ENGINEERING TRIPOS PART IIA

Monday 9 May 2011 9 to 10.30

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

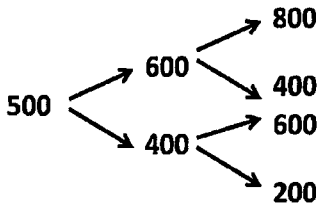
STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Final Version

1 (a) Assume that you own a firm that can produce 150,000 units of a good at a cost of £100 per unit. The retail price of your good was £500 per unit recently, but you now expect it to go up or down £100 per unit this year, i.e., either to £400 per unit or £600 per unit. The year thereafter, you expect it to go up or down by £200 per unit. These price scenarios can be shown in a simple tree below.



All price changes are equally likely. The annual fixed costs of running the plant are £50 million, and the annual rent (regardless of whether you run the plant or not) is £10 million. The world is risk-neutral and the interest rate is 10% per year. Moreover, assume that you know at the beginning of each year what the price over the whole year will be, because you receive customer orders at this point. Calculate the value of flexibility for each one of the following scenarios.

(i) You can open or shut down the plant at any point in time. [20%]
(ii) You can delay your decision whether to open or not the plant for one year. That is, instead of making the decision now, you can postpone it for year 1 [20%]
(iii) You can open the plant at any time in the future, but once you open it you cannot shut it down. [20%]

(b) A stock is currently selling for £25 per share. In a year's time, the price will be either £20 or £30. Also, in a year's time, a bond will pay a 10% return rate.

(i) What is the value of a call option with a £20 exercise price? [20%]
(ii) What is the value of a call option with a £26 exercise price? [20%]

2 (a) The Copy Shop has three identical copying machines that are run by employees of the shop. Only two operators are kept on duty to run the machines, so the third machine is a spare that is used only when one of the other machines breaks down. When a machine is being used, the time until it breaks down has an exponential distribution with a mean of 2 weeks. If one machine breaks down while the other two are operational, a service representative is called in to repair it, in which case the total time from the breakdown until the repair is completed has an exponential distribution with a mean of 1/5 weeks. However, if a second machine breaks down before the first one has been repaired, the third machine is shut off while the two operators work together to repair this second machine quickly, in which case its repair time has an exponential distribution with a mean of only 1/15 weeks. If the service representative finishes repairing the first machine before the two operators complete the repair of the second, the operators go back to running the two operational machines while the representative finishes the second repair, in which case the remaining repair time has an exponential distribution with a mean of 1/5 weeks.

(i) Letting the state of the system be the number of machines not working, construct the rate diagram for this queueing system. [20%]

(ii) Use the balance equations to find the steady-state distribution of the number of machines not working. [20%]

(iii) What is the expected number of operators available for copying? [20%]

(b) Airplanes arrive for take-off at the runway of an airport according to a Poisson process at a mean rate of 20 per hour. The time required for an airplane to take off has an exponential distribution with a mean of 2 minutes, and this process must be completed before the next airplane can begin to take-off. Because a brief thunderstorm has just begun, all airplanes which have not commenced take-off are temporarily prevented from doing so. However, airplanes continue to arrive at the airport during the thunderstorm and join the queue to land until the airport reopens. Assuming steady-state operation before the thunderstorm, determine the expected number of airplanes that will be waiting to take off at the end of the thunderstorm if it lasts 30 minutes. [40%]

3 (a) A local property company wants to better understand the housing market in the Cambridge and Huntingdon area. The company hires you to do some analysis. Suppose that you have collected information on the selling price of 138 houses that are randomly selected from recent transactions. In addition to the selling prices you also have data on some other characteristics of these 138 houses. The information available on each house is as follows:

PRICE	=	selling price (in £ 000s)
SIZE	=	gross living space (in square meters)
AGE	=	age of the house (in years)
BDRM	=	number of bedrooms
TOWN	=	0 if the house is in Cambridge, 1 if in Huntingdon.

You run a multiple regression analysis on this data using SIZE, AGE, BDRM and TOWN to predict PRICE, which produces the following report.

Multiple R	0.881
R Square	0.776
Standard Error	86.772
Observations	138

	Coefficients	Standard Error	t-Stat	Lower 95%	Upper 95%
Intercept	180.833	531.862	0.340	-871.169	1232.834
SIZE	3.07	0.83	3.699	1.44	4.70
AGE	-3.571	2.062	-1.732	-7.650	0.508
BDRM	45.421	48.840	0.930	-51.183	142.024
TOWN	-175.341	74.171	-2.364	-322.048	-28.634

(i) What do you consider to be the strengths of the current model? [15%]

(ii) What do you consider to be the weaknesses of the current model? What might you do to improve the current model? [15%]

(iii) A customer is looking for a house and is interested in a particular three-bedroom house in Cambridge, which is 57 years old and has 175 square meters of gross living space. The asking price for the house is £630,000. Using the regression model, would you say the asking price is higher or lower than its fair market value, in the absence of any additional information? Justify your answer. [15%]

(iv) Based on the regression analysis, estimate how much 30 square meters of additional living space is worth, as far as the selling price is concerned. In addition, compute a 99% confidence interval for your estimate. [15%]

(b) You live in an area that has a possibility of incurring a flood, so you are considering buying flood insurance on your home at an annual cost of £2200. The probability of a flood damaging your home during a year is 0.01. If this happens, you estimate that the cost of the damage (fully covered by flood insurance) will be £200,000. Your home is worth £250,000.

(i) Evaluate the decision of whether or not to buy the insurance assuming you are a risk-neutral decision maker. [10%]

(ii) Assuming you are a risk-neutral decision maker, calculate the expected value of perfect information. [15%]

(iii) Evaluate your decision of whether or not to buy the insurance if your utility function is $U(x) = x^{1/2}$. [15%]

4 (a) Consider the Markov chain with state space E={1,2,3,4,5,6} and transition matrix

$$\mathbf{P} = \begin{Vmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 \\ 0 & 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0.2 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.2 & 0.3 \end{Vmatrix}$$

(i) Construct a graph to find the communicating classes. [10%]

(ii) Classify the states as transient or recurrent. [15%]

(b) On any given day Jane is happy (H), neutral (N) or sad (S). If she is happy today then she will be (H), (N), or (S) tomorrow with respective probabilities 0.7, 0.2 and 0.1. If she is neutral today then she will be (H), (N), or (S) tomorrow with respective probabilities 0.4, 0.3 and 0.3. If she is sad today then she will be (H), (N), or (S) tomorrow with respective probabilities 0.2, 0.4 and 0.4.

(i) What proportion of time is Jane happy in the long run? [20%]

(ii) If she is sad today, what is the average number of days that it will take her to become happy? [20%]

(c) Briefly explain the concept of arbitrage and how it affects options pricing. Also, explain in what sense is CAPM a pricing model. [35%]

END OF PAPER