1.

(a.i) From the discharge record, the base flow is 10 m^3/s.
So, the runoff cue to 1 hour excess rainfall is:

Duration (h)	0-1	1-2	2-3	3-4	4-5	5-6	...
Discharge ($m^3\ s^{-1}$)	6	18	12	4	0	0	...

(a.ii)
First, calculate the excess rainfall in each hour:

Infiltration: $\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)(e^{-K_f t_2} - e^{-K_f t_1})$

Infiltration in the 1st hour: $2(1-0) - \frac{1}{0.5}(10-2)(e^{-0.5\times1} - e^{-0.5\times0}) = 8.3$ mm

Infiltration in the 2nd hour: $2(2-1) - \frac{1}{0.5}(10-2)(e^{-0.5\times2} - e^{-0.5\times1}) = 5.8$ mm

Excess rainfall in the 1st hour: 20-8.3=11.7 mm
Excess rainfall in the 2nd hour: 10-5.8=4.2 mm

According to the unit hydrograph theory:

Duration (h)	0-1	1-2	2-3	3-4	4-5	5-6	6-7
Direct runoff in 1st hour	6×1.17	18×1.17	12×1.17	4×1.17	0	0	0
Direct runoff in 2nd hour		6×0.42	18×0.42	12×0.42	4×0.42	0	0
Base flow	10	10	10	10	10	10	10
Total ($m^3\ s^{-1}$)	17.02	33.58	31.6	19.72	11.68	10	10

The peak flow rate is 33.58 $m^3\ s^{-1}$.

(b.i)
Construct the S curve:

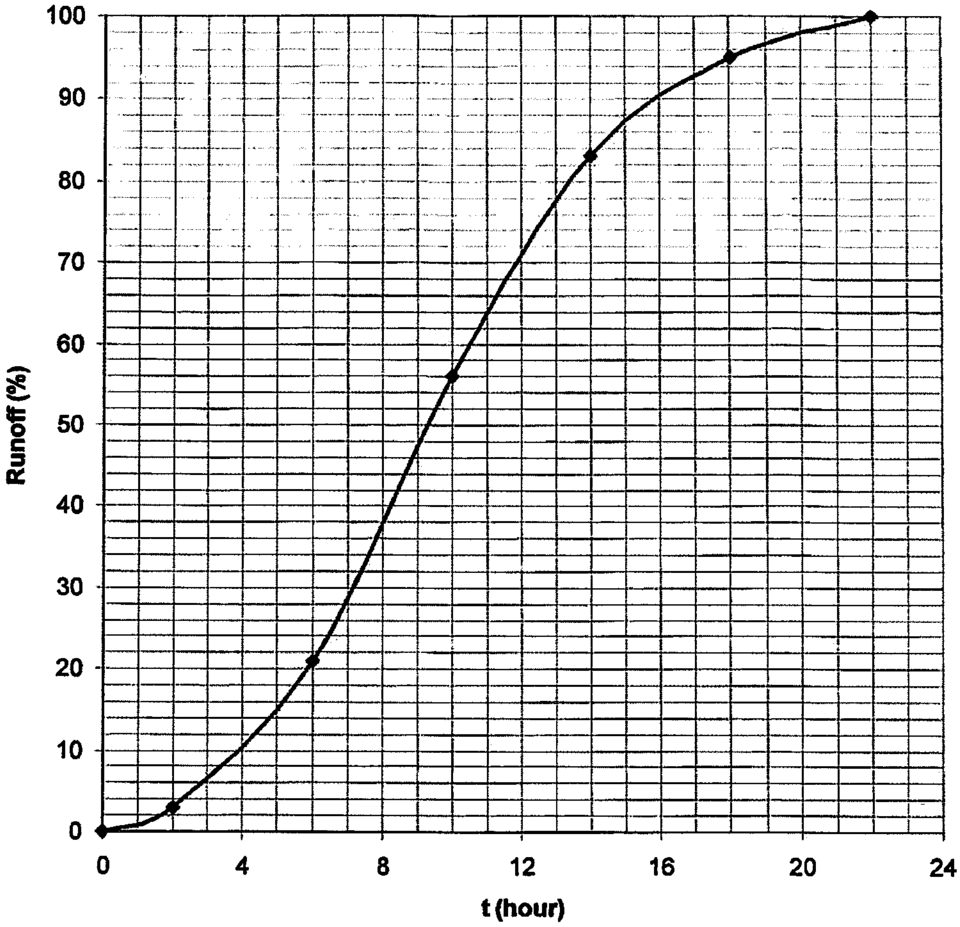
Time (hour)	0	2	6	10	14	18	22
Runoff (%)	0	3	21	56	83	95	100

The time of the concentration is 22 hour.

(b.ii)
Need to find the runoff proportions due to the 3 hour uniform rainfall.
Read from S curve:

Time (hour)	0	1.5	4.5	7.5	10.5	13.5	16.5	19.5	22.5	...
S-curve value	0	2	13	33	60	80	92	97	100	...
Shift S-curve by 3 hour		0	2	13	33	60	80	92	97	...
3-hour hydrograph	0	2	11	20	27	20	12	5	3	0

So, the peak flow rate occurs 10.5 hour (between 9 hours and 12 hours) after the rainfall starts.



100
90
80
70
60
50
40
30
20
10
0
Runoff (%)
0
4
8
12
16
20
24
t (hour)

2 (a.i)

Manning formula: $U=\frac{1}{n}\cdot R_h^{2/3}\cdot S_b^{1/2}=\frac{1}{n}\cdot\left(\frac{B\cdot h_0}{B+2h_0}\right)^{2/3}\cdot S_b^{1/2}$

$$Q=U\cdot B\cdot h_0=\frac{1}{n}\cdot\frac{(B\cdot h_0)^{5/3}}{(B+2h_0)^{2/3}}\cdot S_b^{1/2}$$

$$10=\frac{1}{0.013}\cdot\frac{(6\cdot h_0)^{5/3}}{(6+2h_0)^{2/3}}\cdot 0.001^{1/2}$$

$h_0=0.88$ m is the solution.

(a.ii)

Critical flow: $U=\sqrt{gh_c}$

$$Q=U\cdot B\cdot h_c=\sqrt{gh_c}\cdot B\cdot h_c=g^{1/2}\cdot h_c^{3/2}\cdot B$$

$$10=9.81^{1/2}\cdot h_c^{3/2}\cdot 6$$

$h_c=0.66$ m

(a.iii)

Gradually varied flow: $\frac{dh}{dx}=\frac{S_b-S_f}{1-Fr^2}=\frac{S_b-\frac{n^2\cdot U^2}{R_h^{4/3}}}{1-Fr^2}$

When h = 0.85 m: $U=\frac{10}{6\times 0.85}\approx 1.96\text{ m/s}$

$$R_h=\frac{6\times 0.85}{6+2\times 0.85}\approx 0.662\text{ m}$$

$$S_f=\frac{n^2\cdot U^2}{R_h^{4/3}}=\frac{0.013^2\cdot 1.96^2}{0.662^{4/3}}\approx 0.001125$$

$$Fr=\frac{U}{\sqrt{gh}}=\frac{1.96}{\sqrt{9.81\times 0.85}}\approx 0.6788$$

$$\frac{dh}{dx}=\frac{S_b-S_f}{1-Fr^2}=\frac{0.001-0.001125}{1-0.6788^2}\approx -2.32\times 10^{-4}$$

When h = 0.75 m: $U=\frac{10}{6\times 0.75}\approx 2.22\text{ m/s}$

$$R_h=\frac{6\times 0.75}{6+2\times 0.75}\approx 0.60\text{ m}$$

$$S_f=\frac{n^2\cdot U^2}{R_h^{4/3}}=\frac{0.013^2\cdot 2.22^2}{0.60^{4/3}}\approx 0.001616$$

$$Fr=\frac{U}{\sqrt{gh}}=\frac{2.22}{\sqrt{9.81\times 0.75}}\approx 0.8184$$

$$\frac{dh}{dx}=\frac{S_b-S_f}{1-Fr^2}=\frac{0.001-0.001616}{1-0.8184^2}\approx -0.001865$$

So $\frac{0.75-0.85}{\Delta x}=\frac{0.001-\frac{0.001125+0.001616}{2}}{1-\left(\frac{0.6788+0.8184}{2}\right)^2}=-8.43\times 10^{-4}$

$$\Delta x = 118.6 \text{ m}$$

Or $$\frac{0.75-0.85}{\Delta x} = \frac{-2.32\times10^{-4}-0.001865}{2} \approx -1.05\times10^{-3}$$

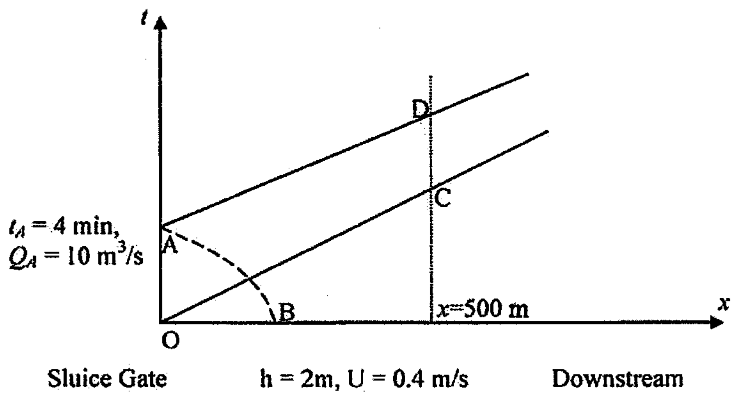
$$\Delta x = 95.2 \text{ m}$$

Or using $$\frac{d}{dx}\left(h+\frac{U^2}{2g}\right) = S_b - S_f$$

$$\frac{\left(0.75+\frac{2.22^2}{2\times9.81}\right)-\left(0.85+\frac{1.96^2}{2\times9.81}\right)}{\Delta x} = 0.001 - \frac{0.001125+0.001616}{2}$$

$$\Delta x = 120.4 \text{ m}$$

(b.i)



Along the negative line AB: $\left(U - 2\sqrt{gh}\right) = const$

$$U_A - 2\sqrt{9.81h_A} = 0.4 - 2\sqrt{9.81\times2} = -8.46$$

Also, $$Q_A = 5U_A h_A = 10$$

It can be easily shown that $h_A = 2.23$ m and $U_A = 0.897$ m/s are the solutions.

(b.ii) According to the straight positive line OC: $\frac{dx}{dt} = U_O + \sqrt{gh_O}$

$$\frac{500}{t_C - 0} = 0.4 + \sqrt{9.81\times2} \Rightarrow t_C = 103.5 \text{ s}$$

(b.iii) According to the straight positive line AD: $\frac{dx}{dt} = U_A + \sqrt{gh_A}$

$$\frac{500}{t_D - 4\times60} = 0.897 + \sqrt{9.81\times2.23} \Rightarrow t_D = 329.7 \text{ s}$$

3 (a.i)

$$U = \frac{Q}{Bh} = \frac{200}{50 \times 2} = 2 \text{ m/s}$$

For uniform flow: $\tau_b = \rho g R_h S_b = \frac{g}{C^2} \rho \cdot U^2$

$$C = \sqrt{\frac{1}{R_h S_b}} \cdot U = \sqrt{\frac{1}{2 \times 0.002}} \times 2 = 31.6 \text{ m}^{1/2} \text{ s}^{-1}$$

Grain-related value: $C' = 7.8 \ln\left(\frac{12h}{k_s'}\right) = 7.8 \ln\left(\frac{12 \times 2}{0.01}\right) = 60.7 \text{ m}^{1/2} \text{ s}^{-1}$

$$\tau_b = \rho g R_h S_b = \frac{g}{C^2} \rho \cdot U^2$$

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d} = \frac{\rho g R_h S_b}{g(\rho_s - \rho)d} = \frac{h S_b}{(s-1)d} = \frac{2 \times 0.002}{(2.65-1) \times 0.01} = 0.24$$

$$\frac{q_b}{\sqrt{g(s-1) \cdot d^3}} = 8\left[\left(\frac{C}{C'}\right)^{1.5} \theta - 0.047\right]^{1.5}$$

$$\frac{q_b}{\sqrt{9.81 \times (2.65-1) \times 0.01^3}} = 8\left[\left(\frac{31.6}{60.7}\right)^{1.5} \times 0.24 - 0.047\right]^{1.5} = 0.0717$$

$$q_b = 2.9 \times 10^{-4} \text{ m}^3/(\text{s·m}) = 2.9 \times 10^{-4} \times 2650 \times 50 = 38 \text{ kg/s}$$

(a.ii)

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{g R_h S_f} = \sqrt{9.81 \times 2 \times 0.002} = 0.198 \text{ m/s}$$

$$D_{ty} = 0.15 h u_* = 0.15 \times 2 \times 0.198 = 0.0594 \text{ m}^2/\text{s}$$

Consider the image of one channel bank:

$$\bar{c}(x,y) = 2 \times \frac{\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{y^2}{4D_y x/U}\right)$$

According to $\bar{c}(x = 1000, y = 0) = 3$ ppm

$$3 = 2 \times \frac{\dot{M}/2}{2 \times \sqrt{4 \times 3.14 \times \frac{1000}{2} \times 0.0594}}$$

$$\dot{M} = 115.6 = c_{source} \cdot Q_{source} = 100 \cdot Q_{source}$$

$$Q_{source} \approx 1.2 \text{ m}^3/\text{s}$$

(b.i)

$$d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.12 \times 10^{-3} \times \left(\frac{9.81 \times (2.65-1)}{10^{-12}}\right)^{1/3} = 3.036$$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot d_*^{\ 3}} - 10.36\right]$$

$$w_s = \frac{10^{-6}}{0.12\times10^{-3}}\left[\sqrt{10.36^2 + 1.049\times3.036^3} - 10.36\right] = 0.0111 \text{ m/s}$$

(b.ii)

$$\bar{c}(z=2) = 2.2 \text{ kg m}^{-3}, \quad \bar{c}(a=1) = 4.2 \text{ kg m}^{-3}$$

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

$$\frac{2.2}{4.2} = \left(\frac{3-2}{2}\cdot\frac{1}{3-1}\right)^{\frac{0.0111}{0.4\times u_*}}$$

$$\ln(0.524) = \ln(0.25)\times\frac{0.0111}{0.4\times u_*}$$

$$u_* = 0.0595 \text{ m/s}$$

$$\tau_b = \rho g h S_b = \rho \cdot u_*^{\ 2}$$

$$9.81\times3\times S_b = 0.0595^2$$

$$S_b = 1.2\times10^{-4}$$

4. (a)

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right), \text{ where } \mathrm{Re} = \frac{UD}{\nu}$$

In hydraulically smooth flow, the friction factor does not depend on k_s, so

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

$$\frac{\sqrt{8g}}{\sqrt{\lambda}} = -2\sqrt{8g}\log_{10}\left(\frac{2.51\upsilon\sqrt{8g}}{UD\sqrt{8g}\sqrt{\lambda}}\right)$$

Note $C = \sqrt{\frac{8g}{\lambda}}$ and $R_h = \frac{D}{4}$, so

$$C = -2\sqrt{8g}\log_{10}\left(\frac{2.51\upsilon C}{4UR_h\sqrt{8g}}\right) = -2\sqrt{8g}\log_{10}\left(\frac{\upsilon \cdot C}{14.1 \cdot UR_h}\right)$$

$$C = 17.72 \cdot \log_{10}\left(\frac{14.1 \cdot UR_h}{\upsilon \cdot C}\right) = 17.72 \cdot \frac{\ln\left(\frac{14.1 \cdot UR_h}{\upsilon \cdot C}\right)}{\ln(10)} = 7.7 \cdot \ln\left(\frac{14.1 \cdot UR_h}{\upsilon \cdot C}\right)$$

(b.i)

$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = 0.025 \times \frac{70}{0.3} \times \frac{\left(\frac{Q}{3.14 \times 0.3^2/4}\right)^2}{2 \times 9.81} = 59.6Q^2$$

$$H_l = \sum \zeta \frac{U^2}{2g} = 2.5 \times \frac{\left(\frac{Q}{3.14 \times 0.3^2/4}\right)^2}{2 \times 9.81} = 25.5Q^2$$

System curve: $H = 15 + H_f + H_l = 15 + 85.1Q^2$

Equating the system and pump curves:

$$15 + 85.1Q^2 = 22.9 + 10.7 \cdot Q - 111.0 \cdot Q^2$$

So, Q = 0.23 m^3/s, H = 19.5 m

(b.ii)

For two pumps in parallel, the pumping head remains the same at double discharge. The characteristic curve can be written as:

$$H = 22.9 + 10.7 \cdot \frac{Q}{2} - 111.0 \cdot \left(\frac{Q}{2}\right)^2$$

Proceeding as before, the duty point is: Q = 0.29 m^3/s, H = 22.2 m or 22.1

For two pumps in series, the pumping head doubles at the same discharge. The characteristic curve can be written as:

$$H = \left(22.9 + 10.7 \cdot Q - 111.0 \cdot Q^2\right) \times 2$$

Proceeding as before, the duty point is: Q = 0.35 m^3/s, H = 25.4 m or 26.1 m

(b.iii)

The dimensionless form of the pump characteristic should be the same for all speeds.

As the impellor size is fixed, $\frac{Q}{N}$ and $\frac{H}{N^2}$ have the unique relationship.

Express the pump characteristic in terms of $\frac{Q}{N}$ and $\frac{H}{N^2}$, where N = 2900 rpm

$$H = 22.9 + 10.7 \cdot Q - 111.0 \cdot Q^2$$

$$2900^2 \times \left(\frac{H}{N^2}\right) = 22.9 + 10.7 \times (2900) \times \left(\frac{Q}{N}\right) - 111.0 \times 2900^2 \times \left(\frac{Q}{N}\right)^2$$

This equation applies to all N. Letting N = 3450 rpm to get the new characteristic.

$$\left(\frac{2900}{3450}\right)^2 \times H = 22.9 + 10.7 \times \left(\frac{2900}{3450}\right) \times (Q) - 111.0 \times \left(\frac{2900}{3450}\right)^2 \times Q^2$$

$$H = 32.41 + 12.73 \cdot Q - 111.0 \cdot Q^2$$