ENGINEERING TRIPOS PART IIA

Tuesday 10 May 2011 9 to 10.30

Module 3B5

SEMICONDUCTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) (i) Find the solution to the time-independent Schroedinger's Equation for a particle in the 1-dimensional potential well of Fig.1, for the case $E \ll V_0$, (i.e. the depth of the well can be considered infinite). State all assumptions and determine all integration constants.

(ii) Show that the particle energy levels are given by [25%]

$$E_n = n^2 \frac{h^2}{8ma^2}$$

(iii) In the potential well of Fig.1 an electron is excited from energy level E_1 to E_2 and returns to E_1 by emitting electromagnetic radiation of wavelength $\lambda = 500$nm in vacuum. Calculate the width a of the well. [15%]

(b) Sketch the wavefunctions $\psi_1(x)$, $\psi_2(x)$ corresponding to the two lowest values of the particle energy. [15%]

(c) Assuming that the probability of finding the particle inside the well is unity, calculate the probabilities that a particle with wavefunction $\psi_2(x)$ is in the region:

(i) $0 < x < a/4$;

(ii) $3/4a < x < a$. [20%]

(d) Find the solution to the time-independent Schroedinger Equation for the case of the potential well of Fig.1, when the depth of the well is finite and $E < V_0$. Do not determine the values of the integration constants unless they are equal to zero. Sketch the wavefunction corresponding to the lowest value of E and comment on the differences with that of part (b). [25%]

Note:
Time-Independent Schroedinger Equation:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

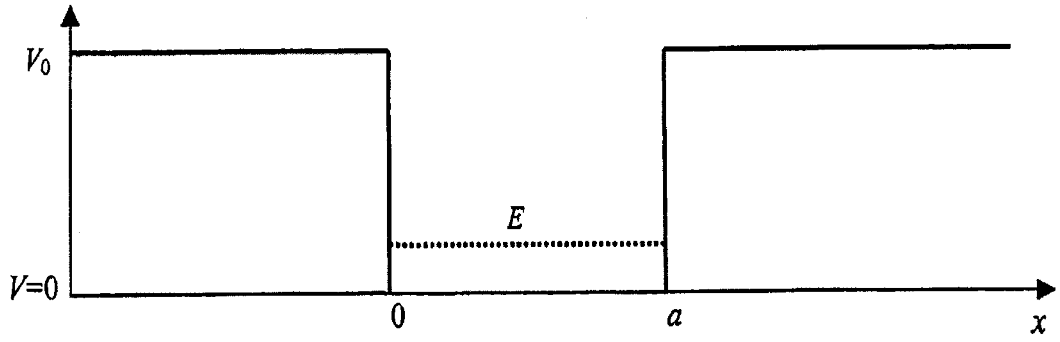

Fig.1

(TURN OVER

2 (a) The quantum numbers n (principal quantum number), l (angular quantum number), m (magnetic quantum number), can take the values:

$$n=1,2\ldots\ldots\ldots\ldots n$$
$$l=0,1\ldots\ldots\ldots\ldots n\text{-}1$$
$$m=0,\pm1\ldots\ldots\ldots\pm l$$

Electrons with $l = 0, l = 1, l = 2, l = 3$ are referred to as "s", "p", "d", "f" electrons respectively. Write the atomic configuration of Helium (He), with a total of 2 electrons, Neon (Ne), with 10 electrons, and Phosphorus (P) with 15 electrons in the form: $1s^{n_{1s}}, 2s^{n_{2s}}, 2p^{n_{2p}}$ ....., where n_{1s}, n_{2s}, n_{2s}, etc are the numbers of 1s, 2s, 2p electrons respectively. Explain why He and Ne do not combine with other elements or among themselves to form molecules (Noble gases). [20%]

(b) Starting from the energy level of a single atom, explain how energy bands arise in crystals and sketch the band structures for conductors, insulators and semiconductors. Explain why a conductor conducts electricity whereas an insulator does not. What is the difference between a semiconductor and an insulator? [20%]

(c) Explain why Sodium (Na), with 11 electrons per atom and Magnesium (Mg) with 12 electrons per atom are both good conductors. [20%]

(d) Explain how the conductivity of silicon can be changed by doping the crystal with donor and acceptor impurities. Explain how one can dope GaAs with donor and acceptor impurities. [20%]

(e) Calculate the Fermi Energy in silicon at room temperature with respect to the Valence Band maximum, when the crystal is doped with a density of boron atoms $N_A = 1 \times 10^{22}\ \text{m}^{-3}$ and $N_A >> N_D$. Assume all acceptors are ionised. [20%]

Note: For silicon at room temperature the effective density of states in the Valence Band is $N_V = 1.04\text{x}10^{25}\ \text{m}^{-3}$.

3 (a) An abrupt p-n junction is fabricated from crystalline silicon. Sketch the space charge density distribution, electric field and electrostatic potential across the unbiased junction. [15%]

(b) The p-n junction is one-sided with a concentration of acceptors in the p region of $N_A = 10^{23}\ \mathrm{m}^{-3}$ and a concentration of donors in the n-region of $N_D = 10^{20}\ \mathrm{m}^{-3}$. Show that the width w of the depletion region may be derived from Gauss' law of Electrostatics to be

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r V_0}{e N_D} \right)^{1/2}$$

where ε_r is the relative permittivity and V_0 the built-in potential. [35%]

(c) A negative bias of -1 V is now applied to the p-side of the junction. Sketch the energy band diagram indicating the applied bias. Calculate the width of the depletion layer and estimate the maximum electric field for the biased junction. Assume $V_0 = 0.63$ V and $\varepsilon_r = 12$. [20%]

(d) Calculate the ratio between the capacitance per unit area of the biased junction and that of the unbiased junction. [20%]

(e) In order to create a photodetector the depletion region thickness is tailored by inserting an intrinsic (undoped) Si layer between the p- and n-region of the diode. Sketch the energy band diagram of the as-formed, unbiased p-i-n diode. [10%]

4 (a) Draw a schematic diagram of a p^+np bipolar junction transistor (BJT) and give the two major components that make up the base current in the active mode of operation. Comment on whether drift or diffusion is the main transport mechanism across an uniformly doped base region. [15%]

(b) Starting from the continuity equation, show that the excess hole concentration in the base as function of distance x from the base edge of the emitter depletion region is given by

$$\Delta p_n(x) = \Delta p_n(0)\frac{\exp\left((W_b - x)/L_h\right) - \exp\left((x - W_b)/L_h\right)}{\exp\left(W_b/L_h\right) - \exp\left(-W_b/L_h\right)}$$

where W_b is the undepleted width of the base, L_h is the hole diffusion length and $\Delta p_n(0)$ is the density of excess holes injected into the base from the emitter junction at $x = 0$. Assume that the excess hole concentration at the base edge of the collector depletion region is zero, i.e. $\Delta p_n(W_b) = 0$. State any further assumptions made. [35%]

(c) Sketch $\Delta p_n(x)$ for $\frac{W_b}{L_h} = 0.5$ and discuss how the BJT design can be optimised to get a base transport factor close to unity. [20%]

(d) In order to improve the frequency response of the BJT the doping density is varied exponentially across the base from a donor concentration $N_{DE} = 10^{23}\ \mathrm{m}^{-3}$ at the emitter junction to $N_{DC} = 10^{20}\ \mathrm{m}^{-3}$ at the collection junction. Estimate the electric field attracting the holes across a base width of $W_b = 1\mu\mathrm{m}$. Assume room temperature operation and state any further assumptions made. [30%]

Note: the Continuity equation for holes is

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} - \mu_h \varepsilon \frac{\partial(\Delta p)}{\partial x} + D_h \frac{\partial^2(\Delta p)}{\partial x^2}$$

END OF PAPER

2011 3B5 Answers to numerical questions

Q1 a(iii): a=6.74x10^{-10}m
c(i):1/4; 1/4

Q2 c: E_F-E_V = 0.174eV

Q3 c: w=4.6 micron
ε_{max} = -694kVm^{-1}
d: 0.63

Q4 d: 177kVm^{-1}