + 201.5 \times 10^6 = 0$$

$$\therefore A_s = \frac{179 \times 10^3 \pm \sqrt{(179 \times 10^3)^2 - 4(201.5 \times 10^6)(20)}}{2(20)}$$

$$= 1321 \text{ mm}^2 \quad \text{(LOWER ROOT)}.$$

$\therefore$ PROVIDE 3T25 BARS (1473 mm²).



AT MID-SPAN (X-X) OPTIMISED CLOSER TO SUPPORTS. (Y-Y)

↳ CALCULATE DISTANCE l_1 FROM SUPPORTS AT WHICH 3T25 BARS MAY BE REDUCED TO 2T25 BARS

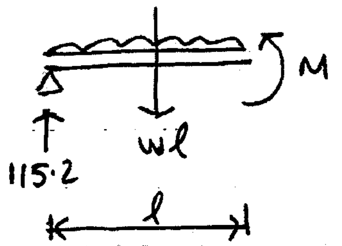
DETERMINE DEPTH OF NEUTRAL AXIS FOR 2T25 BARS:

$$x = \frac{491 \times 2 \times 460 / 1.15}{0.6 \times 40 \times 250 / 1.5} = 98.2 \text{ mm}.$$

$$\therefore M_u = \frac{A_s f_y}{\gamma_s}\left(d - \frac{x}{2}\right) = \frac{491 \times 2 \times 460}{1.15}\left(447.5 - \frac{98.2}{2}\right)$$

$$= 156.4 \text{ kNm}.$$

FROM B.M.D. : $M = -\frac{wl^2}{2} + 115.2\,l$



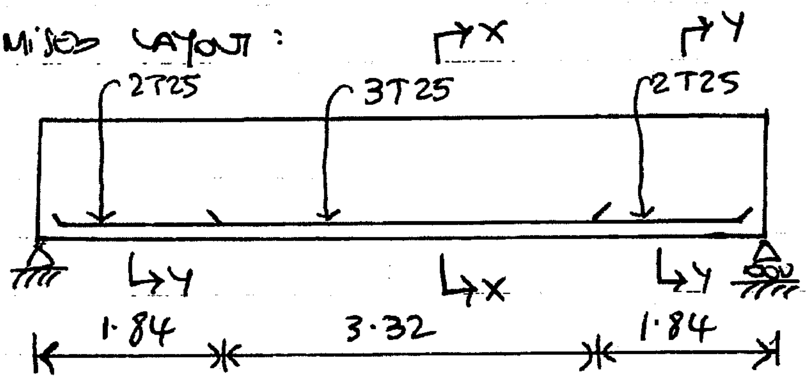
$= -\frac{32.9\,l^2}{2} + 115.2\,l$

WHEN $M = M_u \,(= 156.4\,kNm)$; $l = l_1$

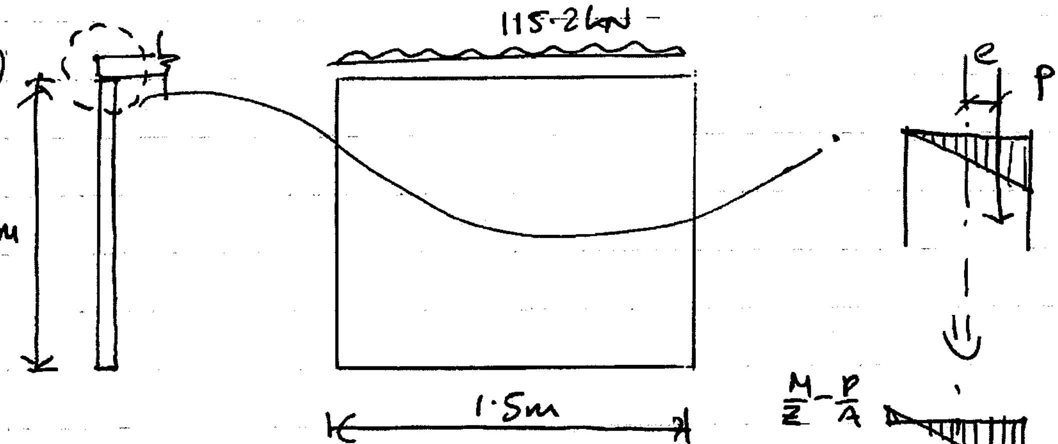
∴ SOLVE FOR $16.45\,l_1^2 - 115.2\,l_1 + 156.4 = 0$

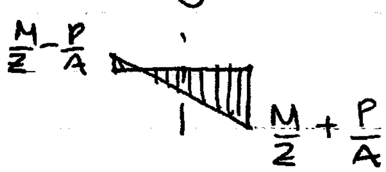
$\Rightarrow l_1 = 1.84m$; $l_1 = 5.16m$

∴ OPTIMISED LAYOUT :



(c)





MODES OF FAILURE :

- FLEXURAL TENSION → $M/Z - P/A$
- " COMPRESSION → $M/Z + P/A$
- BUCKLING.

- FLEXURAL COMPRESSION $\frac{f_k}{\gamma_m} \geq \frac{P}{bt} + \frac{M}{Z}$ BUT $M = \frac{Pe}{Z} = \frac{P.t}{6} . \frac{6}{bt^2}$ $= P/bt$

∴ $\frac{f_k}{\gamma_m} = 2P/bt$

$$\therefore\ t \geq \frac{2P\gamma_m}{b f_k} \geq \frac{2 \times 115.2 \times 10^3 \times 3.5}{1500 \times 4.5}$$

i.e. $t \geq 119.5$ mm

- FLEXURAL TENSION = 0.

- BUCKLING. $P = 115.2$ kN, TRY $t_{ef} = 119.5$ mm

$$\frac{h_{ef}}{t_{ef}} = \frac{2000}{119.5} = 16.7 \Rightarrow \beta \simeq 0.66$$

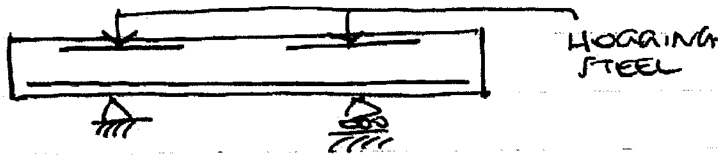
$$\therefore\quad P_b = \frac{\beta A f_k}{\gamma_M} = \frac{0.66 \times 1500 \times 119.5 \times 4.5}{3.5}$$

$$= 152.1\text{ kN} > 115.2\text{ kN}$$

$\therefore$ FLEXURAL COMPRESSION DOMINATES

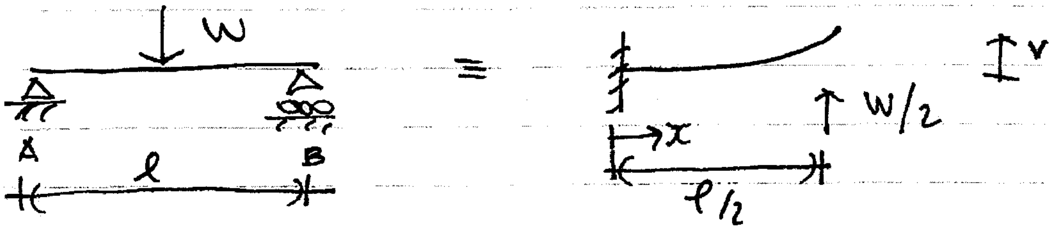
PROVIDE $t > 119.5$ mm, SAY 120mm

d) • REDUCTION IN SAGGING B.M. IN BEAM, BUT INTRODUCES HOGGING B.M. OVER SUPPORTS



- REDUCTION IN ECCENTRICITY AT TOP OF WALL $\therefore$ WALL THICKNESS CAN BE REDUCED.

4) a)



FLEXURAL DEFLECTION

$$v_f = \frac{\frac{W}{2}\cdot\left(\frac{\ell}{2}\right)^3}{3EI} = \frac{W\ell^3}{4Eh^3 b}$$

WHERE h = DEPTH
b = BREADTH.

SHEAR DEFLECTION:

$$\tau_{MAX} = \frac{SA\bar{y}}{Ib} = \frac{W/2 \,.\, bh/2 \,.\, h/4}{\frac{bh^3}{12}\,.\,b} = \frac{3W}{4bh} = \gamma_{NA} G$$

$$dv_s = \gamma_{NA}\, dx$$

$$\therefore\ v_s = \int_0^{\ell/2} \frac{3W}{4bhG}\,.\,dx = \frac{3Wx}{4bhG} + C$$

@ $x = 0 \rightarrow v_s = 0$ $\therefore$ $C = 0$

$\therefore$ WHEN $x = \ell/2$ $\quad v_s = \dfrac{3W\ell}{8bhG}$

$$\therefore\ v_{TOT} = v_f + v_s = \underline{\underline{\frac{W\ell^3}{4Eh^3 b}\left[1 + \frac{3Eh^2}{2G\ell^2}\right]}}$$

b) i BENDING

$$\text{MAX BM} = W\ell/4$$
$$= 18 \times 1.5 \times 5/4$$
$$= 33.75 \text{ kNm}$$

$$\sigma_{MAX} = \frac{M}{Z} = 6M/bh^2$$

$\therefore h = \sqrt{\dfrac{6M}{\sigma b}}$ WHERE $\sigma = f_{m,d} = \dfrac{27\text{ MPa}}{1.3}$

$= 20.8\text{ MPa}$

$= \sqrt{\dfrac{6 \times 33.75 \times 10^6}{20.8 \times 200}}$

$= \underline{220.6\text{ mm}}$.

SHEAR.

MAX. SHEAR FORCE $= 18 \times 1.5 = 27\text{kN}$
(AT MID SPAN)

$\tau_{max} = S/bh$ WHERE $\tau = f_{v,d} = 2.8/1.3$

$\therefore h = S/\tau b$ $= 2.15\text{MPa}$

$= 27 \times 10^3 / 2.15 \times 200$

$= \underline{62.8\text{mm}}$.

DEFLECTION.

CHECK $h = 220\text{ mm}$. (NOTE SLS $\therefore$ NO γ ON LOAD)

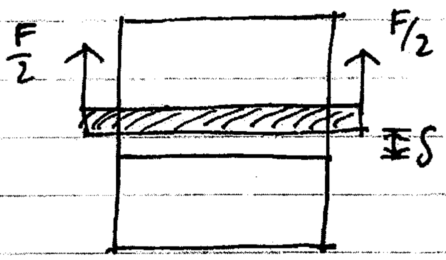
$$V_{TOT} = \frac{18 \times 10^3 \times 5000^3}{4 \times 12 \times 10^3 \times 200 \times 220^3}\left[1 + \frac{3 \times 12 \times 10^3 \times 220^2}{2 \times 0.75 \times 10^3 \times 5000^2}\right]$$

$= 22.9\text{ mm}. < \dfrac{\text{SPAN}}{200}\ (25\text{mm}) \therefore$ OK.

$\therefore$ BENDING GOVERNS $\underline{\underline{h > 220.6\text{mm}.}}$

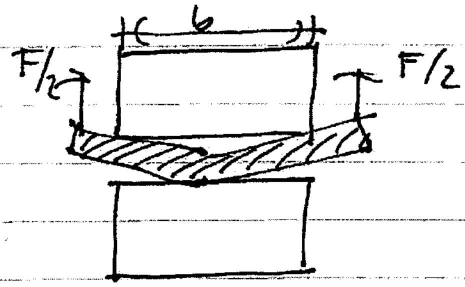
b ii) TWO MODES TO CONSIDER:

MODE (b)



BEARING FAILURE IN TIMBER.

MODE (j)



PLASTIC HINGE IN BOLT.

$$F\delta = f_{h1d} \cdot t \cdot d \cdot \delta$$

$$\therefore F = f_{h1d}\, t\, d$$

$$\therefore d = F / f_{hd} \cdot t$$

$$= \frac{18 \cdot \times 10^3 \times 1.5/2}{14.85 \times 200}$$

$$= \underline{4.54\,\text{mm}}$$

$$= \frac{f_{h,0,k}}{k_{90} \sin^2 90 + \cos^2 90} \cdot \frac{1}{\gamma_m}$$

$$= \frac{f_{h,0k}}{1.1 \times 1.3}$$

TRY 30mm BOLT $f_{h,0,k} = 0.082(1 - 0.01 \times 30)\,370 = 21.24\,\text{MPa}.$

$$\therefore f_{h,90,d} = 14.85\,\text{MPa}.$$

$$M_{yd} = 0.8\, \sigma_y\, d^3 / 6\gamma_m$$

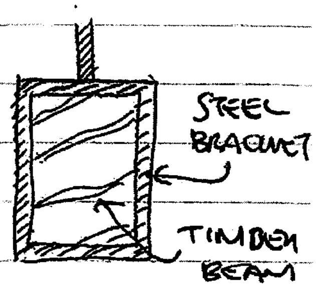
$$Fb/8 = 0.8 \times 275 \times d^3 / 6 \times 1.15$$

$$\therefore d = \sqrt[3]{\left(\frac{9 \times 10^3 \times 200}{8}\right)\left(\frac{6 \times 1.15}{0.8 \times 275}\right)}$$

$$= \underline{\underline{19.1\,\text{mm}}}$$

$\therefore$ PROVIDE 1 20mm ϕ BOLT.

(iii) CONNECTION GOVERNED BY YIELDING OF BOLT ∴ CONSIDER USING STEEL BOLT WITH HIGHER σ_y OR USING BRACKET →



ENGINEERING TRIPOS PART IIA 2011

NUMERICAL ANSWERS
MODULE 3D3: STRUCTURAL MATERIALS & DESIGN

1 (b) i) 203x102x23 UB; 406x178x54 UB
iii) 203x203x46 UC

2 (b) i) M20 bolts are adequate, but beam web has in sufficient bearing capacity and requires strengthening with 4.7mm thick steel plate.
ii) 6mm
(c) 1636kN

3 (a) Max BM = 201.5kNm, Max SF = 115.2kN.
(b) 3T25 bottom reinforcement at mid span, 2T25 bottom reinforcement at supports.
(c) 120mm

4 (b) i) 220.6mm
ii) 1x20mm diameter bolt.