2012 IIA Paper 3A3 (Fluid Mechanics II) Answers

Q1: a) steady, isentropic, irrotational, linearised.

b) $\left.\dfrac{\partial \phi}{\partial y}\right|_{y=0} = \dfrac{\tau}{c} g' u_\infty$

c) $k = -2\dfrac{\partial \widetilde{\phi}}{\partial \widetilde{x}}$ where $\widetilde{x} = x/c$ and $\widetilde{\phi} = \dfrac{\sqrt{1-M_\infty^2}}{\tau u_\infty}\phi$

d) $u_\infty = 180.7 \text{ m/s} \quad (M_\infty = 0.531)$

e) $u_\infty = 168.3 \text{ m/s} \quad (M_\infty = 0.495)$

Q2: b) $drag/length = 3.82\times10^{-3}\, p_\infty \text{ N/m}$

c) $163.8 \text{ kW} \quad (drag = 277.5 \text{ N} \quad \text{for 6 m long bump})$

Q3: b) i) $0.0582 \text{ m}^2 \quad (M_{exit} = 2.11)$

ii) 7026 N

c) i) just choked with isentropic subsonic flow in divergent section $p_{exit} = 1.667 \text{ bar}$

ii) $M_{ahead} = 2.075 \quad (p_{0exit}/p_{0inlet} = 0.6856)$

iii) 2397 N

Q4: b) i) $M_{inlet} = 0.3800,\ c_f = 0.0090$ or $M_{inlet} = 1.9792,\ c_f = 0.0010$

c) i) $1\text{ m}:\ M_{ahead-shock} = 1.7909,\ M_{after-shock} = 0.6186,\ \left.\dfrac{4c_f L}{D}\right|_{exit} = 0.0322$

$2\text{ m}:\ M_{ahead-shock} = 1.6180,\ M_{after-shock} = 0.6631,\ \left.\dfrac{4c_f L}{D}\right|_{exit} = -0.0398$

ii) $1.45 \text{ m},\ M_{ahead-shock} = 1.71$

Q5: c) $h_1/h_0 = 0.444$ (Not a small height change.)

d) $d = 179 \text{ m} \quad (\Delta t = 104.4 \text{ s})$

Q6: c) Get u_j^{i+1} from momentum equation, then v_{j+1}^{i+1} from continuity

Q7: a) i) Second order in space, first order in time.

ii) $0 < \beta < 0.5$

b) ii) $Power|_{test} = 0.761 \text{ MW},\ \eta|_{test} = 82.8\%,\ Head|_{test} = 184 \text{ m} \quad (\dot{Q}|_{test} = 0.350 \text{ m}^3\text{s}^{-1})$

Q8: a) $\phi = 0.500$

b) $\psi = 1.145$

c) 8.20 bar

d) 53.8 mm

e) 40.4 kg/s

f) 89.2%