ENGINEERING TRIPOS PART IIA

Tuesday 26 April 2011 9 to 10.30

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

This paper consists of three sections.

Answer **one** *question from each section.*

Answers to questions in each section should be tied together and handed in separately.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Medical Image Acquisition*

Answer **one** *question from this section*

1 An X-ray phantom consists of four homogeneous rods with unknown X-ray linear attenuation coefficients. The rods in the phantom are exposed to X-rays from three different directions, as shown in Fig. 1. Let $Q = -\ln(I/I_0)$, where I_0 is the X-ray intensity incident on the phantom and I is the transmitted intensity. Q is recorded for various paths through the phantom and the values observed are shown in Fig. 1.

(a) Let the X-ray linear attenuation coefficients in the rods be a, b, c, and d, as shown in Fig. 1. Based on the values of Q in Fig. 1, write down a set of linear simultaneous equations that must be satisfied by a, b, c, and d. [10%]

(b) Based on Fig. 1, write down a 4×4 matrix M, such that

$$M \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 18 \\ 24 \\ 30 \\ 26 \end{pmatrix}$$

[10%]

(c) Find the values of a, b, c, and d. Explain the method that you use. [20%]

(d) Given that this problem can be solved using simple linear algebra, what is different about medical computed tomography that necessitates the use of algorithms such as filtered backprojection or maximum-likelihood expectation-maximisation (ML-EM)? [40%]

(e) Consider a different set of acquired data, illustrated in Fig. 2. If this data alone is available, is it possible to determine the linear attenuation coefficients in the rods? If so, show how it can be done. If not, explain why not. [20%]

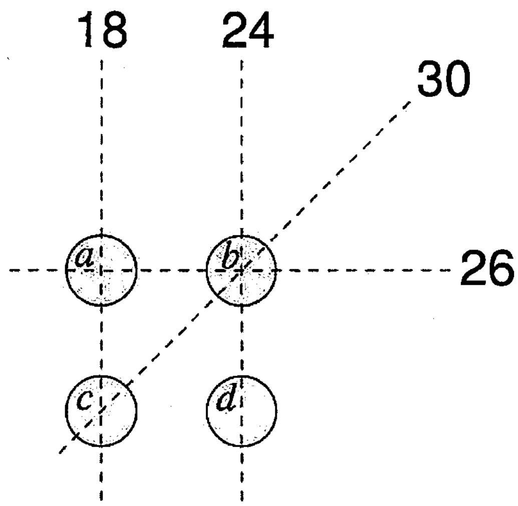

Fig. 1

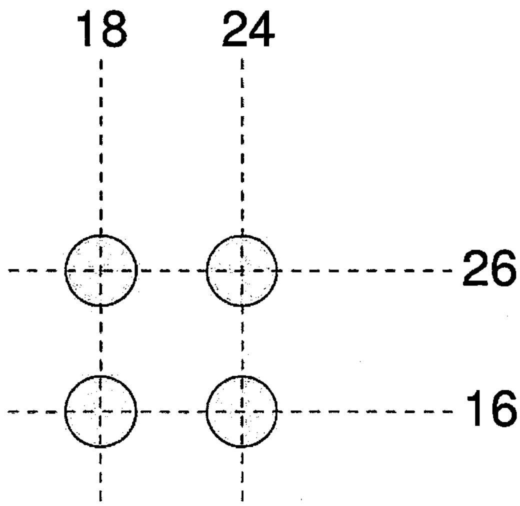

Fig. 2

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2 (a) For each of the medical imaging modalities listed below, explain what determines the resolution of the acquired data. If the resolution varies as a function of spatial location, you should describe this variation and explain why it occurs. If the resolution is not isotropic in all three dimensions, you should describe how it varies as a function of direction and explain the cause of this variation.

(i) Ultrasonic imaging [25%]

(ii) X-ray computed tomography (CT) [25%]

(iii) Single photon emission computed tomography (SPECT) [25%]

(b) In magnetic resonance imaging, explain the role of *slice selection, phase selection* and *readout* as part of a simple spin-echo sequence. Hence describe how the gradient field can be used to create a unique response from each individual point in three-dimensional space. [25%]

SECTION B *Curves, Surfaces and Interpolation*

*Answer **one** question from this section*

3 (a) A surface comprises a closed mesh of triangles, each with three vertices with coordinates (x, y, z). The mesh is stored as: A) a list of triangles each containing nine floating point numbers defining the coordinates of each vertex, or B) a list of vertices each containing three floating point numbers defining the coordinates, and a list of triangles each containing three integers referring to entries in the vertex list.

Outline the steps involved in each of the following operations for both storage schemes A and B. Assume that each process is started by a user clicking at a point whose location (x_c, y_c, z_c) is known in the surface coordinate system. Comment on the advantages and disadvantages of each scheme in each case.

(i) Moving a vertex to a new location by click-and-drag. [15%]

(ii) Deleting a point and removing any redundant triangles. [15%]

(iii) Deleting an edge and removing any redundant points and triangles. [30%]

(b) A polygon is represented by a set of lines joining each neighbouring pair of an ordered set of N vertices with coordinates (x_n, y_n), where $1 \leq n \leq N$. The polygon is closed with a line connecting vertex N to vertex 1.

(i) Explain how you would calculate the area enclosed by the polygon. [10%]

(ii) In order to display the polygon as a smooth curve, it is represented by a set of cubic B-spline segments. For a polygon with $N = 4$, write down the geometry matrices in terms of x and y for each segment. In what scenarios might this representation be desirable? [10%]

(iii) B-spline segment n is defined by vector $\mathbf{v_n}(t)$, where $0 \leq t \leq 1$. How can the area enclosed by the contour's cubic B-spline representation be calculated in terms of $\mathbf{v_n}(t)$? [20%]

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4 (a) A radial basis function $s(x, y)$ is used to interpolate N measurements f_n taken at locations (x_n, y_n), where:

$$s(x,y) = \sum_{n=1}^{N} \lambda_n \phi(r_n(x,y)) + p(x,y)$$

$p(x,y) = c_0 + c_1 x + c_2 y$ is a first order polynomial function of x and y, and $r_n(x,y)$ is the Euclidean distance from (x,y) to (x_n, y_n). $s(x,y)$ is computed by considering the following matrix equation:

$$\begin{bmatrix} \phi(r_1(x_1,y_1)) & \cdots & \phi(r_N(x_1,y_1)) & 1 & x_1 & y_1 \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ \phi(r_1(x_N,y_N)) & \cdots & \phi(r_N(x_N,y_N)) & 1 & x_N & y_N \\ 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & \cdots & x_N & 0 & 0 & 0 \\ y_1 & \cdots & y_N & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_N \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(i) What are the purposes of the terms λ_n, $\phi(r_n(x,y))$ and $p(x,y)$? [10%]

(ii) Explain carefully how the matrix equation is derived, and how it can be used to find $s(x,y)$. [20%]

(b) The following data are interpolated by $s(x,y)$, with $\phi(r_n) = r_n$:

n	x	y	f
0	0	0	0
1	1	0	1
2	0	1	1
3	1	1	0

(i) Set up the specific matrix equation for this case. [10%]

(ii) By substitution or otherwise, solve the matrix equation for the values $\lambda_1 \ldots \lambda_4$ and $c_0 \ldots c_2$. [30%]

(iii) Evaluate $s(x,y)$ at the locations $(0.5, 0.5)$, $(0, 0.5)$ and $(0.5, 0)$. [15%]

(c) For the data in (b), comment on the differences between the interpolation method described above, and linear interpolation of f across a Delaunay triangulation of the measurement locations. [15%]

SECTION C *3D Graphical Rendering*

Answer ***one*** *question from this section*

5 (a) Explain, with the aid of diagrams, what is meant by *recursive ray tracing.* Discuss the relative advantages and disadvantages of recursive ray tracing compared with standard surface rendering. [50%]

(b) Is it safe to enable *back-face culling* when ray tracing? Why? [10%]

(c) Figure 3 illustrates a technique called *environment mapping*, which approximates ray tracing in some respects. Six images of the environment are obtained by an imaginary camera pointing in each direction along each of the principal axes. These images form six texture maps, one for each face of the cube in Fig. 3. Objects are then surface rendered in the usual manner, including 2D texture mapping, with the texture coordinates obtained by the intersections of the rays R with the cube.

Discuss the relative advantages and disadvantages of environment mapping compared with recursive ray tracing. [40%]

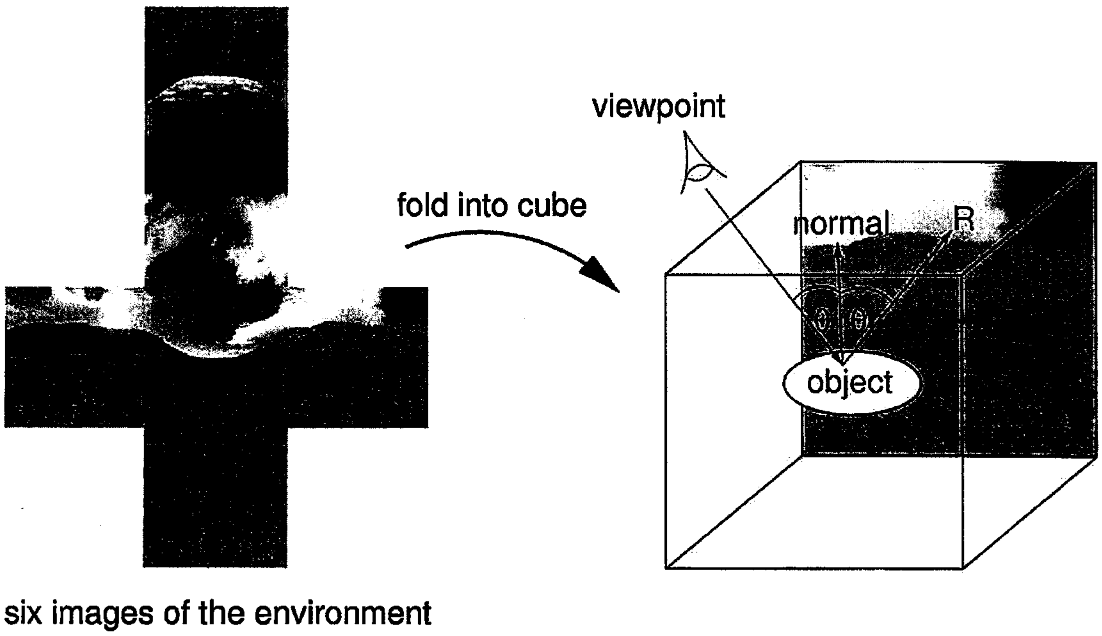

Fig. 3

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6 Figure 4(a) shows a square chessboard as it appears in a rendering. Perspective effects result in its far edge ($z_v = -20$) appearing shorter than its near edge ($z_v = -10$). The chessboard is modelled as two triangles. Points A and B are two of the triangles' vertices, while C lies midway (in the rendering) between A and B.

(a) Explain how intensity and depth (z_s) values within the interior are derived from vertex attributes. Consider both Gouraud and Phong shading. [25%]

(b) Figure 4(b) shows a 2D texture map that is to be applied to the chessboard. The programmer associates vertex A with the texture map point (s_0, t_0) and vertex B with (s_0, t_1). Two different interpolation techniques are considered for calculating texture coordinates along the edge AB:

$$t_\alpha = (1-\alpha)t_0 + \alpha t_1$$

$$t_\alpha = \frac{(1-\alpha)(t_0/z_0) + \alpha(t_1/z_1)}{(1-\alpha)(1/z_0) + \alpha(1/z_1)}$$

where α is the fraction of the distance along the rendered edge from A to B, and z_0 and z_1 are the z_v values at A and B respectively.

(i) Calculate the t texture coordinate at C using each of the above equations. Comment on the correctness of the rendering in each case. [25%]

(ii) Which of the two interpolation methods should be used for depth (z_s) values? Why? [20%]

(c) Discuss how the visual appearance of the rendering might be optimized if the chessboard appears relatively small in the rendering, covering far fewer pixels than the texture map itself. [30%]

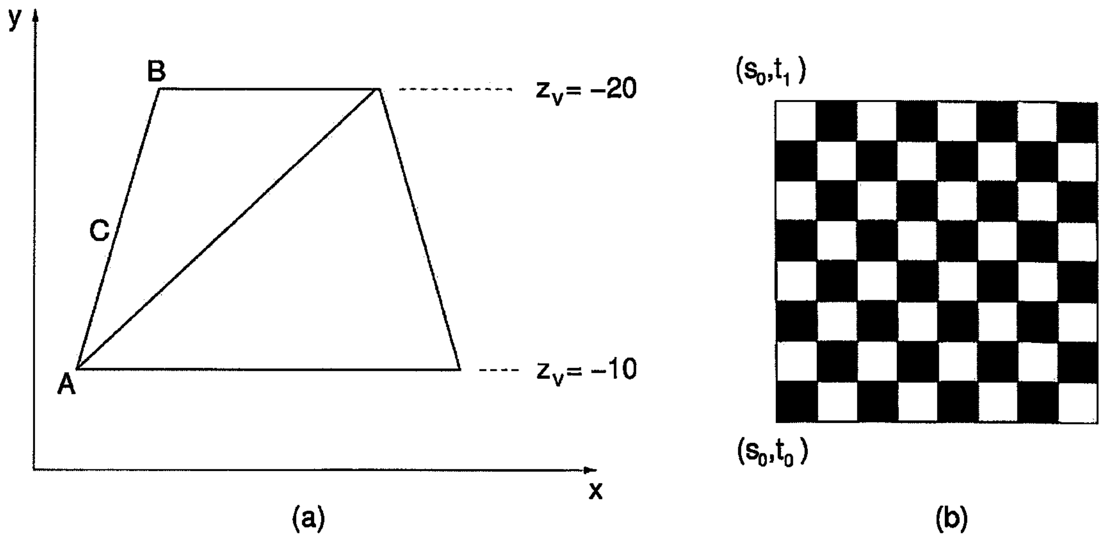

Fig. 4

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