ENGINEERING TRIPOS PART IIA

Tuesday 26 April 2011 9 to 10.30

Module 3D3

STRUCTURAL MATERIALS AND DESIGN

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Where indicated, "ULS" and "SLS" denote Ultimate Limit State and Serviceability Limit State respectively.

Attachments: 3D3 Structural Materials and Design Data Sheets (12 pages)

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

mo03

1 (a) The simply supported beam in Fig. 1 is subjected to a vertical load of total magnitude W that is distributed as indicated. Show that the mid-span deflection is $WL^3/60EI$. [30%]

(b) The multi-storey building shown in Fig. 2(a) and Fig. 2(b) consists of a grade S355 steel frame with simple beam-to-column connections and negligible self-weight. The building is subjected to factored wind loads at ULS as shown in the figure. The external cladding spans horizontally between the floor beams. The floor at each level consists of two-way spanning 300 mm thick concrete slabs, of density 2400 kgm^{-3}. A uniformly distributed live load of 5 kNm^{-2} is applied to the floor and the load combination at ULS is: (1.4 x dead load) + (1.6 x live load). The deflection of the steel beams must not exceed span / 200.

(i) By assuming that the steel beams may achieve fully plastic behaviour and that they are laterally restrained by the concrete slab, select UB sections that will satisfy ULS and SLS requirements for the 6 m spans and the 3 m spans. [30%]

(ii) Without carrying out further calculations explain how your answers to (b)(i) would be affected if the concrete slab is unable to provide lateral restraint to the floor beams. [10%]

(iii) By taking vertical loads and horizontal loads into account determine the axial loads on the four internal pin-ended columns at ground floor level and select a suitable UC section for these columns. [30%]

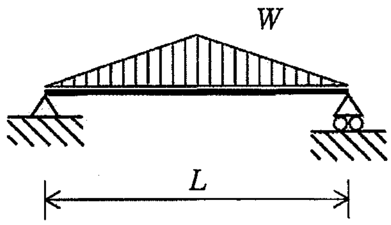
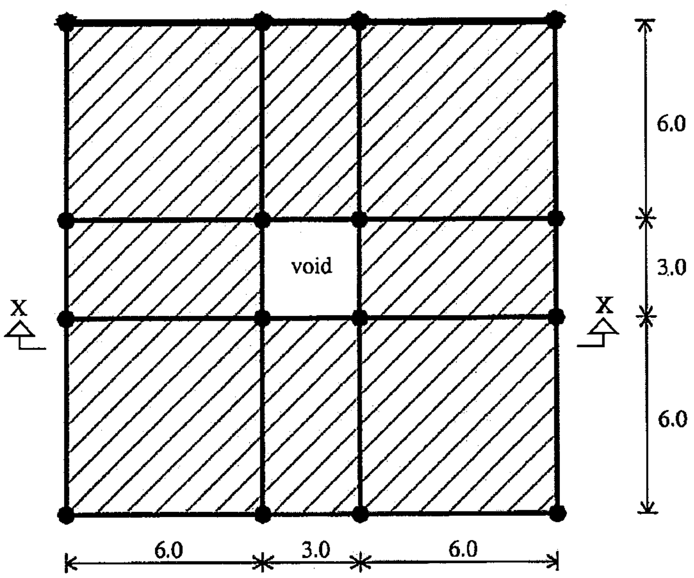
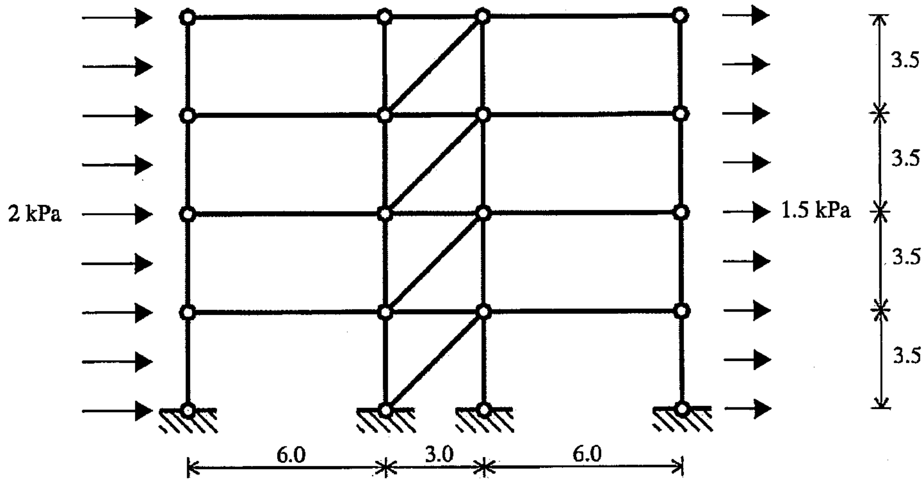


Fig. 1



Typical Floor Plan
(all dimensions in metres; hatched area denotes concrete slab)
(a)



Section XX
(all dimensions in metres)
(b)

Fig. 2

2 (a) Describe, with sketches where appropriate, the limit state design approach in terms of characteristic loads and characteristic resistances. [15%]

(b) Fig. 3 (a) shows a horizontal and vertical section through a bolted connection between a 356 x 171 x 51 UB and a 254 x 254 x 89 UC. The shear force at ULS between the steel beam and the column is 250 kN. The connection uses 280 mm right-angled cleats and bolts in standard clearance holes. All steel sections are grade S355 and all bolts have yield strength of 460 MPa.

(i) Given that the resultant shear force on a given bolt is a function of both the vertical force and the overall transmitted couple. Check whether M20 bolts are satisfactory for both the cleat–to-beam and the cleat-to-column connections. Comment on any modifications that are necessary. [40%]

(ii) By considering the average shear stress, the maximum elastic bending stress and the maximum bolt bearing stress in the cleat, determine the thickness of cleat required. [20%]

(c) Fig. 3(b) shows an alternative design of the connection. Two pairs of 6 mm thick load bearing stiffeners with a yield strength of 355 MPa are provided in the beam. Calculate the shear force that may be safely transmitted from the beam to the column. [15%]

(d) With reference to your answers to (b) and (c), comment briefly on the benefits and limitations of the two connections. [10%]

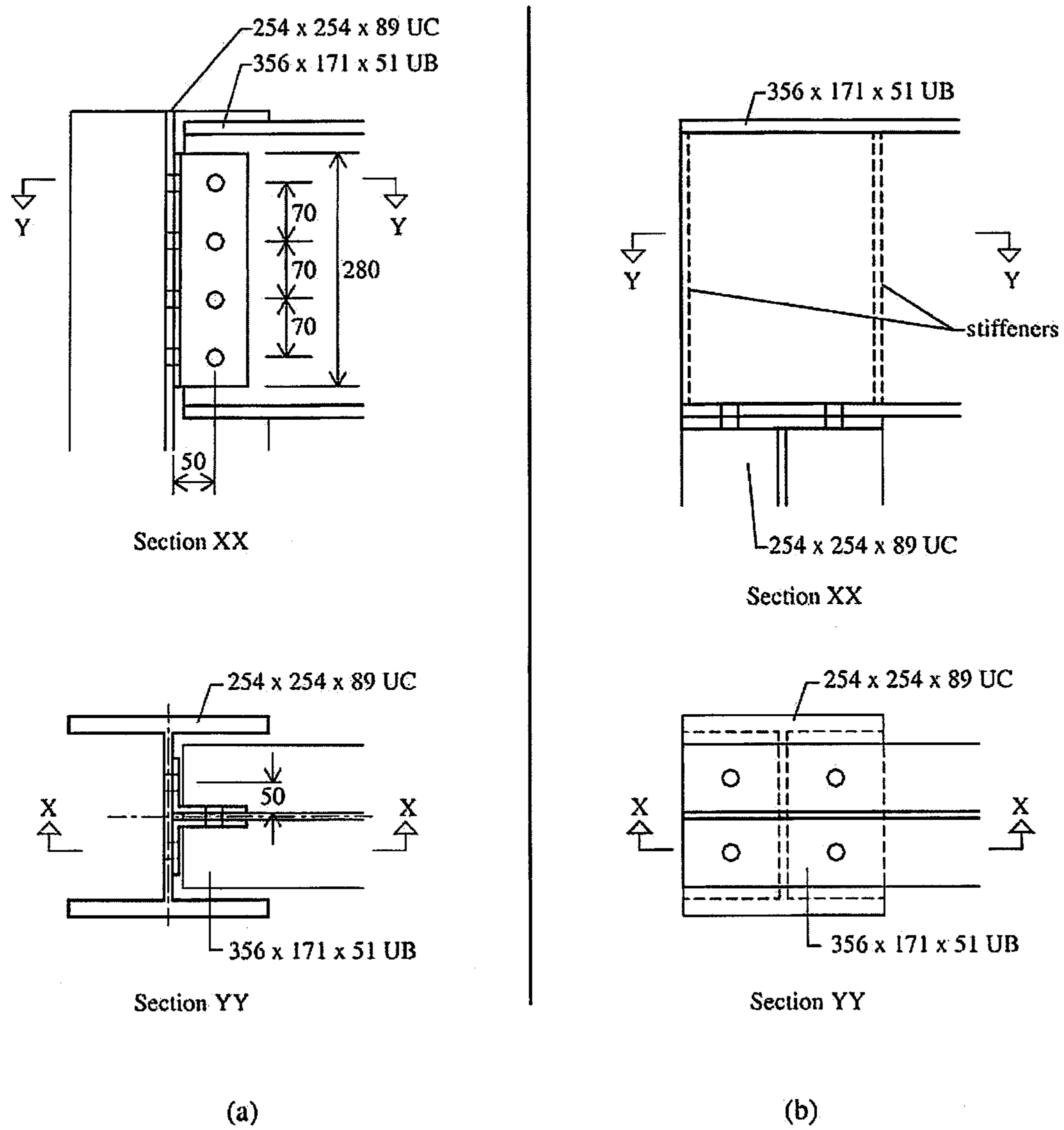


Fig. 3
(bolts not shown for clarity)

3 The 250 mm wide by 500 mm deep by 7 m long reinforced concrete beam shown in Fig. 4(a) is supported by 2 m high brick walls at A and B. The beam supports a uniformly distributed working live load of 18 kNm^{-1} and its self weight. The concrete cube strength is 40 MPa. The longitudinal reinforcement bars have a diameter of 25 mm, a yield stress of 460 MPa and a cover of 40mm. The partial safety factors for concrete and steel are 1.5 and 1.15, respectively, and the load factors for dead and live loads are 1.4 and 1.6 respectively. The partial safety factor for the masonry wall is 3.5.

(a) Sketch the bending moment and shear force diagrams for the concrete beam and identify the locations at which maxima occur. [10%]

(b) Design a layout for the longitudinal reinforcement based on the critical cross-section. By considering the variations in bending moment along the beam, sketch an optimised longitudinal reinforcement layout. [50%]

(c) Assuming that the load from the beam is carried uniformly by 1.5 m long brick walls at A and B and that the compressive strength of the masonry is 4.0 MPa and the flexural strength is 0.5 MPa, determine the minimum thickness of the walls. [30%]

(d) The brick walls at A and B are now located as shown in Fig. 4 (b). Without carrying out further calculations describe briefly how this would affect your answers above. [10%]

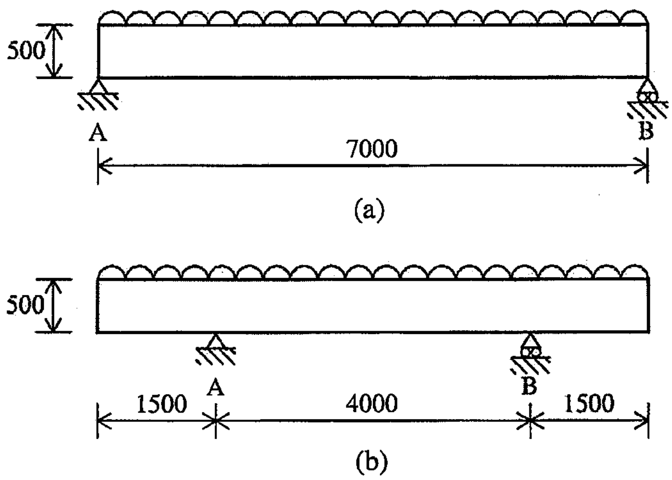


Fig 4

4 (a) By considering both shear and flexural deformations, show that the maximum deflection of a simply supported beam of depth h and width b subjected to a central point load W at mid-span is given by:

$$v_{tot} = \frac{WL^3}{4Ebh^3}\left[1 + \frac{3Eh^2}{2GL^2}\right]$$ [30%]

(b) A 5000 mm long by 200 mm wide C27 timber beam shown in Fig. 5(a) carries a characteristic point load of 18 kN. The beam is restrained laterally. The factors $k_{mod} = k_h = k_{ls} = 1.0$, $\gamma_m = 1.3$ and the load factor for ultimate limit state is 1.5.

(i) Determine the minimum depth h of a timber beam that satisfies shear strength requirements, bending strength requirements and an instantaneous deflection limit of span/200 at mid span. [20%]

(ii) The support at B is actually provided by a steel tie that is connected to the timber beam by means of a single steel bolt as shown in Fig. 5(b). The yield strength and the material safety factor for the bolt are 275 MPa and 1.15, respectively. By assuming that any plastic hinges in the bolt can only form within the breadth of the beam, use Johanson's theory to determine a suitable bolt diameter. In your calculation you may assume that $k_{c90} = 1.1$. [40%]

(iii) Without carrying out further calculations suggest ways of improving the connection shown in Fig. 5(b). [10%]

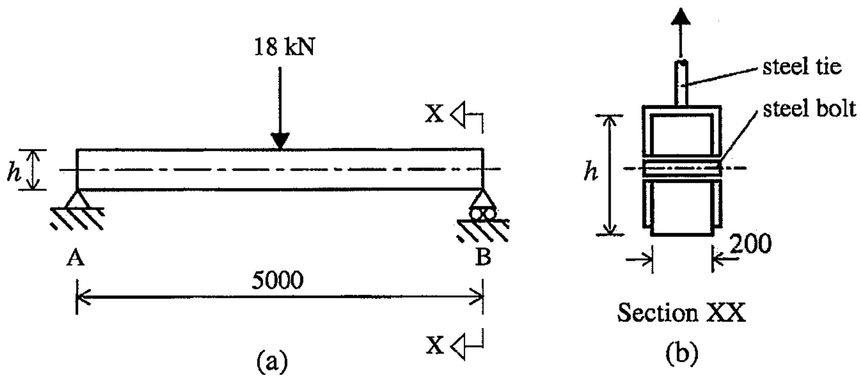


Fig. 5

END OF PAPER

mo03

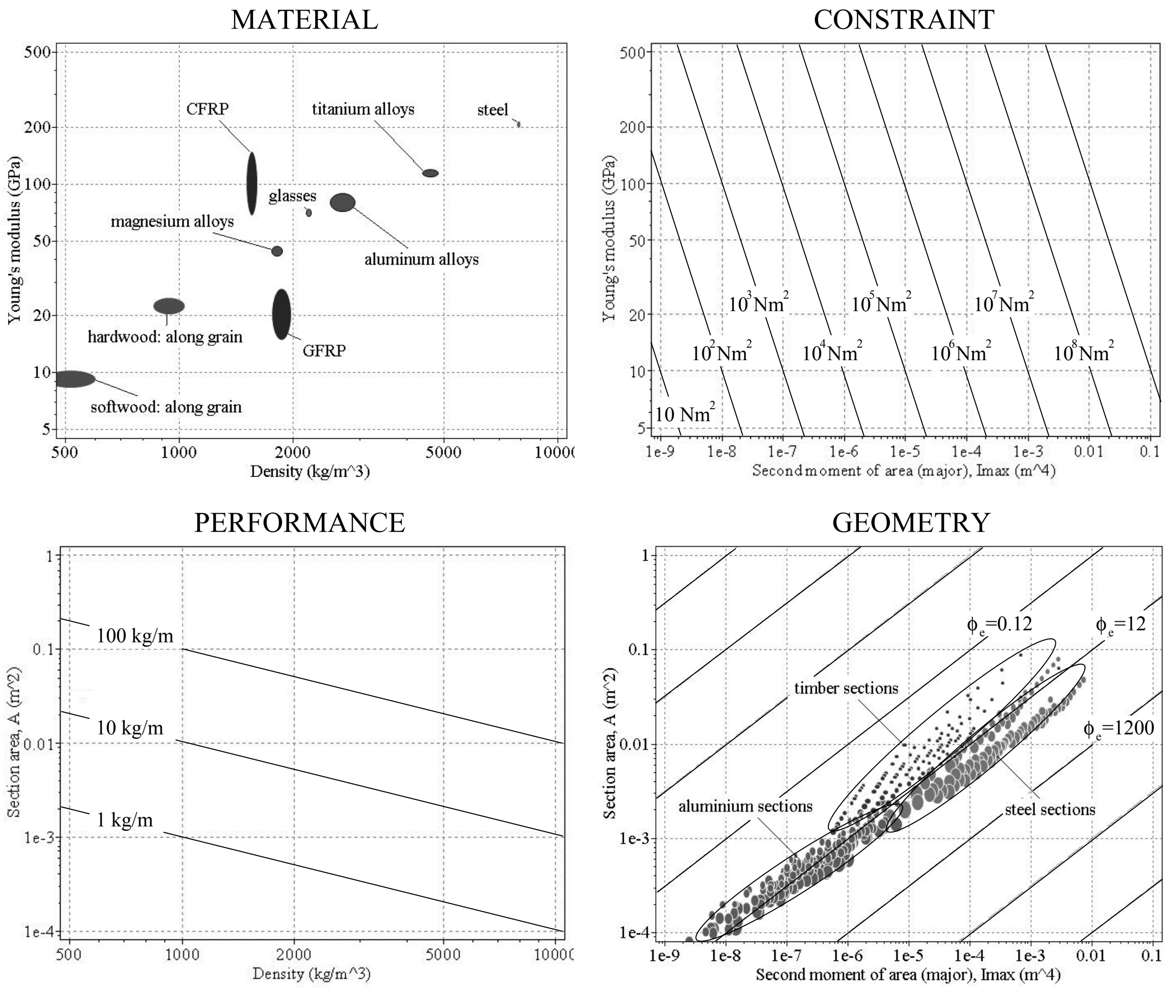
3D3: STRUCTURAL MATERIALS AND DESIGN

Data Sheets: 2009/10

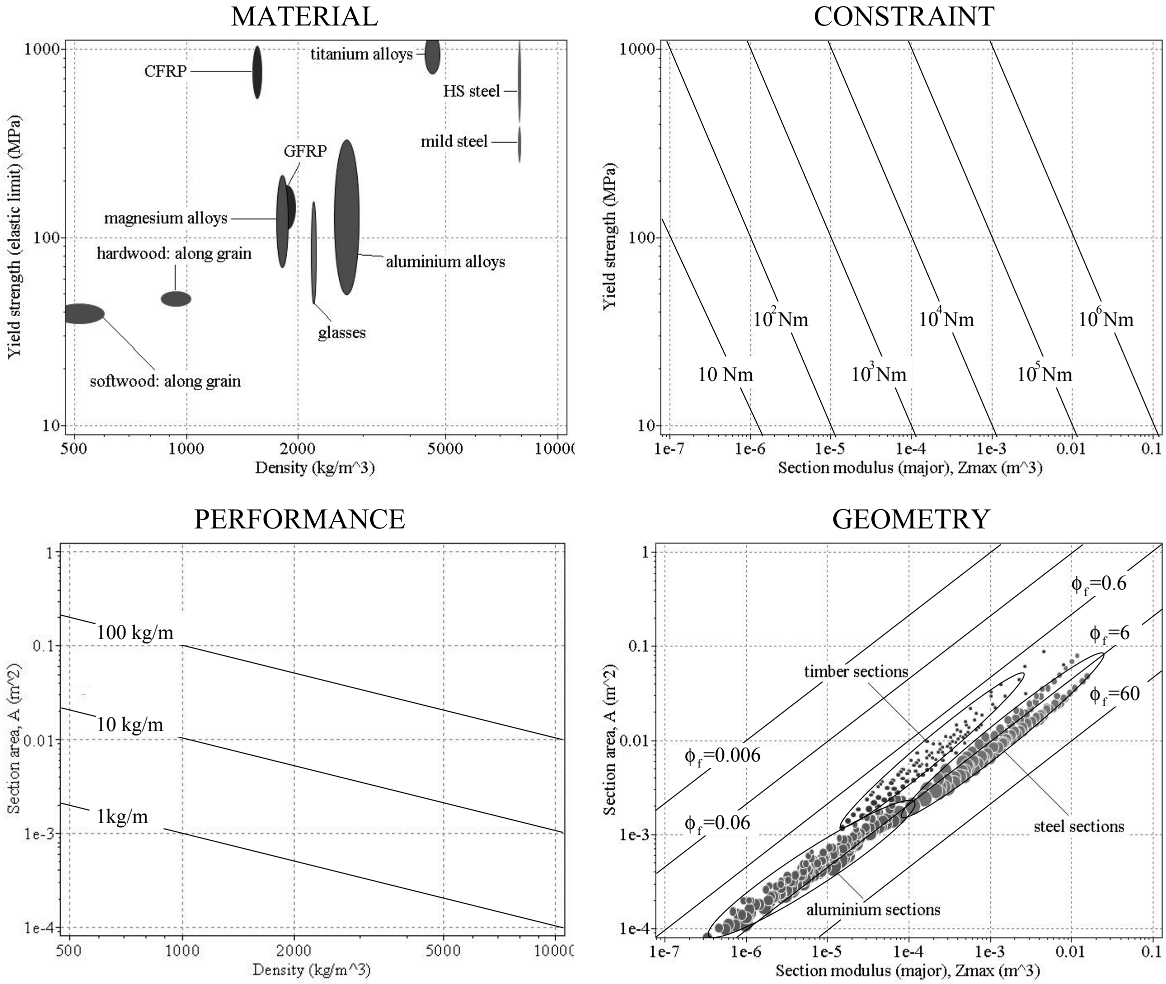
K A Seffen
M. Overend

Selection of Material and Shape: Design for Bending

Stiffness. Use in conjunction with the cyclic method described in notes: note that axes are shared.



Strength. Use in conjunction with the cyclic method described in notes: note that axes are shared.

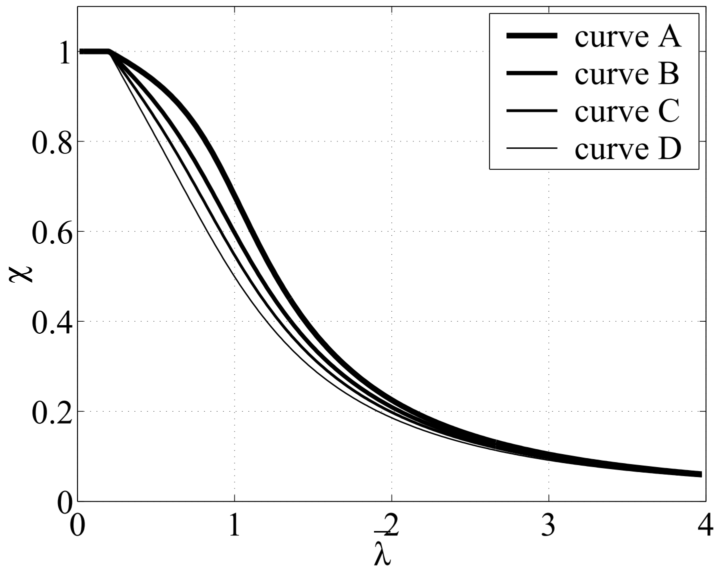


Steel Data Sheet

Material. The two most common steels are S275 and S355, with characteristic yield strengths, σ_y, of 275 MPa and 355 MPa, respectively. The design strength divides σ_y by a specified partial safety factor, γ_m. Partial safety factors for loads at ULS are often 1.4 for dead loads and 1.6 for live loads.

Tension members (axial force only). Gross cross-section area is A; net area, A_n, is A subtract hole(s). Effective section is KA_n but not greater than A, where K is 1.2 for S275 or 1.1 for S355. For eccentric connection, with area, A_{out}, not connected at the joint, the effective area is $A_e - cA_{\text{out}}$, where c is 0.5 bolts or 0.3 for welds.

Compression members (axial force only). Radius of gyration is r, extreme fibre distance is y, effective length of column is L, and $\lambda = L/r$. Define $\bar{\lambda} = \lambda/\lambda_0$ where $\lambda_0 = \pi\sqrt{E/\sigma_y}$ and a reduction factor, χ, on the full axial yield strength, equal to σ_c/σ_y where σ_c is the critical buckling stress. Buckling performance given by:



Select curve A: $r/y > 0.7$; B: $0.5 < r/y < 0.7$; C: $0.5 < r/y$; D: only when the flange thickness is larger than 40 mm.

Beams (without axial force).

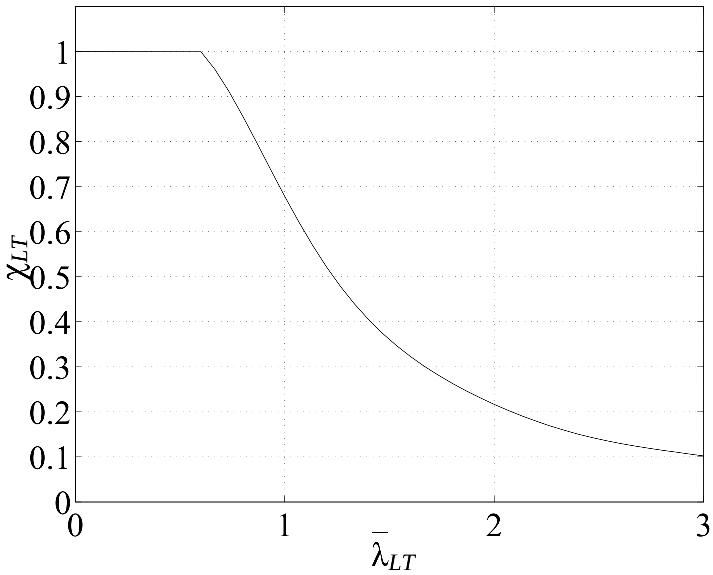
Moment: check maximum moment is less than $\sigma_y Z_p$. Beware local buckling for thin-walled sections.

Shear: yield strength, q_w, given by $0.6\sigma_y$. Check buckling stress capacity, q_b, is not exceeded in thin webs with thickness, t, and panel aspect ratio, a/b ($\leftrightarrow$ / $\updownarrow$), where $q_b = \left[3/4 + b^2/a^2\right] \cdot 1000/(b/t)^2$ in MPa.

Lateral torsional buckling, (LTB); theoretical elastic critical moment, M_c, for a beam of span L under constant moment (and supported at its ends only where lateral deflection and twist are prevented), then

$$M_c = \frac{\pi}{L}\left[EI_{yy}\left(GJ + \frac{\pi^2}{L^2}EC_w\right)\right]^{0.5}$$

where C_w is a constant due to the restraining (stiffening) effect of *warping*, equal to $D^2 I_{yy}/4$, D being the distance between flange centres. Design curve is given by



where $\bar{\lambda}_{LT} = \sqrt{M_p/M_c}$ and $\chi_{LT} = M_{cr}/M_p$. M_{cr} is the critical moment, which must be greater than the maximum moment in practice: $M_{cr} > M_{\max}$ for uniform bending moment case; $M_{cr} > 0.8M_{\max}$ for centrally loaded, simply supported case.

Joint design. Ductility of steel allows a reasonably simple equilibrium system to be envisaged for initial design, often with a transmitted force uniformly distributed across the various fasteners involved. For a bolted joint in shear, a couple, C, about its centre can be taken simply by extra shear forces, F_i, on each bolt perpendicular to the line to the centre of the bolt group and proportional to the distance, d_i from the centre, so that $F_i = Cd_i/\sum d_i^2$.

Applied shear forces are commonly checked against the shear strength ($0.6\sigma_y$) of the bolt, depending on the number of active shear planes; and against bearing strength in each plate, $\sigma_y dt$ where $d \times t$ is the bolt diameter times plate thickness.

3D3 – Structural Materials and Design – Masonry Datasheet

Bearing or crushing resistance per unit length

$$P_b = \frac{f_k t}{\gamma_m}$$

Buckling resistance per unit length

$$P_b = \frac{\beta f_k t}{\gamma_m}$$

Table for capacity reduction factor β from BS 5628

Slenderness ratio h_{ef}/t_{ef}	Eccentricity at top of wall, e_x			
	Up to 0.05t	0.1t	0.2t	0.3t
0	1.00	0.88	0.66	0.44
6	1.00	0.88	0.66	0.44
8	1.00	0.88	0.66	0.44
10	0.97	0.88	0.66	0.44
12	0.93	0.87	0.66	0.44
14	0.89	0.83	0.66	0.44
16	0.83	0.77	0.64	0.44
18	0.77	0.70	0.57	0.44
20	0.70	0.64	0.51	0.37
22	0.62	0.56	0.43	0.30
24	0.53	0.47	0.34	
26	0.45	0.38		
27	0.40	0.33		

Flexural resistance per unit length

$$M = \frac{f_{kx}\ Z}{\gamma_m}$$

3D3 – Structural Materials and Design – Concrete Datasheet

Structural system	Span/effective depth ratio	
	EC2*	
	high	light
1. Simply supported beam, one-way or two-way spanning simply supported slab	14	20
2. End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	18	26
3. Interior span of beam or one-way or two-way spanning slab	20	30
4. Slab supported on columns without beams (flat slab), based on longer span	17	24
5. Cantilever	6	8

highly stressed $\rho = 1.5\%$ and lightly stressed $\rho = 0.5\%$ (slabs are normally assumed to be lightly stressed) *Table 7.4N, NA.5 [10.2]

Table 10.1 Span versus depth ratio

Member	Fire resistance	Minimum dimension, mm		
		4 hours	2 hours	1 hour
Columns fully exposed to fire	width	450	300	200
Beams	width cover	240 70	200 50	200 45
Slabs with plain soffit	thickness cover	170 45	125 35	100 35

Extracts from Table 4.1 [10.1]

Table 10.2 Minimum member sizes and cover (to main reinforcement) for initial design of continuous members

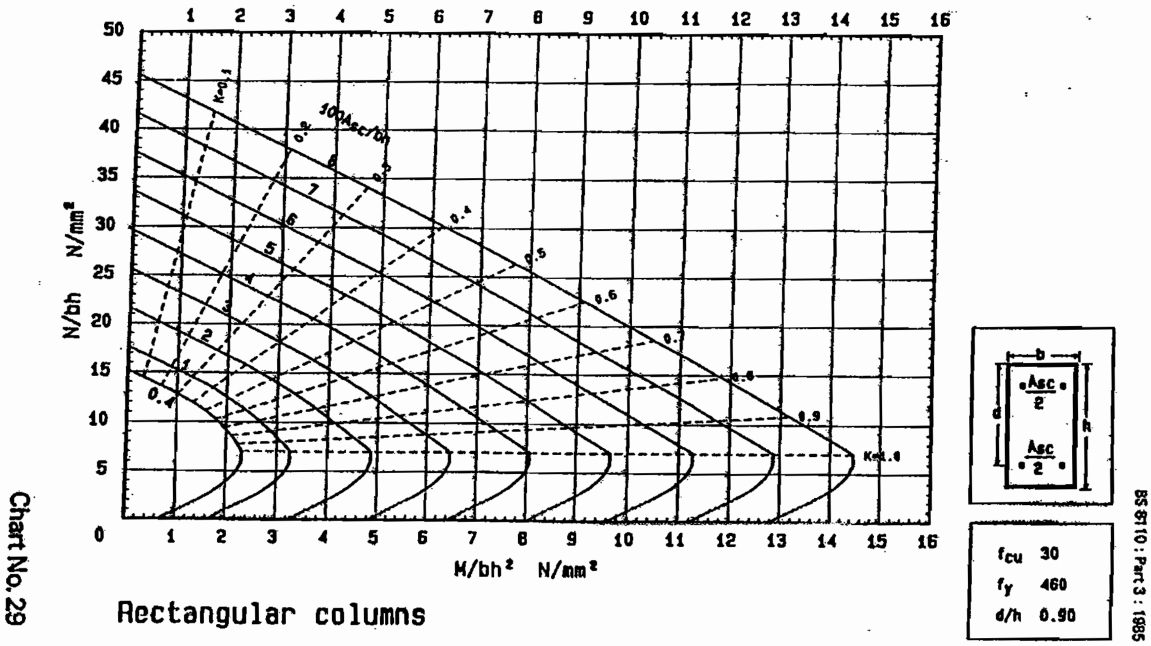


Fig 10.1 Interaction diagram from [10.3]

[10.1] Manual for the design of reinforced concrete building structures to EC2, IStructE, ICE, March 2000 - FM 507
[10.2] Eurocode 2: Design of concrete structures, EN 1992-1-1:2004, UK National Annex –NA to BS EN 1992-1-1:2004
[10.3] Structural design. Extracts from British Standards for Students of Structural design. PP7312:2002, BSi

Flexure

Under-reinforced – singly reinforced

$$M_u = \frac{A_s f_y d(1-0.5x/d)}{\gamma_s}$$

$$\frac{x}{d} = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} bd}$$

if x/d =0.5

$$M_u = 0.225 f_{cu} bd^2 / \gamma_c$$

Balanced section

$$\rho_b = \frac{A_s}{bd} = \frac{\gamma_s 0.6 f_{cu}}{\gamma_c f_y} \cdot \frac{\varepsilon_{cu}}{\varepsilon_y + \varepsilon_{cu}}$$

Bond anchorage

$$l_b = \frac{f_y \phi}{4(2.25\eta_1 \eta_2 f_{ctd})}$$

where: η_1 is 1.0 for good bond, 0.7 otherwise

η_2 is 1.0 for $\phi \leq 32$

Cracking

$$w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{r,\max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff}$$

where: k_1 is 0.8 for high bond, 1.6 for plain bars

k_2 is 1.0 for pure tension, 0.5 for bending

k_3, k_4 factors (in UK k_3=3.4 and k_4=0.425)

$\rho_{p,eff}$ is the effective steel ratio A_s/A_{ceff}

Shear

Without internal stirrups

$$V_{Rd,c} = [\frac{0.18}{\gamma_c} k(100\rho_1 f_{ck})^{1/3}]b_w d \geq (0.035 k^{3/2} f_{ck}{}^{1/2}) b_w d$$

where: f_{ck} is the characteristic concrete compressive cylinder strength (MPa).

$k = 1 + \sqrt{200/d} \leq 2.0$ (d in mm)

$\rho_l = A_s/b_w d \leq 0.02$

With internal stirrups

- Concrete resistance

$$V_{Rd,\max} = f_{c,\max}(b_w 0.9d)/(\cot\theta + \tan\theta)$$

where: $f_{c,\max} = 0.6(1 - f_{ck}/250) f_{cd}$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d)(\cot\theta)/(s\gamma_s)$$

Columns – axial loading only

$$\sigma_u = 0.6\frac{f_{cu}}{\gamma_c} + \rho_c \frac{f_y}{\gamma_s}$$

Standard steel diameters (in mm) - 6, 8, 10, 12, 16, 20, 25, 32 and 40

3D3 – Structural Materials and Design – Timber Datasheet

			C14	C16	C18	C22	C24	C27	C40
$f_{m,k}$	bending	MPa	14	16	18	22	24	27	40
$f_{t,0,k}$	tens $\parallel$	MPa	8	10	11	13	14	16	24
$f_{t,90,k}$	tens $\perp$	MPa	0.3	0.3	0.3	0.3	0.4	0.4	0.4
$f_{c,0,k}$	comp $\parallel$	MPa	16	17	18	20	21	22	26
$f_{c,90,k}$	comp $\perp$	MPa	4.3	4.6	4.8	5.1	5.3	5.6	6.3
$f_{v,k}$	shear	MPa	1.7	1.8	2.0	2.4	2.5	2.8	3.8
$E_{0,mean}$	tens mod $\parallel$	GPa	7	8	9	10	11	12	14
$E_{0,05}$	tens mod	GPa	4.7	5.4	6	6.7	7.4	8	9.4
$E_{90,mean}$	tens mod $\perp$	GPa	0.23	0.27	0.3	0.33	0.37	0.4	0.47
G_{mean}	shear mod	GPa	0.44	0.50	0.56	0.63	0.69	0.75	0.88
ρ_k	density	kg/m^3	290	310	320	340	350	370	420
ρ_{mean}	density	kg/m^3	350	370	380	410	420	450	500

Table 11.2 Selected strength classes - characteristic values according to EN 338 [11.3]– Coniferous Species and Poplar (Table 1)

Table 3.1.7 Values of k_{mod}

Material/ load-duration class	Service class 1	Service class 2	Service class 3
Solid and glued laminated timber and plywood			
Permanent	0.60	0.60	0.50
Long-term	0.70	0.70	0.55
Medium-term	0.80	0.80	0.65
Short-term	0.90	0.90	0.70
Instantaneous	1.10	1.10	0.90

Selected Modification Factors for Service Class and Duration of Load [11.2]

[11.2] DD ENV 1995-1-1 :1994 Eurocode 5: Design of timber structures – Part 1.1 General rules and rules for buildings
[11.3] BS EN 338:1995 Structural Timber – Strength classes

Flexure - Design bending strength

$$f_{m,d} = k_{mod} k_h k_{crit} k_{ls} f_{m,k} / \gamma_m$$

Shear – Design shear stress

$$f_{v,d} = k_{mod} k_{ls} f_{v,k} / \gamma_m$$

Bearing – Design bearing stress

$$f_{c,90,d} = k_{ls} k_{c,90} k_{mod} f_{c,90,k} / \gamma_m$$

Stability – Relative slenderness for bending

$\lambda_{rel,m} = \sqrt{f_{m,k}/\sigma_{m,crit}}$

"For beams with an initial lateral deviation from straightness within the limits defined in chapter 7, k_{crit} may be determined from (5.2.2 c-e)"

$$k_{crit} = \begin{cases} 1 & \text{for} \quad \lambda_{rel,m} \le 0.75 & (5.2.2c) \\ 1.56 - 0.75\lambda_{rel,m} & \text{for} \quad 0.75 < \lambda_{rel,m} \le 1.4 & (5.2.2d) \\ 1/\lambda_{rel,m}^2 & \text{for} \quad 1.4 < \lambda_{rel,m} & (5.2.2e) \end{cases}$$

Extract from [11.2] - k_{crit}

Joints

For bolts and for nails *with* predrilled holes, the characteristic embedding strength $f_{h,0,k}$ is:

$f_{h,0,k} = 0.082(1-0.01d)\rho_k$ N/mm²

For bolts up to 30 mm diameter at an angle α to the grain:

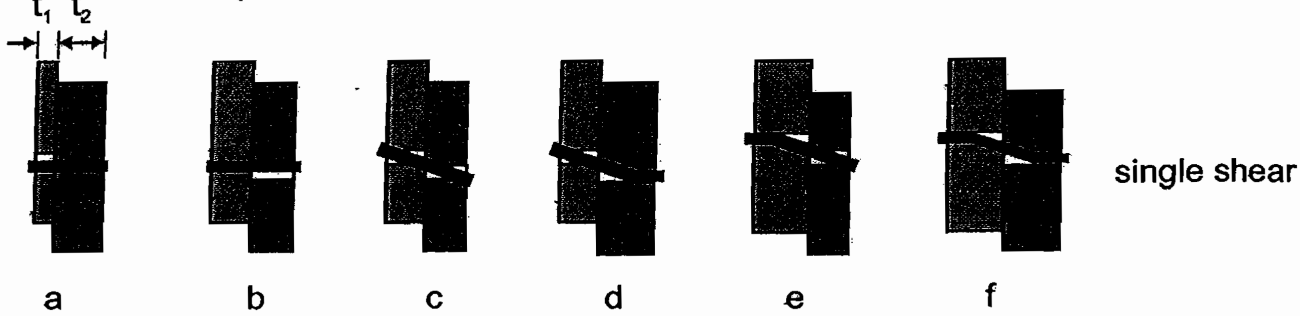
$$f_{h,\alpha,k} = \frac{f_{h,0,k}}{k_{90}\sin^2\alpha + \cos^2\alpha} \qquad \begin{array}{ll} \text{for softwood} & k_{90} = 1.35 + 0.015d \\ \text{for hardwood} & k_{90} = 0.90 + 0.015d \end{array}$$

Design yield moment for round steel bolts: $M_{y,d} = (0.8 f_{u,k} d^3)/(6\gamma_m)$

Design embedding strength e.g. for material 1: $f_{h,1,d} = (k_{mod,1} f_{h,1,k})/\gamma_m$

Design load-carrying capacities for fasteners in single shear

$$R_d = \min. \begin{cases} f_{h,1,d}\, t_1 d & (6.2.1a) \\ f_{h,1,d}\, t_2 d\beta & (6.2.1b) \\ \dfrac{f_{h,1,d}\, t_1 d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \dfrac{t_2}{t_1} + \left(\dfrac{t_2}{t_1}\right)^2\right] + \beta^3\left(\dfrac{t_2}{t_1}\right)^2} - \beta\left(1 + \dfrac{t_2}{t_1}\right)\right] & (6.2.1c) \\ 1.1\dfrac{f_{h,1,d}\, t_1 d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta)M_{y,d}}{f_{h,1,d}\, d t_1^2}} - \beta\right] & (6.2.1d) \\ 1.1\dfrac{f_{h,1,d}\, t_2 d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta) + \dfrac{4\beta(1+2\beta)M_{y,d}}{f_{h,1,d}\, d t_2^2}} - \beta\right] & (6.2.1e) \\ 1.1\sqrt{\dfrac{2\beta}{1+\beta}}\sqrt{2M_{y,d} f_{h,1,d}\, d} & (6.2.1f) \end{cases}$$



Extract from [11.2] – Timber-to-timber and panel-to-timber joints

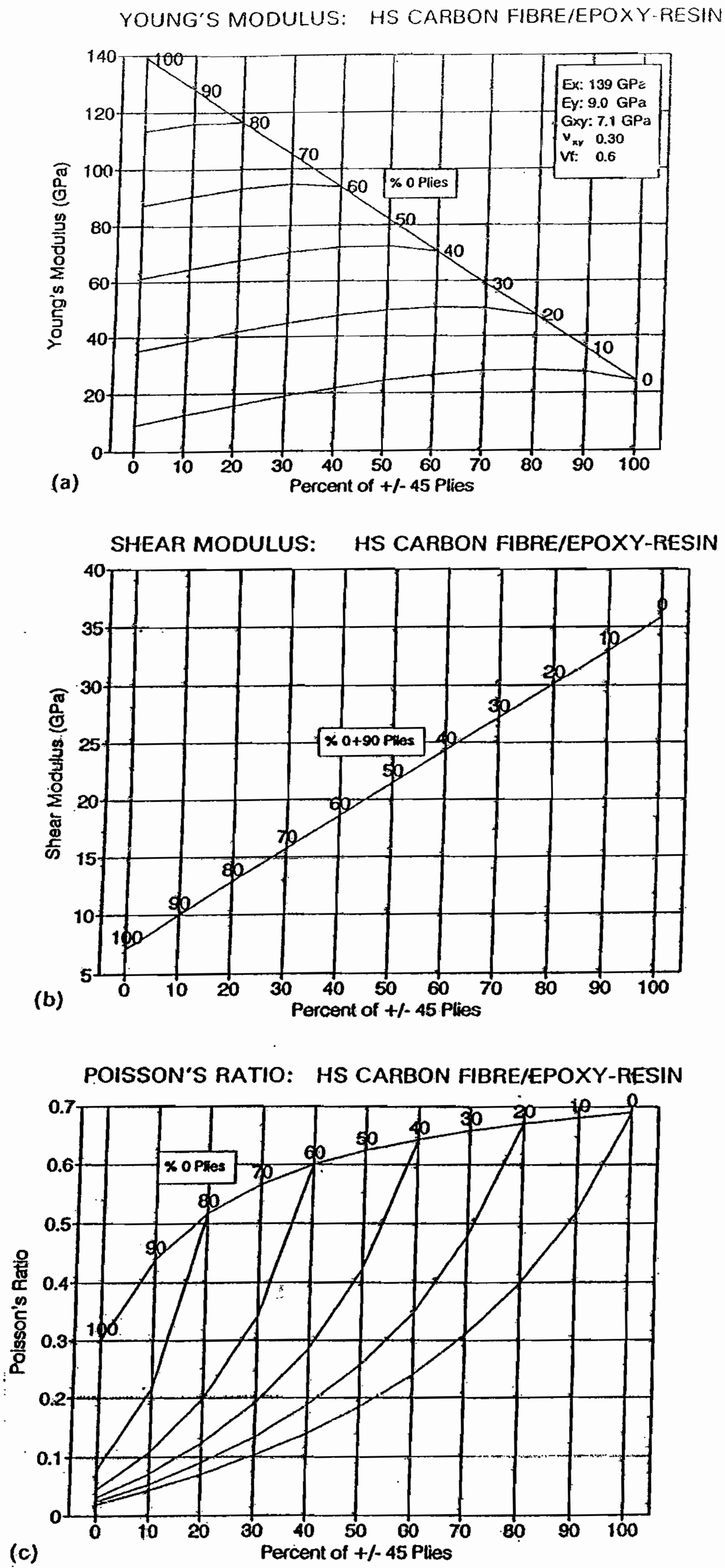


Figure 1. HS carbon/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

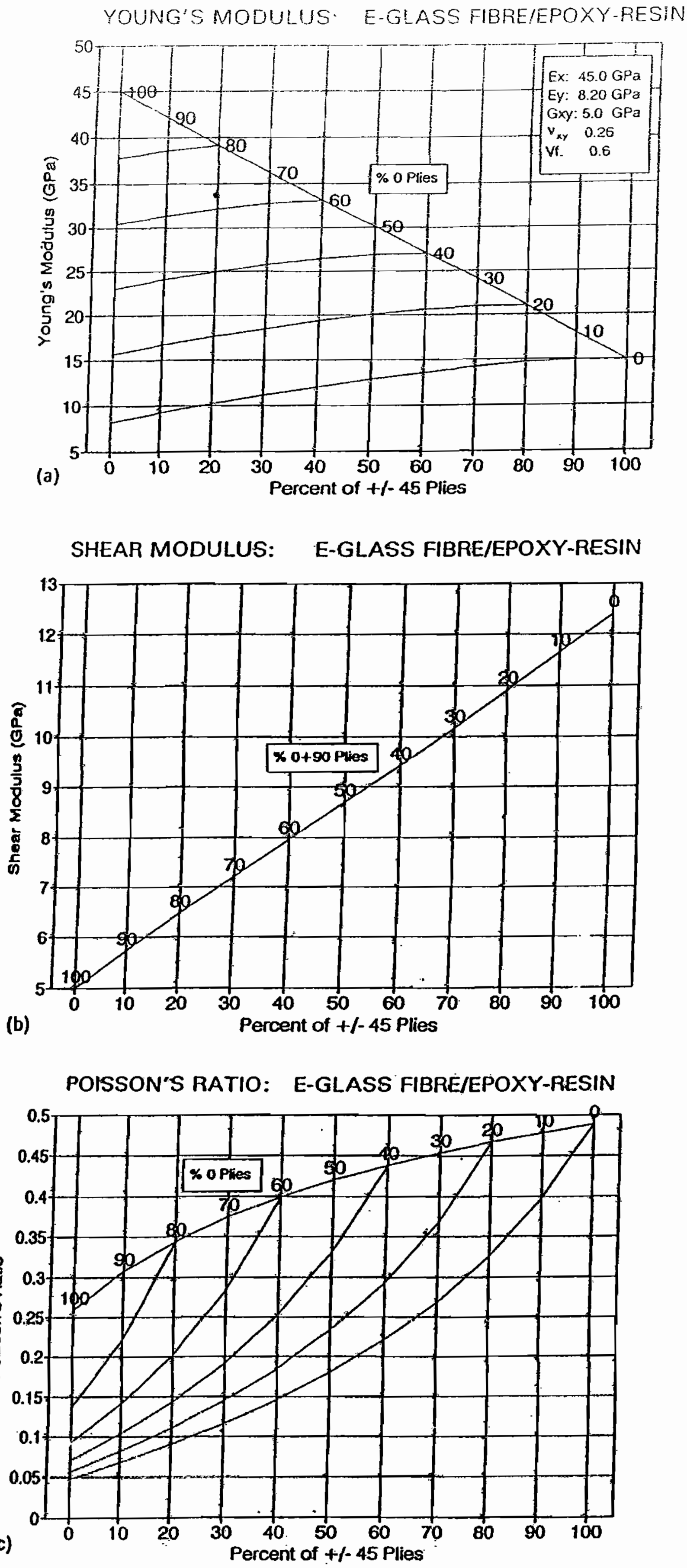


Figure 2. E-glass/epoxy plots (a) Young's modulus (b) shear modulus and (c) Poisson's ratio [12.3]

[12.3] M.G. Bader, Materials selection, preliminary design and sizing for composite laminates. Journal Paper: Composites Part A, 27A, (1996), pp 65-70.)

ENGINEERING TRIPOS PART IIA 2011

NUMERICAL ANSWERS
MODULE 3D3: STRUCTURAL MATERIALS & DESIGN

1 (b) i) 203x102x23 UB; 406x178x54 UB
iii) 203x203x46 UC

2 (b) i) M20 bolts are adequate, but beam web has in sufficient bearing capacity and requires strengthening with 4.7mm thick steel plate.
ii) 6mm
(c) 1636kN

3 (a) Max BM = 201.5kNm, Max SF = 115.2kN.
(b) 3T25 bottom reinforcement at mid span, 2T25 bottom reinforcement at supports.
(c) 120mm

4 (b) i) 220.6mm
ii) 1x20mm diameter bolt.