ENGINEERING TRIPOS PART IIA

Tuesday 3 May 2011 9 to 10.30

Module 3F1

SIGNALS AND SYSTEMS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

VERSION: 2

1 (a) Consider a signal $\{y_k\}$ with z-transform

$$Y(z) = \frac{z}{(z-1)(z-3)}$$

(i) Find the signal $\{y_k\}$. [20%]

(ii) Could the final value theorem be used to find the value of y_k as $k \to \infty$? Justify your answer. [10%]

(b) The signal $\{y_k\}$ above is the output of a linear time invariant system when its input $\{u_k\}$ is the unit step

$$u_k = \begin{cases} 0 & \text{for } k < 0 \\ 1 & \text{for } k \geq 0. \end{cases}$$

Find the z-transfer function $G(z)$ of this system. Find also a difference equation that can represent it. [15%]

(c) Consider now the feedback interconnection below where λ is a constant.

(i) Find the z-transfer function from r to y. [10%]

(ii) Find the values of λ for which the closed loop transfer function in (c)(i) is stable. [15%]

(iii) How would your answer in (c)(ii) change if the feedback control law is instead given by $u_k = \lambda y_{k-1}$. [30%]

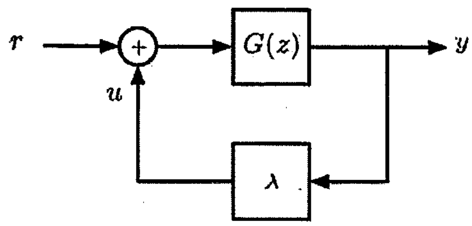

Fig. 1

VERSION: 2

2 (a) A continuous time system with transfer function $G(s)$ will be transformed into a discrete time system using:

(i) The Euler method, where s is replaced by $\frac{z-1}{T}$.

(ii) The Tustin transformation, where s is replaced by $\frac{2}{T}\left(\frac{z-1}{z+1}\right)$.

Prove in each of these two cases whether the following statement is true or false: "a stable continuous time system will always be transformed into a stable discrete time system". [35%]

If the discrete time system is obtained such that its step response is the same as the sampled step response of the continuous time system (with sampling period T), derive the corresponding z-transfer function. [15%]

(b) Rifle bullets are fired at a target and the displacement of the shots from the centre of the target is distributed as a normal (Gaussian) distribution with standard deviation σ_1 in the horizontal direction and a normal distribution with standard deviation σ_2 in the vertical direction. The horizontal and vertical displacements are assumed to be independent.

(i) Obtain an expression for the 2-dimensional probability distribution function (pdf) $f(r,\theta)$ of the shots in polar coordinates, r and θ. [25%]

(ii) The target comprises a central bullseye of diameter 20mm, worth 100 points, and two circular rings outside of this of width 10mm each, worth 50 and 25 points, decreasing from the centre. If the pdf is as above with $\sigma_1 = \sigma_2 = 15$mm, calculate the average score per shot. [25%]

3 (a) What is meant by an *ergodic* random signal? Briefly discuss whether *stationarity* is a necessary or a sufficient condition for a signal to be ergodic. [20%]

(b) How does the power spectral density (PSD) of an ergodic random signal depend on its auto-correlation function (ACF)? [20%]

(c) When an ergodic random signal is passed through a linear system with impulse response $h(t)$ and frequency-domain transfer function $H(\omega)$, derive an expression for the PSD of the output $S_Y(\omega)$ in terms of the PSD of the input $S_X(\omega)$. [20%]

(d) Show how this may be converted to an expression which relates the ACF of the output, $r_{YY}(\tau)$, to the the ACF of the input, $r_{XX}(\tau)$. [20%]

(e) Discuss how these results allow the frequency response of a system to be determined while it is operating online and is subject to random disturbances. [20%]

4 (a) Show that $\log_e x \leq x - 1$. Find the value of x that achieves equality in this equation. [20%]

(b) Hence show that the maximum entropy of a source with an alphabet of N symbols is $\log_2 N$ bit/symbol and find the source probability mass function (pmf) for which this entropy is attained. [20%]

(c) A ternary (three-level) communication channel transmits symbols $A = +1$, $B = 0$ and $C = -1$. Channel noise causes a finite error probability on the channel, such that the transition probability matrix for the output Y, given the input X, is:

$P(Y|X)$:

Y \ X	A	B	C
A	$1-b$	b	0
B	b	$1-2b$	b
C	0	b	$1-b$

If the probabilities for all three states of X are equal, determine the entropy of Y, $H(Y)$, and the conditional entropy of Y given X, $H(Y|X)$, as a function of the error probability parameter, b. [30%]

(d) Hence obtain an expression for the mutual information from input to output of this communication channel, and calculate the channel capacity in bits per transmitted symbol if $b = 0.1$. [15%]

(e) What does this tell us about the amount of redundancy of the error correction code that is likely to be needed to ensure error-free communication over this channel? [15%]

END OF PAPER