ENGINEERING TRIPOS PART IIA

Thursday 12 May 2011 2.30 to 4

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

FINAL version

1 In a polar binary baseband communication system with a bit rate of $1/T$ Hz, the transmitter consists of a modulator that outputs a train of random impulses having values of $+1$ and -1 with equal probability, followed by a transmit filter $H_T(\omega)$. The receiver consists of a filter $H_R(\omega)$, followed by a data slicer. The required spectrum of the signal at the slicer input in response to a unit impulse at the modulator output is $P_R(\omega)$, where $kP_R(\omega) = H_T(\omega)H_C(\omega)H_R(\omega)$, $H_C(\omega)$ is the channel frequency response and k is an arbitrary positive gain factor.

(a) If the noise in the system is modelled as additive white Gaussian noise at the input to the receiver filter having a 2-sided PSD of N_0, show that $H_T(\omega)$ and $H_R(\omega)$ that minimize the bit error rate (BER) for a given transmitted power are each proportional to

$$\left|\frac{P_R(\omega)}{H_C(\omega)}\right|^{\frac{1}{2}}$$

[40%]

(b) If the channel frequency response is given by

$$H_C(\omega) = \begin{cases} \cos\left(\dfrac{\omega T}{2}\right) & , \quad \dfrac{-\pi}{T} \le \omega \le \dfrac{\pi}{T} \\ 0 & , \quad \text{elsewhere} \end{cases}$$

and the desired pulse spectrum at the slicer input is

$$P_R(\omega) = \begin{cases} T\cos^2\left(\dfrac{\omega T}{2}\right) & , \quad \dfrac{-\pi}{T} \le \omega \le \dfrac{\pi}{T} \\ 0 & , \quad \text{elsewhere,} \end{cases}$$

(i) Determine $H_T(\omega)$ and $H_R(\omega)$ to minimise the BER. [20%]

(ii) Determine an expression for the ratio of the signal to noise power ratio at the slicer input divided by the power at the output of the transmit filter. [40%]

Note: the Schwartz inequality is

$$\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \ge \left|\int_{-\infty}^{\infty} F(\omega)G(\omega)d\omega\right|^2$$

2 (a) Describe how an eye diagram is produced and is used to assess the performance of a digital baseband transmission system. Explain its limitations as a method of bit error rate (BER) estimation. [20%]

(b) In a binary transmission system, the sampled signal is either $V_0 + v(t)$ with probability P_0 or $V_1 + v(t)$ with probability P_1 where, $V_1 > V_0$; $v(t)$ is zero mean Gaussian noise of rms amplitude σ . Derive an expression for the BER if the receiver data slicer threshold is V_T , making use of the function

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\left(x^2/2\right)} dx$$

[30%]

(c) A baseband channel can be modelled as shown in Fig. 1. The channel is used to transmit binary data at a rate of $1/T$ bit/s and the input to the channel has the form of rectangular pulses of duration T and amplitude of either 0 V or 1 V with equal probability. The receiver data slicer threshold is 0.4 V, $v(t)$ is zero mean white Gaussian noise of rms amplitude 0.1 V and $T = 2RC$.

(i) Sketch the eye diagram. [20%]

(ii) Estimate the BER stating any assumptions made. [20%]

(iii) What are the optimum slicer threshold and the associated BER? [10%]

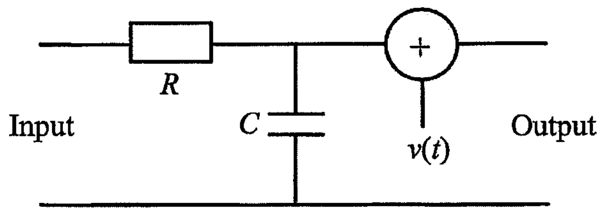


Fig. 1

Note: the Gaussian error integral approximation is

$$Q(x) \approx \frac{e^{-x^2/2}}{1.64x + \sqrt{0.76x^2 + 4}}$$

3 (a) Explain how a modulated signal $s(t) = Re[\,p(t)e^{j\omega_C t}\,]$ may be demodulated to recover $u(t)$ and $jv(t)$, the real and imaginary parts of $p(t)$ respectively. Draw the block diagram and derive the mathematical expressions which show how the quadrature demodulator works. [20%]

(b) Figure 2 shows the block diagram of an ideal demodulator for binary phase shift keyed (BPSK) signals that are in phasor form. Explain how this system works, including the meaning of symbols, and draw a block diagram showing how it should be connected to the quadrature demodulator discussed in part (a) in order to produce a demodulator for real BPSK signals, $s(t)$. [40%]

(c) If the signals $s(t)$ are instead modulated with quadrature phase shift keying (QPSK), show how the block diagram obtained in part (b) should be extended in order to produce a practical QPSK demodulation system. [15%]

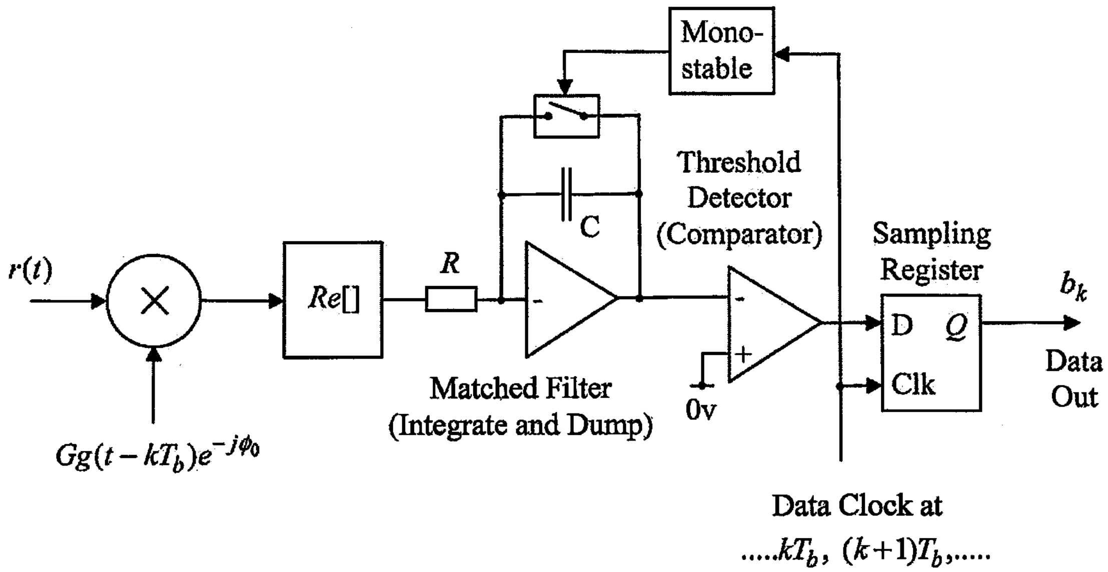
(d) A voltage-controlled oscillator (VCO) is available with outputs, $2\cos(\omega_C t + \phi_0)$ and $-2\sin(\omega_C t + \phi_0)$, which connect to the two local oscillator inputs of the quadrature demodulator. The input signal $s(t)$ to the demodulator is given by:

$$s(t) = V_s \cos(\omega_C t + \phi_k)$$

where

$$e^{j\phi_k} = (b_{2k} + jb_{2k+1})/\sqrt{2}$$

according to the polarities of the data bits, $b_{2k} = \pm 1$ and $b_{2k+1} = \pm 1$. Explain the operation of a phase-locked loop, operating with the VCO and the QPSK demodulator in part (c), that will allow the loop to lock correctly to the input signal $s(t)$. [25%]



Mono-
stable
Threshold
Detector
(Comparator)
C
Sampling
Register
r(t)
Re[]
R
-
-
+
D
Q
Clk
b_k
Data
Out
Matched Filter
(Integrate and Dump)
0v
Gg(t − kT_b)e^{−jφ_0}
Data Clock at
.....kT_b, (k+1)T_b,.....

Fig. 2

4 (a) Explain the significance of the quantity:

U = (Energy per bit) / (noise power spectral density)

in the analysis of digital communication systems. What does it tell us about the way that the transmitter power needs to be varied with data rate in a system, if the bit error probability is to remain constant? [20%]

(b) Sketch the phasor constellation for quadrature phase shift keying (QPSK), 16-level PSK and 16-level quadrature amplitude modulation (QAM). For a given bit rate and signal power at the receiver, explain why the energy per symbol is greater for the 16-level modulation schemes than for QPSK, and also why the probability or error is expected to be worse for such schemes, despite this. In addition, comment on the relative merits of 16-QAM versus 16-PSK. [40%]

(c) Given that the bandwidth of a PSK or QAM modulation scheme is proportional to the transmitted symbol rate, discuss the trade-offs that must be made when selecting suitable modulation schemes for digital video and digital audio broadcasting. [20%]

(d) What additional technique of modulation is usually employed in order to mitigate the problem of multiple transmission paths, whose delay spread substantially exceeds the bit period of the transmitted data? Briefly explain how this technique overcomes the problem. [20%]

END OF PAPER

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Engineering Triops Part 2A
Module 3F4. Data Transmission, May 2011 - Answers

1.

a)

b) (i) $|H_R(\omega)| = \sqrt{\frac{T}{\sqrt{N_0}}\cos\left(\frac{\omega T}{2}\right)}$

$|H_T(\omega)| = k\sqrt{T\sqrt{N_0}\cos\left(\frac{\omega T}{2}\right)}$

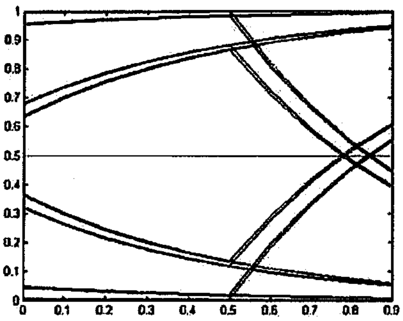
(ii) $\frac{3\pi^2 T}{32 N_0}$

2.

a)

b)

c) (i)



(ii) 2.01×10^{-3}

(iii) 0.5V, 1.31×10^{-4}

3.

a)

b)

c)

d)

4.

a)

b)

c)

d)

I. J. Wassell

May 2011