ENGINEERING TRIPOS PART IIA

Thursday 12 May 2:30 - 4:00

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS
Engineering Data Book.
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 A solid rod of circular cross section of radius R and length L is shown in Fig. 1. Reactions within the rod lead to a uniform volumetric heat release rate $\dot{Q}$. The ends of the rod are held at temperatures T_1 and T_2. Assume that the rod material has a uniform and constant thermal conductivity k and the system is in steady state.

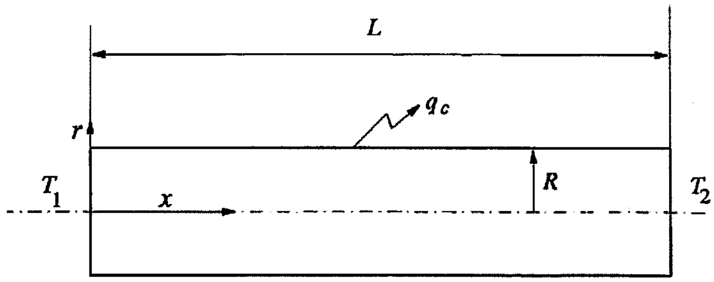

Fig. 1

(a) Assume that the rod diameter is small, so that the temperature along the rod can be represented by a mean temperature T. The marked heat loss per unit area by convection is given as $q_c = h(T - T_\infty)$, where T_∞ is the temperature of the surroudings and h is the convective heat transfer coefficient. Consider a differential element along the rod from distance x to $x + dx$.

(i) Perform an energy balance on the differential element, neglecting higher order terms, to show that the mean temperature is given by:

$$\frac{d^2T}{dx^2} + \frac{\dot{Q}}{k} - \frac{2h}{kR}(T - T_\infty) = 0$$

Show all steps and assumptions in your derivation. [20%]

(ii) Establish a criterion involving a dimensionless quantity to determine whether the volumetric heat release is large or small relative to the convection loss. [15%]

(iii) Obtain an expression for $T(x)$ as a function of the given parameters when the convection loss is small compared to the volumetric heat release. [15%]

(b) Consider a differential element within the rod from distance x to $x+dx$ and from radius r to $r+dr$.

(i) Perform an energy balance on the differential element to show that the governing equation for temperature is:

$$\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{Q}}{k}=0$$

when higher order terms are neglected. [20%]

(ii) Assume that conduction along the x-direction is negligible and that the surface temperature is T_s. Determine the radial temperature profile $T(r)$ as a function of the given parameters. [15%]

(iii) Sketch the variation of $T(r)$ with r and discuss the change in the surface temperature and the surface temperature gradient if the heat loss at the surface is convective: $q_c = h(T_s - T_\infty)$, or radiative: $q_r = \sigma\varepsilon(T_s^4 - T_\infty^4)$, where σ is the Stefan-Boltzmann's constant and ε is the grey body emissivity of the surface. [15%]

2 (a) Considering a single grey diffuse surface i of area A_i and total emissivity ε_i, show that the net rate of radiative heat loss $\dot{Q}_i$ is given by:

$$\dot{Q}_i = \frac{\varepsilon_i A_i (E_{bi} - J_i)}{(1 - \varepsilon_i)}$$

where E_{bi} is the black body radiation and J_i is the radiosity of the surface. [15%]

(b) Considering the same grey diffuse surface i radiating to N other surfaces, show that the explicit expression for the radiosity of the i-th surface is given by:

$$J_i = \frac{1}{1 - F_{ii}(1 - \varepsilon_i)} \left[\varepsilon_i E_{bi} + (1 - \varepsilon_i) \sum_{j=1,(j \neq i)}^{N} F_{ij} J_j \right]$$

where F_{ij} is the view factor from the i-th to the j-th surface. [20%]

(c) Figure 2 shows the cross section of a black body simulator made using a small circular cylindrical hole in a large metal block. The length L is 2 cm and the diameter D is 1 cm. The plate is held at a constant temperature of 1000 K in a black enclosure at 300 K. The metal surfaces within the hole should be considered as grey, diffuse and having a total emissivity of 0.9 . The radiosity of the internal surface varies along the hole. An estimate of the effective emissivity of the hole can be obtained by dividing the internal surface into two sections, the base 1 and the side wall 2 as indicated in Fig. 2.

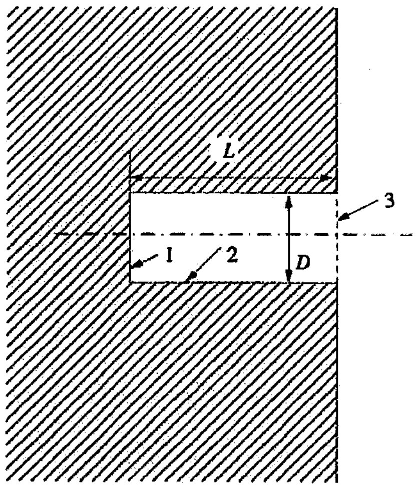

Fig. 2

(i) For each of the three surfaces marked as 1, 2 and 3 in Fig. 2, calculate the view factors F_{ij} and the black body energy emitted E_{bi}. The view factor between a pair of parallel circular discs at a distance to diameter ratio of 2 can be taken as 0.6. The Stefan-Boltzmann constant is $5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. [30%]

(ii) Formulate the nodal equations for the radiosity of the three surfaces using the result in part (b). Write your equations in terms of the black body radiation and radiosity of the other surfaces. Do not evaluate the numerical values for the radiosity. [20%]

(iii) Calculate the rate of total radiative heat loss from the hole in terms of J_1 and J_2. Using this total radiative heat loss, calculate the effective emissivity of the hole in terms of J_1 and J_2. [15%]

3 A counterflow heat exchanger is made of a copper tube of inner radius r, thickness t and length L, surrounded by a jacket. The thermal conductivity of copper is κ. A cold fluid at temperature $T_{c,i}$ enters the tube at one end and a hot fluid at temperature $T_{h,i}$ enters the heat exchanger jacket at the other end. The flow rates of the fluids are $\dot{m}_c$ and $\dot{m}_h$. Treat the specific heat capacities c_c and c_h as constant. The cold and hot side convective heat transfer coefficients are h_c and h_h respectively. Some fouling is expected on the cold side with fouling factor R_f. The heat exchanger operates under steady state.

(a) Carefully sketch the temperature variation along the length of the tube. Mark the directions for fluids flow and heat transfer. [5%]

(b) Draw an equivalent resistance diagram for the heat transfer between the hot and cold fluids. Clearly mark all heat transfer resistances. Write an expression for the overall heat transfer coefficient U based on the inside area of the tube. Express your answer in terms of h_c, h_h, R_f, κ, r, t, and L. [10%]

(c) Fins are used to enhance the heat transfer rate on the cold side. For this case, draw an equivalent resistance diagram and explain it. [10%]

(d) Consider control volumes on the hot and cold sides. By applying an appropriate energy balance across these control volumes, develop a differential equation for $\Delta T = (T_h - T_c)$ in terms of the overall heat transfer coefficient, the mass flow rates and the specific heat capacities. The symbols T_h and T_c respectively denote the bulk mean temperature on the hot and cold sides. Show that ΔT remains constant along the length of the tube when the heat capacities on the hot and cold sides are matched. If U is constant, what can you say about the variation of T_h or T_c along the tube length for the matched condition? [45%]

(e) Deduce an expression for the tube length L required to achieve a temperature $T_{c,o}$ at the outlet of the cold side using your solution for the matched condition given in (d). Using this expression, identify possible methods to reduce L and explain your answers. [30%]

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4 Consider a steady and fully developed laminar flow of air with a small amount of nitric oxide, NO, inside a pipe of radius R. The bulk mean velocity and mass fraction of nitric oxide in the air are U and Y_{NO} respectively. NO from the bulk flow diffuses towards the pipe wall which is coated with a catalyst to convert NO to N_2 and O_2. The rate of this conversion per unit area is given by $k \rho Y_{NO,s}$, where k is a rate constant, ρ is the air density and $Y_{NO,s}$ is the NO mass fraction on the surface. Neglect the change in the bulk mean temperature of the mixture due to this chemical reaction. The convective mass transfer coefficient is $h_m = 2.18\mathscr{D}/R$, where $\mathscr{D}$ is the mass diffusivity.

(a) By considering the mass balance of NO across an appropriate control volume, show that the variation of Y_{NO} along the pipe length, x, is governed by:

$$\frac{dY_{NO}}{dx} = \frac{h_m P}{UA}\left(Y_{NO,s} - Y_{NO}\right)$$

where A and P denote an appropriate area and perimeter. [30%]

(b) Develop a relationship between Y_{NO} and $Y_{NO,s}$ at a given x location in terms of k and h_m. Sketch the variation of $\left(Y_{NO,s}/Y_{NO}\right)$ with the distance from the surface when the reaction is diffusion limited and when the reaction is kinetic limited. [30%]

(c) Integrate the differential equation obtained in (a) for the diffusion limited case to obtain $Y_{NO}(x)$ in terms of h_m, U, R and the NO mass fraction at the inlet, $Y_{NO,i}$. [15%]

(d) For the case in (c), calculate the length of the pipe required to achieve 1% of $Y_{NO,i}$ at the exit, when the volumetric flow rate of the mixture is 2 litres per minute. Take $\mathscr{D} = 1.12 \times 10^{-5}\ \mathrm{m^2\,s^{-1}}$. [25%]

END OF PAPER

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Numerical Answers

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2) c)(i)

$F_{11} = 0$	$F_{12} = 0.4$	$F_{13} = 0.6$
$F_{21} = 0.05$	$F_{22} = 0.9$	$F_{23} = 0.05$
$F_{31} = 0.6$	$F_{32} = 0.4$	$F_{33} = 0$

$E_{b1} = E_{b2} = 5.67E04$ W/m^2, Eb3 = 4.593E02 W/m^2

4) d) 1 m

N. Swaminathan

1/June/2011