ENGINEERING TRIPOS PART IIA

Friday 29 April 2011 2.30 to 4

Module 3D5

WATER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

The values of relevant parameters are listed at the end of the data book unless otherwise noted in the question.

Attachment: 3D5 Data Book (5 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
Graph paper	CUED approved calculator allowed

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL version

1 (a) After a one-hour excess rainfall of 10 mm over a small catchment, the following hourly-averaged flow rates are recorded at the catchment outlet

Duration (h)	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	...
Discharge (m^3 s^{-1})	16	28	22	14	10	10	10	10	...

(i) What is the base flow rate of the catchment? [10%]

(ii) What is the peak discharge at the catchment outlet generated by a different two-hour rainfall event, whose overall intensity is 20 mm in the first hour and 10 mm in the second hour? Assume that the infiltration-related coefficients are $f_0 = 10$ mm h^{-1}, $f_c = 2$ mm h^{-1}, and $K_f = 0.5$ h^{-1}. [50%]

(b) A four-hour uniform excess rainfall produces the following successive distribution percentages of the runoff above the base flow at the catchment outlet: 3, 18, 35, 27, 12, 5 over four-hour intervals starting from the start of the rain.

(i) Draw the S-curve on graph paper. Estimate the time of concentration for this catchment. [20%]

(ii) Uniform excess rain falls over the same catchment for three hours, and runoff is generated. Estimate the time when the flow rate peaks at the catchment outlet. [20%]

2 (a) Steady flow of 10 $m^3 s^{-1}$ occurs in a rectangular channel 6 m wide, lined with concrete (Manning's $n = 0.013$ s $m^{-1/3}$) and laid on a slope of 0.001. The flow in the upstream reach of the channel is uniform. The flow at the downstream end undergoes a freefall, where a critical flow condition is noticed.

(i) Show that the uniform water depth is around 0.88 m. [10%]

(ii) Show that the critical water depth is around 0.66 m. [10%]

(iii) Calculate the distance over which the water depth drops from 0.85 m to 0.75 m, assuming a linear variation of the water depth in between. [30%]

(b) Water flows from a sluice gate into a long channel of rectangular cross section with constant width 5 m. Initially, the flow is uniform with depth 2 m and flow rate 4 $m^3 s^{-1}$. Then at $t = 0$, the sluice gate is controlled in such a way that the flow rate at the upstream end of the channel increases linearly from 4 $m^3 s^{-1}$ to 10 $m^3 s^{-1}$ in 4 min and remains at 10 $m^3 s^{-1}$ thereafter. Ignore the influence of the bed slope and bed friction.

(i) Show that the water depth at the upstream end of the channel reaches 2.23 m at $t = 4$ min and remains at this value thereafter. [20%]

(ii) 500 m downstream of the sluice gate, when does the water depth start to deviate from 2 m? [10%]

(iii) 500 m downstream of the sluice gate, when does the water depth first reach 2.23 m? [20%]

3 (a) A channel is 50 m wide and 2 m deep. The flow rate is 200 $m^3 s^{-1}$, and the bed slope is 0.002. The channel bed is composed of sediments of diameter 10 mm.

(i) Taking the grain-related roughness height of the bed to be the same as the grain size and assuming the channel banks to be smooth, predict the bedload sediment transport rate in kg s^{-1} using the Meyer-Peter and Müller formula. (Because the banks are smooth, the hydraulic radius is the same as the water depth.) [30%]

(ii) On one side of the channel, pollutant is discharged with waste water through a vertical line source at a concentration of 100 ppm. 1000 m downstream of the line source, there is a water intake on the same side of the river. The maximum allowable concentration of the pollutant is 3 ppm for the intake water. Ignoring the influence of the other channel bank, calculate the maximum permissible discharge of the waste water from the line source. [30%]

(b) A very wide river has depth 3 m. The diameter of the suspended sediments is 0.12 mm. The sediment concentration is found to be 2.2 kg m^{-3} at 1 m below the free surface and 4.2 kg m^{-3} at 2 m below the free surface.

(i) Show that the fall speed of the sediment is 0.011 m s^{-1}. [10%]

(ii) Estimate the bed slope of the river. [30%]

4 (a) Based on the Colebrook-White formula, show that the Chezy coefficient C, hydraulic radius R_h, flow velocity U and fluid kinematic viscosity ν are related by the following equation when the flow is hydraulically smooth

$$C \approx 7.7 \ln\left(\frac{14.1 R_h U}{C \nu}\right)$$ [30%]

(b) Water is pumped from a lower reservoir to an upper reservoir through a pipeline with internal diameter 300 mm, length 70 m. The water level difference between the two reservoirs is 15 m. In the following computation, assume that the friction factor, λ, has a constant value of 0.025 and that the local head losses sum to $2.5U^2/(2g)$.

(i) The pump characteristic is approximated by the formula

$$H = 22.9 + 10.7Q - 111.0Q^2$$

where H is in m and Q in $m^3\ s^{-1}$. Determine the discharge and pumping head. [20%]

(ii) There are two identical pumps, with the characteristic of each one being described by the equation in (i). Determine the discharges and pumping heads when the two pumps are in parallel, and in series, respectively. [30%]

(iii) If the pump characteristic given in (i) corresponds to a pump speed of 2900 rpm, derive the characteristic of the same pump running at 3450 rpm. [20%]

END OF PAPER

UNIVERSITY OF
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Engineering Tripos Part IIA — Third Year

Module 3D5: Water Engineering

Data Book (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity)
$$f = f_c + (f_0 - f_c)e^{-K_f t}$$
$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flow with a free surface (uniform flow in a very wide channel):

Eddy viscosity coefficient
$$\nu_t = \kappa u_* z\left(1 - \frac{z}{h}\right)$$

Velocity in hydraulically smooth regime ($u_* k_s/\nu < 5$)
$$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$$

Velocity in hydraulically rough regime ($u_* k_s/\nu > 70$)
$$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$$

Open Channel Flow

Chézy coefficient in large Reynolds-number flows
$$C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$$

Froude number for rectangular channels
$$Fr = \frac{U}{\sqrt{gh}}$$

Steady flow momentum equation
$$\sum F = \rho Q(U_{out} - U_{in})$$

Bed shear stress
$$\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$$

Uniform flows:

Chézy formula
$$U = C\sqrt{R_h S_b}$$

Manning formula
$$U = \frac{1}{n} \cdot R_h^{1/6}\sqrt{R_h S_b} = \frac{1}{n} \cdot R_h^{2/3} \cdot S_b^{1/2}$$

Gradually varied flows:
$$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

or
$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \dfrac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \dfrac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$$

Characteristics for unsteady flows in rectangular channels:

$$\frac{d}{dt}\left(U+2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U+\sqrt{gh}$$

$$\frac{d}{dt}\left(U-2\sqrt{gh}\right)=g\left(S_b-S_f\right) \text{ along } \frac{dx}{dt}=U-\sqrt{gh}$$

Pollutant Transport

Analytical values of the mixing coefficients: $D_{tx}=D_{ty}=0.15hu_*$, $D_{tz}=0.067hu_*$, $D_L=5.86hu_*$

For instantaneous release from origin at $t=0$ in uniform flows along x direction:

One-dimensional $\bar{c}(x,t)=\frac{M/A}{\sqrt{4\pi tD_x}}\exp\left(-\frac{(x-Ut)^2}{4D_xt}\right)$

Two-dimensional $\bar{c}(x,y,t)=\frac{M/h}{4\pi t\sqrt{D_xD_y}}\exp\left(-\frac{(x-Ut)^2}{4D_xt}-\frac{y^2}{4D_yt}\right)$

Three-dimensional $\bar{c}(x,y,z,t)=\frac{M}{(4\pi t)^{3/2}\sqrt{D_xD_yD_z}}\exp\left(-\frac{(x-Ut)^2}{4D_xt}-\frac{y^2}{4D_yt}-\frac{z^2}{4D_zt}\right)$

For continuous release from origin in uniform flows along x direction:

Two-dimensional $\bar{c}(x,y)=\frac{\dot{M}/h}{U\sqrt{4\pi\frac{x}{U}D_y}}\exp\left(-\frac{y^2}{4D_y\,x/U}\right)$

Three-dimensional $\bar{c}(x,y,z)=\frac{\dot{M}}{4\pi x\sqrt{D_yD_z}}\exp\left(-\frac{y^2}{4D_y\,x/U}-\frac{z^2}{4D_z\,x/U}\right)$

Sediment Transport

Definitions of Shields parameter, non-dimensional grain size and transport stage parameter:

$$\theta=\frac{\tau_b}{g(\rho_s-\rho)d},\quad d_*=d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3},\quad T=\frac{\tau_b'-\tau_{bc}}{\tau_{bc}}=\frac{\theta'-\theta_c}{\theta_c}$$

Critical Shields parameter $\theta_c=\frac{0.30}{1+1.2d_*}+0.055\left[1-\exp(-0.02d_*)\right]$

Fall velocity $w_s=\frac{\nu}{d}\left[\sqrt{10.36^2+1.049\cdot d_*^3}-10.36\right]$

Shear stress partition: $C'=7.8\ln\left(\frac{12h}{k_s'}\right)$, $\tau_b'=\rho g\frac{U^2}{C'^2}$

$C=7.8\ln\left(\frac{12h}{k_s}\right)$, $\tau_b=\rho g\frac{U^2}{C^2}$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=8\left[\left(\frac{C}{C'}\right)^{1.5}\theta-0.047\right]^{1.5}$

van Rijn $\frac{q_b}{\sqrt{g(s-1)\cdot d^3}}=0.053\frac{T^{2.1}}{d_*^{0.3}}$

Rouse profile of suspended sediment concentration $$\frac{\bar{c}(z)}{\bar{c}(a)}=\left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

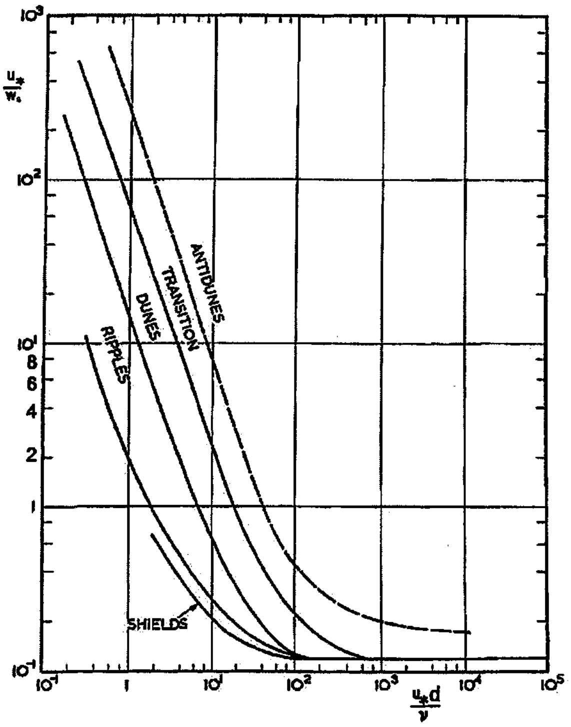
Reference volumetric concentration close to the bed:

Zyserman and Fredsøe $$\bar{c}(2d)=\frac{0.331\cdot(\theta'-0.045)^{1.75}}{1+0.72\cdot(\theta'-0.045)^{1.75}}$$

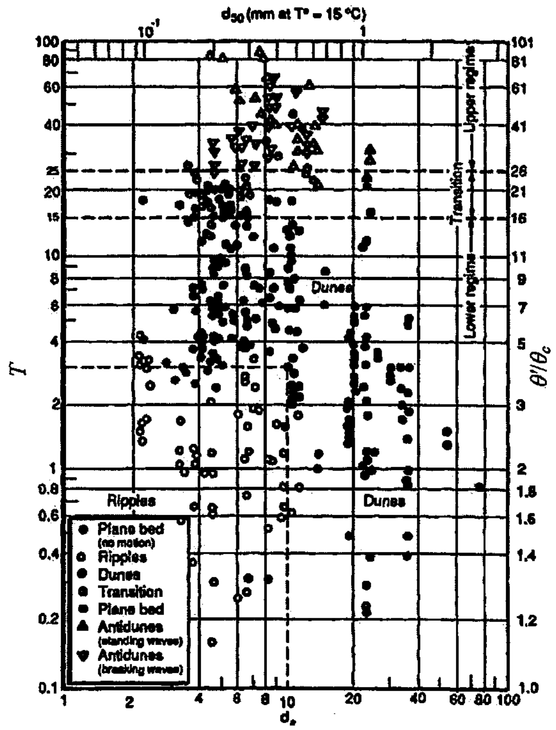
van Rijn $$\bar{c}(a)=0.015\frac{d\cdot T^{1.5}}{a\cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s=\int_a^h\bar{c}(z)\bar{u}(z)dz=11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right)+I_2\right]$$

a/h	$w_s/(\kappa u_*)=0.2$		$w_s/(\kappa u_*)=0.6$		$w_s/(\kappa u_*)=1.0$		$w_s/(\kappa u_*)=1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

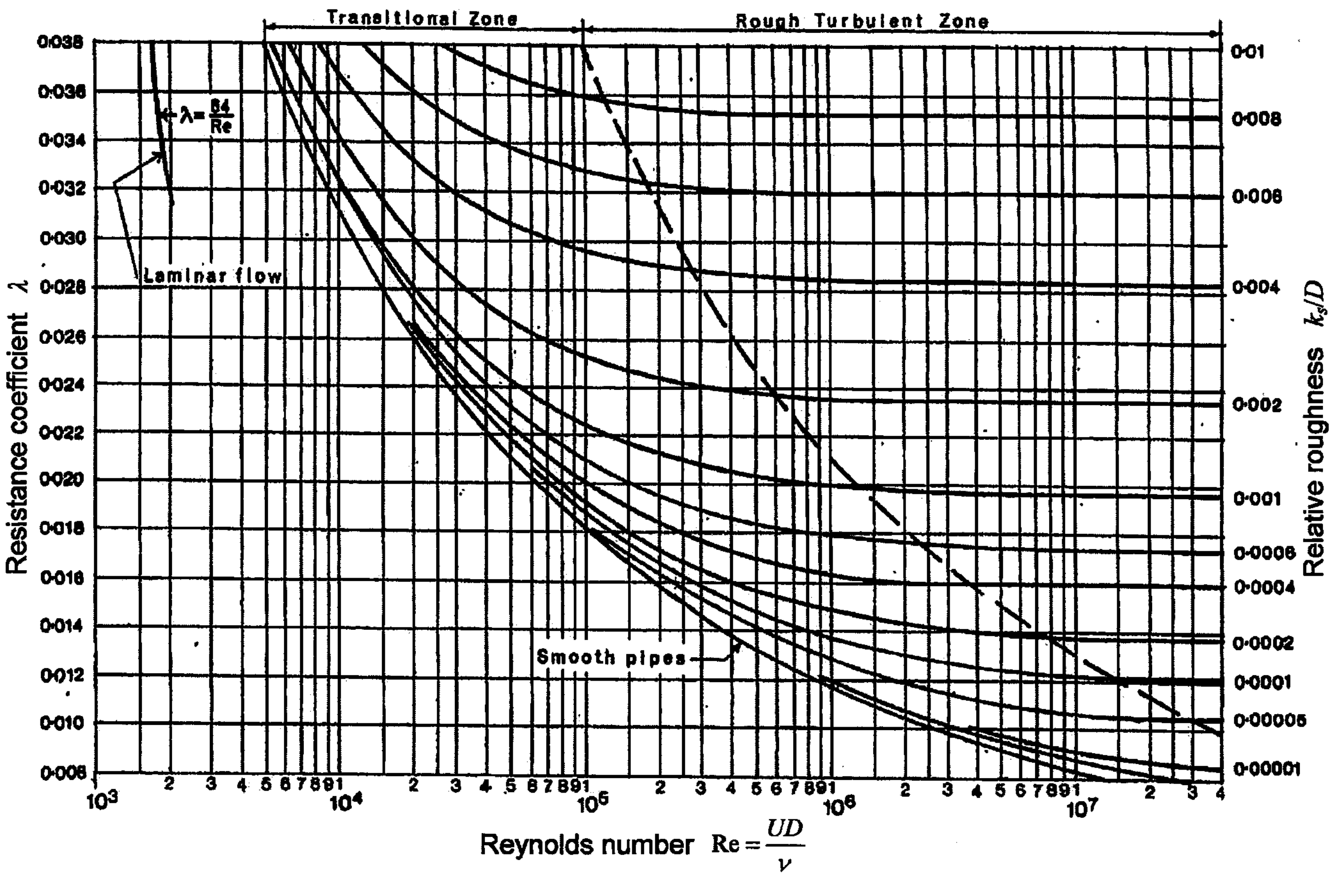
Darcy-Weisbach Equation $H_f = \lambda \dfrac{L}{D}\dfrac{U^2}{2g}$

Colebrook-White formula $\dfrac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\dfrac{k_s}{3.7D} + \dfrac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$ with $\mathrm{Re} = \dfrac{UD}{\nu}$

Power consumption $P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\dfrac{Q_p}{N_p \cdot D_p{}^3}, \quad \dfrac{gH_p}{N_p{}^2 \cdot D_p{}^2}, \quad \dfrac{P_p}{\rho \cdot N_p{}^3 \cdot D_p{}^5}$

Specific speed $N_S = \dfrac{N_p \cdot Q_p{}^{1/2}}{H_p{}^{3/4}}$



Symbols

A	area
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient
c	concentration

D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x, D_y, D_z	diffusion coefficients in x, y and z directions respectively
D_{tx}, D_{ty}, D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.81 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	rate of decrease of f capacity
k_s	roughness height, also called equivalent or Nikuradse's sand roughness height
M	amount of the pollutant released
$\dot{M}$	rate of the pollutant release
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity, also called relative density or density ratio (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity
u_*	shear velocity
w_s	fall velocity
x, y, z	spatial coordinates
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
$\overline{}$	Reynolds-averaged value
$'$	effective shear-stress component, also called grain-related shear-stress component
$_p$	pump

D Liang, Oct 2010