ENGINEERING TRIPOS PART IIA

Tuesday 1 May 2012 9 to 10.30

Module 3F1 SIGNALS AND SYSTEMS

Short answers

1. (a) $\dfrac{1}{z^2-z+\alpha}$;

(b) $z=0, z=1$, unstable; $z=\frac{1}{2}, z=\frac{1}{2}$, stable;
$z=\frac{1\pm i}{2}$, stable; $z=\frac{1\pm i\sqrt{7}}{2}$, unstable.

(c) $y_k=\{0,0,1,1,1,\cdots\}$;
$y_k=0$ for $k=0,1$, $y_k=2(1-k)(1/2)^{k-1}$ for $k\geq 2$;
$y_k=0$ for $k=0,1$, $y_k=(1/\sqrt{2})^{k-3}\sin(\frac{\pi}{4}(k-1))$ for $k\geq 2$.

(d) (i) $\dfrac{K}{z^2-z+\alpha+K}$;
(ii) $0<K<1$; $-\frac{1}{4}<K<\frac{3}{4}$; $-\frac{1}{2}<K<\frac{1}{2}$; $-2<K<-1$; No.

2. (b) (i) $f_Y(y)=\displaystyle\int_{-\infty}^{\infty} f_2(y-x_1)\,f_1(x_1)\,dx$ (convolution)
(ii) $\Phi_Y(u)=\Phi_{X_2}(u)\,\Phi_{X_1}(u)$

3. (b) $S_X(\omega)=P\,T_0\,\mathrm{sinc}^2\left(\dfrac{\omega T_0}{2}\right)=\dfrac{2P}{T_0}\left(\dfrac{1-\cos(\omega T_0)}{\omega^2}\right)$

(c) $S_Y(\omega)=|H(\omega)|^2\,P\,T_0\,\mathrm{sinc}^2\left(\dfrac{\omega T_0}{2}\right)$

(d) $|H(\omega)|=\sqrt{\dfrac{S_Y(\omega)}{S_X(\omega)}}$

(e) $H(\omega)=\dfrac{S_{XY}(\omega)}{S_X(\omega)}$

4. (a) (i) no, yes, yes; (ii) Huffman only for third code.

(b) (i) $H(X)=h(p)$; $H(Y|X)=p$; $H(XY)=h(p)+p$; $H(Y)=h(p/2)$;
$H(X|Y)=h\left(\frac{p}{2-p}\right).(1-\frac{p}{2})=h(p)+p-h(p/2)$
$I(X;Y)=h(p/2)-p$
(iii) $p_{\max}=0.4$; $I_{\max}(X;Y)=0.3219$ bit per use.