ENGINEERING TRIPOS PART IIA 2012

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

SOLUTIONS TO TRIPOS QUESTIONS

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Final version

1. (a) Neglecting changes in KE and PE, the SFEE and SFSE are :

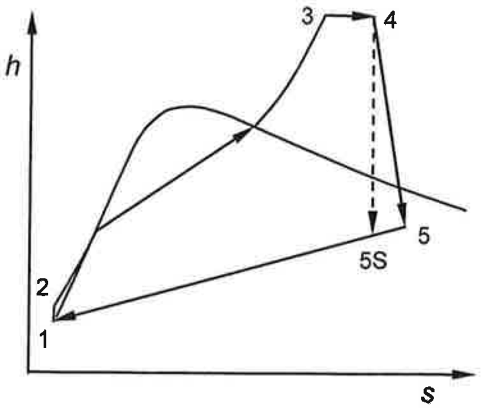
$$\dot{m}(h_2 - h_1) = \dot{Q}_S - \dot{Q}_0 - \dot{W}_X = \int_{CS} d\dot{Q}_S - \int_{CS} d\dot{Q}_0 - \dot{W}_X$$

$$\dot{m}(s_2 - s_1) = \int_{CS} \frac{d\dot{Q}_S}{T} - \int_{CS} \frac{d\dot{Q}_0}{T} + \dot{S}_{irrev}$$

Multiplying the SFSE by T_0 and subtracting from the SFEE gives,

$$\dot{m}\left[(h_2 - T_0 s_2) - (h_1 - T_0 s_1)\right] = \dot{m}(e_2 - e_1) = \int_{CS}\left(1 - \frac{T_0}{T}\right)d\dot{Q}_S - \dot{W}_X - \int_{CS}\left(1 - \frac{T_0}{T}\right)d\dot{Q}_0 - T_0\dot{S}_{irrev}$$ [20%]

(b) (i)



[5%]

(ii) Turbine power output $= \dot{m}(h_4 - h_5)$

$= 200 \times (3502.0 - 2400.2) \times 10^{-3} = 220.36$ MW [5%]

The expansion is adiabatic so there is no lost power due to heat transfer loss. Hence,

Turbine lost power $= T_0\dot{S}_{irrev} = \dot{m}T_0(s_5 - s_4)$

$= 200 \times 298.15 \times (7.7910 - 7.1624) \times 10^{-3} = 37.48$ MW [5%]

Turbine power + lost power $= 220.36 + 37.48 = 257.84$ MW

Isentropic power output $= \dot{m}(h_4 - h_{5S})$

$= 200 \times (3502.0 - 2205.8) \times 10^{-3} = 259.24$ MW [5%]

The isentropic expansion involves no lost power but the isentropic power does not equal the sum of the actual turbine power and lost power. This is because the end states 5 and 5S are different and therefore $(e_5 - e_{5S}) \neq 0$. The power output for a reversible process between states 4 and 5 would indeed be equal to the sum of the actual power and lost power. [10%]

(iii) Lost power due to entropy creation (irreversibilities) in the condenser $= T_0\dot{S}_{irrev}$

SFSE for the condenser is,

$$\dot{m}(s_1 - s_5) = -\int_5^1 \frac{d\dot{Q}_0}{T} + \dot{S}_{irrev}$$

From the SFEE and the *Tds* equation (being careful with the signs),

$$d\dot{Q}_0 = -\dot{m}\,dh = -\dot{m}(Tds + v\,dp)$$

There is no pressure drop in the condenser so $dp = 0$. Hence,

$$\dot{m}(s_1 - s_5) = \dot{m}\int_5^1 ds + \dot{S}_{irrev} \quad \rightarrow \quad \dot{S}_{irrev} = 0$$ [15%]

This result follows because of the assumption that there is no pressure drop in the condenser. [5%] From the exergy equation, the 'lost power' associated with the heat transfer to the cooling water is thus,

$$\int_5^1 \left(1 - \frac{T_0}{T}\right) d\dot{Q}_0 = \dot{m}(e_5 - e_1) = \dot{m}\left[(h_5 - T_0 s_5) - (h_1 - T_0 s_1)\right]$$

$$= 200\times[(2400.2 - 151.5) - 298.15\times(7.7910 - 0.5210)]\times10^{-3} = 16.23 \text{ MW}$$ [5 %]

(iv) Exergy supply rate $= \dot{m}(e_3 - e_2) = \dot{m}[(h_3 - T_0 s_3) - (h_3 - T_0 s_2)$

$= 200\times[(3502.0 - 162.6) - 298.15\times(6.7585 - 0.5210)]\times10^{-3}$

$= 295.94$ MW [5%]

Feed pump power output $= \dot{m}(h_1 - h_2) = \dot{m}[(h_3 - T_0 s_3) - (h_3 - T_0 s_2)$

$= 200\times(162.6 - 151.5)\times10^{-3} = -2.22$ MW [5%]

No lost power in feed pump because the compression is reversible and adiabatic.
Throttle is adiabatic hence,

Throttle lost power $= \dot{m}T_0(s_4 - s_3)$

$= 200\times298.15\times(7.1624 - 6.7585)\times10^{-3} = 24.08$ MW [5%]

Total power + lost power $= (220.36 - 2.22) + (24.08 + 37.48 + 16.23) = 295.93$ MW [5%]
= Exergy supply rate

Rational efficiency $= \dfrac{\text{Net power output}}{\text{Exergy supply rate}} = \dfrac{220.36 - 2.22}{295.94} = 0.737$ [5%]

2. (a) The given characteristic equation of state is,

$$g = A(T - T\ln T) - \frac{BT^2}{2} + RT\ln p - \frac{Kp}{T^3} + h_0 - Ts_0$$

From the definition $g = h - Ts$ we have $dg = dh - Tds - sdT$.
Substituting $Tds = dh - vdp$ then gives $dg = vdp - sdT$. Hence, [10%]

$$v = \left(\frac{\partial g}{\partial p}\right)_T = \frac{RT}{p} - \frac{K}{T^3}$$ [10%]

$$-s = \left(\frac{\partial g}{\partial T}\right)_p = A(1 - 1 - \ln T) - BT + R\ln p + \frac{3Kp}{T^4} - s_0$$

Therefore,

$$s = A\ln T + BT - R\ln p - \frac{3Kp}{T^4} + s_0$$ [10%]

From $g = h - Ts$ we find,

$$h = g + Ts = AT + \frac{BT^2}{2} - \frac{4Kp}{T^3} + h_0$$ [5%]

Finally,

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = A + BT + \frac{12Kp}{T^4}$$ [5%]

(b) The equation for complete combustion would be,

$$CO + 1.5\, O_2 \rightarrow CO_2 + O_2$$

Allowing for dissociation, the actual reaction is,

$$CO + 1.5\, O_2 \rightarrow (1-\alpha)\, CO_2 + \alpha\, CO + \left(\frac{2+\alpha}{2}\right) O_2$$ [10%]

Total number of moles of products $= n_2 = \dfrac{(4+\alpha)}{2}$

If the pressure after combustion is p_2, then the partial pressures are given by,

$$p_{CO2} = \frac{(1-\alpha)}{n_2} p_2\,, \qquad p_{CO} = \frac{\alpha}{n_2} p_2\,, \qquad p_{O2} = \frac{(2+\alpha)}{2n_2} p_2\,.$$ [10%]

Reaction (7) in the Thermofluids Data Book is,

$$-\,CO \;-\; \frac{1}{2}O_2 \;+\; CO_2 \;=\; 0$$

so the equilibrium equation is,

$$\left(\frac{p_{CO2}}{p_0}\right)^{+1}\left(\frac{p_{CO}}{p_0}\right)^{-1}\left(\frac{p_{O2}}{p_0}\right)^{-0.5} = K_{p7}(T_2)$$ [10%]

where $p_0 = 1$ bar is 'standard pressure'. Substituting the expressions for the partial pressures :

$$\left(\frac{1-\alpha}{n_2}\right)^{+1}\left(\frac{\alpha}{n_2}\right)^{-1}\left(\frac{2+\alpha}{2n_2}\right)^{-0.5}\left(\frac{p_2}{p_0}\right)^{-0.5} = K_{p7}(T_2)$$

From the Data Book at 2600 K, $\ln(K_{p7}) = 2.800$, so $K_{p7} = 16.445$. Substituting for n_2 gives,

$$\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{4+\alpha}{2+\alpha}\right)^{0.5}\left(\frac{p_2}{p_0}\right)^{-0.5} = 16.445$$ [10%]

Now $p_2V = n_2\overline{R}T_2$ and $p_1V = n_1\overline{R}T_1$ with $n_1 = 5/2$ and $p_1 = p_0 = 1$ bar. Hence,

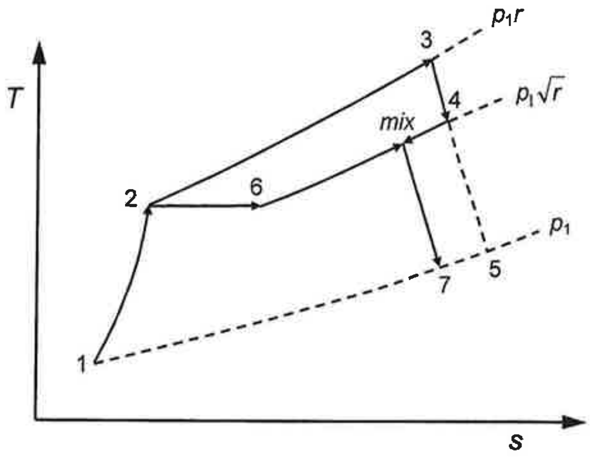
$$\frac{p_2}{p_0} = \frac{n_2}{n_1}\frac{T_2}{T_1} = \frac{(4+\alpha)}{5}\times\frac{2600}{298.15}$$ [10%]

Substituting for (p_2/p_0) gives,

$$\frac{(1-\alpha)}{\alpha(2+\alpha)^{0.5}} = \left(\frac{2600}{5\times 298.15}\right)^{0.5}\times 16.445 = 21.718$$

This is satisfied by the given value of $\alpha = 0.0313$. [10%]

3. (a)



[15%]

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/(\gamma\eta_p)} = T_1 r^{(\gamma-1)/(\gamma\eta_p)} = T_1 r_t^{1/\eta_p}$$

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\eta_p(\gamma-1)/\gamma} = T_3\left(\frac{\sqrt{r}}{r}\right)^{\eta_p(\gamma-1)/\gamma} = T_1\,\theta\, r_t^{-\eta_p/2}$$

SFEE applied to the mixing process (note that $T_6 = T_2$ because there is no temperature change in the throttling process) :

$$c_p T_{mix} = m c_p T_2 + (1-m) c_p T_4$$

Hence,

$$\frac{T_{mix}}{T_1} = m r_t^{1/\eta_p} + (1-m)\theta r_t^{-\eta_p/2}$$

[25%]

(b) Compressor work input :

$$w_c = c_p(T_2 - T_1) = c_p T_1 (r_t^{1/\eta_p} - 1)$$

Turbine work output :

$$w_t = (1-m)c_p(T_3 - T_4) + c_p(T_{mix} - T_7)$$

For the turbine expansion :

$$T_7 = T_{mix}\left(\frac{p_7}{p_{mix}}\right)^{\eta_p(\gamma-1)/\gamma} = T_{mix}\left(\frac{1}{\sqrt{r}}\right)^{\eta_p(\gamma-1)/\gamma} = T_{mix}\, r_t^{-\eta_p/2}$$

Hence,

$$w_t = (1-m)c_pT_1\theta(1-r_t^{-\eta_p/2}) + c_pT_{mix}(1-r_t^{-\eta_p/2})$$

$$= c_pT_1(1-r_t^{-\eta_p/2})[(1-m)\theta + mr_t^{1/\eta_p} + (1-m)\theta r_t^{-\eta_p/2}]$$

$$= c_pT_1(1-r_t^{-\eta_p/2})[(1-m)\theta(1+r_t^{-\eta_p/2}) + mr_t^{1/\eta_p}]$$

Heat input : $q = (1-m)c_p(T_3-T_2) = (1-m)c_pT_1(\theta - r_t^{1/\eta_p})$

Hence, overall GT efficiency is,

$$\eta_{GT} = \frac{w_t - w_c}{q} = \frac{(1-r_t^{-\eta_p/2})[(1-m)\theta(1+r_t^{-\eta_p/2}) + mr_t^{1/\eta_p}] - (r_t^{1/\eta_p}-1)}{(1-m)(\theta - r_t^{1/\eta_p})}$$

Substituting the given values :

$$r_t = r^{(\gamma-1)/\gamma} = 40^{0.37/1.37} = 2.70815$$

$$r_t^{-\eta_p/2} = 2.70815^{-0.9/2} = 0.63870\ ; \quad r_t^{1/\eta_p} = 2.70815^{1/0.9} = 3.02515$$

$$\eta_{GT} = \frac{(1-0.63870)[(1-0.1)\times5\times(1+0.63870) + 0.1\times3.02515] - (3.02515-1)}{(1-0.1)(5-3.02515)} = 0.421$$ [30%]

Setting $m = 0$,

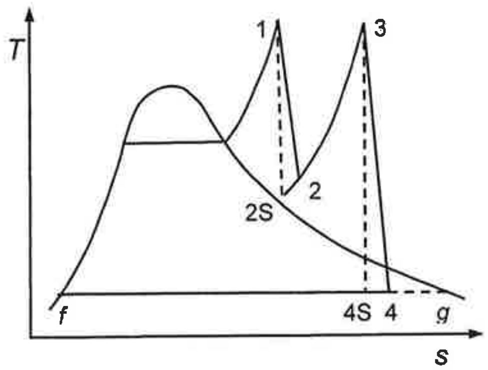
$$(\eta_{GT})_{m=0} = \frac{(1-0.63870)\times5\times(1+0.63870) - (3.02515-1)}{(5-3.02515)} = 0.474$$

Hence, there is a reduction of 5.3 percentage points due to the cooling. [10%]

Note: it is also possible to use: $\eta_{GT} = 1 - q_{out}/q$ where $q_{out} = c_pT_1(T_7/T_1 - 1)$.

(c) The overall gas turbine efficiency increases with increase in turbine inlet temperature. Unfortunately, the material strength of the turbine blades is such that they cannot withstand such high temperatures. The first and (often) second stage blades of the turbine must therefore be cooled using air taken from the compressor. Although this incurs a substantial penalty (as shown by the calculation above), the overall efficiency is still better than if the turbine inlet temperature were reduced to a level where cooling was not required.
Higher turbine inlet temperature also leads to a higher specific work output (and therefore a smaller gas turbine for a required power output). [20%]

4.



(a) (i) For the LP turbine,

$$\eta_{LPT} = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{h_3 - [y_4 h_f + (1 - y_4) h_g]}{h_3 - h_{4S}} = 0.9 - y_4$$

Hence we need the value of h_{4S}. Thus,

$$y_{4S} = \frac{s_g - s_{4S}}{s_g - s_f} = \frac{s_g - s_3}{s_g - s_f} = \frac{8.329 - 7.086}{8.329 - 0.521} = 0.1592$$

$$\therefore \quad h_{4S} = y_{4S} h_f + (1 - y_{4S}) h_g = 0.1592 \times 151.5 + (1 - 0.1592) \times 2566.6 = 2182.1 \text{ kJ/kg}$$

Rearranging the equation for η_{LPT} gives,

$$y_4 = \frac{0.9(h_3 - h_{4S}) - (h_3 - h_g)}{(h_g - h_f) + (h_3 - h_{4S})} = \frac{0.9(3344.8 - 2182.1) - (3344.8 - 2566.6)}{(3344.8 - 2182.1) + (2566.6 - 151.5)} = 0.07497$$

Hence,

$$h_4 = y_4 h_f + (1 - y_4) h_g = 0.07497 \times 151.5 + (1 - 0.07497) \times 2566.6 = 2385.5 \text{ kJ/kg}$$ [40 %]

(ii) For the HP turbine we need to find h_2. We have,

$$s_{2S} = s_1 = 6.303 \text{ kJ/kg.K}$$

Interpolating in the tables (or from the steam chart), h_{2S} = 2858.3 kJ/kg. Then,

$$h_2 = h_1 - \eta_{HPT}(h_1 - h_{2S}) = 3209.8 - 0.9 \times (3209.8 - 2858.3) = 2893.4 \text{ kJ/kg}$$

Work output from HP and LP turbines,

$$w = (h_1 - h_2) + (h_3 - h_4) = (3209.8 - 2893.4) + (3344.8 - 2385.5) = 1275.7 \text{ kJ/kg}$$ [15 %]

Heat input in steam generator and reheater (neglecting change in enthalpy across feed pump),

$$q = (h_1 - h_f) + (h_3 - h_2) = (3209.8 - 151.5) + (3344.8 - 2893.4) = 3509.7 \text{ kJ/kg}$$

Steam cycle efficiency (neglecting the feed pump work input),

$$\eta_{\text{ST}} = \frac{w}{q} = \frac{1275.7}{3509.7} = 0.363$$ [10 %]

(b) SFEE applied to the HRSG :

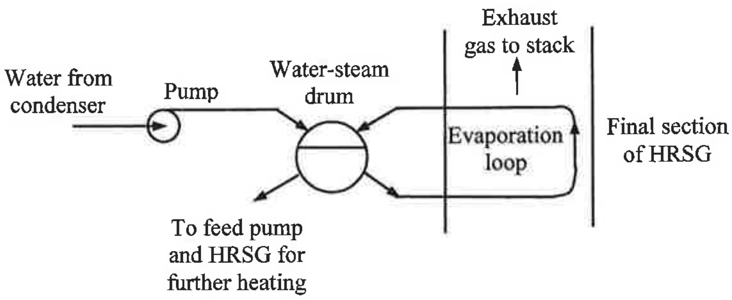
$$\dot{m}_g c_p (T_{in} - T_{ex}) = \dot{m}_s[(h_1 - h_f) + (h_3 - h_2)] = \dot{m}_s q$$

$$\therefore \quad T_{ex} = T_{in} - \frac{\dot{m}_s \, q}{\dot{m}_g \, c_p} = 540 - \frac{3509.7}{8 \times 1.10} = 141.2 \text{ °C}$$

HRSG efficiency,

$$\eta_{\text{B}} = \frac{\dot{m}_g c_p (T_{in} - T_{ex})}{\dot{m}_g c_p (T_{in} - T_{amb})} = \frac{540 - 141.2}{540 - 20} = 0.770$$ [15 %]

(c)



The diagram shows how condensation in the final section of the HRSG might be avoided. Without the water-steam drum the water enters at condenser temperature which is below the dewpoint of the H_2O in the flue gas so condensation on the outside of the tubes occurs. If the pump compresses the water to, say, atmospheric pressure then the water-steam mixture in the drum must be at the saturation temperature and hence saturated liquid water enters the evaporation loop at 100 °C. This is above the dewpoint of the H_2O in the flue gas so condensation on the outside of the tubes is avoided. [20 %]

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MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

1. (b) (ii) 220.4 MW, 37.5 MW, 259.2 MW (isentropic turbine)
   (iv) 295.9 MW, −2.2 MW (feed pump), 24.1 MW (throttle), 0.737

2. (a) $c_p = A + BT + 12KpT^{-4}$

3. (b) 0.421, 0.474

4. (a) (i) 2385.5 kJ/kg
   (ii) 1275.7 kJ/kg, 0.363
   (b) 141 °C, 0.770

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Draft Version 8 – for read-through meeting