ENGINEERING TRIPOS PART IIA

Wednesday 11 May 2011 2.30 to 4

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version: final

1 Electrical power P_g is generated according to the equation

$$\tau_g \frac{dP_g}{dt} = p - c_g P_g - kE \qquad (1)$$

where τ_g is a time constant, p is the price, and E is 'accumulated energy'. c_g and k are positive constants. There are M consumers. The power P_{c_i} consumed by the ith consumer behaves according to

$$\tau_c \frac{dP_{c_i}}{dt} = c_c P_{c_i} - p \qquad (2)$$

where τ_c is a time constant and c_c is a positive constant. The accumulated energy E is defined by

$$\frac{dE}{dt} = P_g - \sum_{i=1}^{M} P_{c_i} \qquad (3)$$

(a) Put equations (1) – (3) into state-space form, considering the price p as an input. [20%]

(b) If the price p is held constant, what are the equilibrium values of P_g, P_{c_i} and E? [20%]

(c) Suppose that there is only one consumer ($M = 1$). Show that this system is unstable. [30%]

(d) In a 'smart grid' the price p is adjusted according to

$$p = -\alpha P_g - \beta E$$

with the intention of driving E to 0. Still assuming that $M = 1$, show that this results in an unstable system if $\beta = -k$. [30%]

2 The speed v of a car is given approximately by

$$m\frac{dv}{dt} + bv = u$$

where u is the forward force provided by the engine, with $m = 1000$ kg and $b = 50$ N sec m^{-1}. A cruise control system is to be designed for maintaining the speed v at a desired set-point v_d determined by the driver. The following specifications are imposed on the system:

(i) The damping factor of all closed-loop poles should be at least 0.8 .

(ii) The dominant time-constant of the closed-loop system should not exceed 1 second.

(iii) There should be no steady-state error in the presence of a constant disturbance force.

(a) A 'proportional plus integral' (PI) feedback controller is proposed, of the form

$$K(s) = k\left(1 + \frac{1}{s}\right) \tag{4}$$

Sketch the root-locus diagram for the system when this controller is used. [20%]

(b) (i) Explain why the 'breakaway points' on a root-locus diagram are given by solutions of $dL(s)/ds = 0$, where $L(s)$ is the return-ratio. [20%]

(ii) Verify that, if the PI controller $K(s)$ defined in (4) is used for the cruise control system, there are breakaway points on the real axis at -1.9747 and -0.0253. [20%]

(c) Explain why placing all the closed-loop poles at -0.0253 would not satisfy the specifications, whereas placing them at -1.9747 would, if a PI controller is used. [20%]

(d) If the PI controller $K(s)$ defined in (4) is used, find the value of k that results in the closed-loop poles being placed at -1.9747. [20%]

3 (a) Derive the formula for the transfer function of a system with state-space realisation (A,B,C,D). [25%]

(b) A linear system S_1 has two inputs u_1 and u_2, one output y, and state-space model

$$S_1: \quad \frac{dx_1}{dt} = A_1x_1 + \begin{bmatrix} B_{11} & B_{12} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad y = C_1x_1 + D_{11}u_1 + D_{12}u_2$$

where x_1 contains n_1 state variables. A second linear system S_2 has one input v, one output w, and state-space model

$$S_2: \quad \frac{dx_2}{dt} = A_2x_2 + B_2v, \quad w = C_2x_2$$

where x_2 contains n_2 state variables.

The two systems are connected together as shown in Fig.1, namely $u_2 = w$ and $v = y$. The resulting system S has state-space realisation (A,B,C,D).

(i) Show that

$$A = \begin{bmatrix} A_1 & B_{12}C_2 \\ B_2C_1 & A_2 + B_2D_{12}C_2 \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} \\ B_2D_{11} \end{bmatrix}$$

and find C and D. [25%]

(ii) Suppose that $n_1 = n_2 = 1$, and the two systems S_1 and S_2 are defined as follows:

$$S_1 \ : \ A_1 = 1, \quad B_{11} = 1, \quad B_{12} = 1, \quad C_1 = 1, \quad D_{11} = 0, \quad D_{12} = 0.$$
$$S_2 \ : \ A_2 = \alpha, \quad B_2 = \beta, \quad C_2 = 1.$$

Show that the system S is controllable if and only if the system S_2 is controllable. [25%]

(iii) Assuming that the systems S_1 and S_2 are defined as in part (b)(ii), find the transfer function of the system S (from input u_1 to output y) in the two cases when S_2 is controllable and when S_2 is not controllable, and comment on the results. [25%]

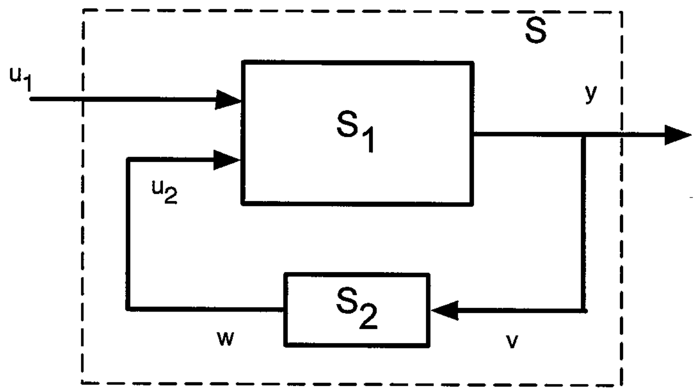

Fig. 1

4 (a) Explain, with the aid of a block diagram, what is meant by a *state observer*. [20%]

(b) Suppose that an algorithm `place(A,B,poles)` is available, which, given matrices A and B, finds a state-feedback matrix K such that the eigenvalues of $A - BK$ are at the locations defined in `poles`. Explain how this algorithm can be used in the design of a state observer. [30%]

What condition must be true for this algorithm to succeed? [10%]

(c) The angular velocity ω of an electric drive system is governed by the equation

$$J\frac{d\omega}{dt} = -B\omega + T - T_L$$

where T is the torque produced by the motor, T_L is a load torque, J is a moment of inertia, and B is a viscous friction coefficient. The angular velocity ω is measured, and the torque T is known. The constants J and B are also known. The load torque T_L is not known, but is assumed to change so rarely that it can be considered to be constant.

Explain how a state observer can be used to estimate T_L. [20%]

(d) Suppose that there is some noise on the measurement of ω in part (c). Discuss how this would affect the design of the state observer. [20%]

END OF PAPER