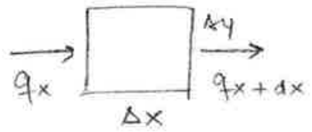
3A6 - 2012

Q1.



ENERGY CONSERVATION IN SOLID :

$$\frac{\partial E}{\partial t} = q$$

$$\Delta V \frac{\partial}{\partial t}(\rho c T) = q_x \Delta y \Delta z - q_{x+dx} \Delta y \Delta z$$

$$= \left(q_x - \frac{\partial q_x}{\partial x}\Delta x\right)\Delta y \Delta z = -\frac{\partial q_x}{\partial x} \underbrace{\Delta x \Delta y \Delta z}_{\Delta V}$$

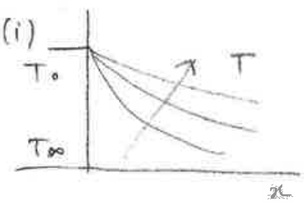
$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}\left(-\lambda \frac{\partial T}{\partial x}\right) = \lambda \frac{\partial^2 T}{\partial x^2}$$

IF WE INCLUDE CONTRIBUTIONS FROM y AND z DIRECTIONS, WE HAVE :

$$\rho c \frac{\partial T}{\partial t} = \lambda \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$

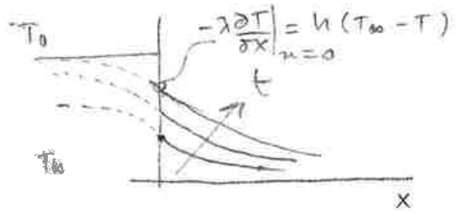
$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] = \alpha \nabla^2 T$$

(b) (i)



(ii)

(iii)



(c)(i) $\dfrac{T_0 - T}{T_0 - T_\infty} = \dfrac{350 - 310}{350 - 300} = 0.8 = \mathrm{erf}(\eta)$

$\eta = \dfrac{x}{2\sqrt{\alpha t}} = 0.9062$ (by interpolation)

$\dfrac{0.015\ \mathrm{m}}{2\sqrt{\alpha (50\ \mathrm{s})}} = 0.9062 \longrightarrow \alpha = 1.26 \times 10^{-7}\ \mathrm{m^2/s}$

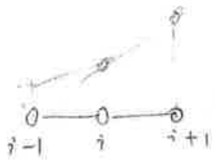
$\alpha = \dfrac{\lambda}{\rho c} = \dfrac{\lambda}{(1320\ \mathrm{kg/m^3})(2160\ \mathrm{J/kg\,K})}$

$\lambda = 0.3617\ \mathrm{W/mK}$

(ii) THERE WOULD BE A LAG RELATIVE TO CASE (i), DUE TO THE TRANSFER OF HEAT BETWEEN ∞ AND WALL, SO THE TEMPERATURE WOULD BE LOWER.

CASE (iii) IS REALISTIC ; CASE (i) WOULD BE DIFFICULT TO REALISE IN PRACTICE.

(d)



$$\frac{\partial T}{\partial t} \cong \frac{T_i^{p+1} - T_i^p}{\Delta t}$$

$$\frac{\partial^2 T}{\partial x^2} \simeq \frac{\left.\frac{\partial T}{\partial x}\right|_{i+1/2} - \left.\frac{\partial T}{\partial x}\right|_{i-1/2}}{\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} \simeq \frac{\frac{T_{i+1}^p - T_i}{\Delta x} - \frac{T_i^p - T_{i-1}}{\Delta x}}{\Delta x} \simeq \frac{T_{i+1}^p - 2T_i^p + T_{i-1}^p}{\Delta x^2}$$

$$\frac{1}{\alpha}\,\frac{T_i^{p+1} - T_i^p}{\Delta t} = \frac{T_{i+1}^p - 2T_i^p + T_{i-1}^p}{\Delta x^2}$$

SOLVING FOR $T_i^{p+1} = \dfrac{\alpha \Delta t}{\Delta x^2}\left(T_{i+1}^p + T_{i-1}^p\right) + \left(1 - 2\dfrac{\alpha \Delta t}{\Delta x^2}\right) T_i^p$

1

(a) MASS CONSERVATION

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} = 0 \qquad \frac{\partial u}{\partial x} = 0 \qquad u = u(y), \quad v = 0$$

MOMENTUM:

$$u\cancel{\frac{\partial u}{\partial x}} + \cancel{v}\frac{\partial u}{\partial y} = \cancel{-\frac{1}{\rho}\frac{\partial p}{\partial x}} + \nu\frac{\partial^2 u}{\partial y^2} + \cancel{\nu\frac{\partial^2 u}{\partial x^2}}$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \rightarrow u = Ay + B$$

$$\left.\begin{aligned} u(0) &= 0 \\ u(\delta) &= U \end{aligned}\right\} \quad u = U\frac{y}{\delta}$$

$$\boxed{u = \omega R \frac{y}{\delta}}$$

(b) (i)

$$\cancel{u\frac{\partial T}{\partial x}} + \cancel{v\frac{\partial T}{\partial y}} = \alpha\left(\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu\phi}{\rho c_p}$$

$$\mu\phi = \tau_{xx}\frac{\partial u}{\partial x} + \tau_{yy}\cancel{\frac{\partial v}{\partial y}} + \tau_{xy}\frac{\partial u}{\partial y} + \tau_{yx}\frac{\partial v}{\partial x} = \mu\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right)^2$$

$$\mu\phi = \mu\left(\frac{\partial u}{\partial y}\right)^2 = \mu\left(\frac{\omega R}{\delta}\right)^2$$

THE GOVERNING EQUATION IS THEREFORE:

$$\alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\phi = 0$$

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{\alpha\rho c_p}\left(\frac{\omega R}{\delta}\right)^2$$

integrating:

$$T(y) = -\frac{\mu}{\alpha\rho c_p}\left(\frac{\omega R}{\delta}\right)^2\frac{y^2}{2} + Ay + B$$

BOUNDARY CONDITIONS:

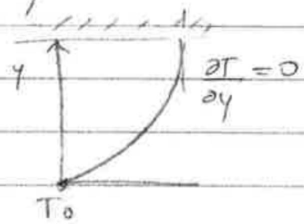
$$T(0) = T_0 \longrightarrow B = T_0$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=\delta} = 0 \rightarrow -\frac{\mu}{\alpha \rho c_p}\left(\frac{\omega R}{\delta}\right)^2 \delta + A = 0$$

$$A = \frac{\mu}{\alpha \rho c_p}\left(\frac{\omega R}{\delta}\right)^2 \delta \qquad \alpha \rho c_p = k$$

$$T(y) = T_0 + \frac{\mu}{k}\left(\frac{\omega R}{\delta}\right)^2 \left(\delta y - \frac{y^2}{2}\right)$$

$$\boxed{T(y) = T_0 + \frac{\mu}{k}(\omega R)^2 \left(\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right)}$$



(ii) $$q_s = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$= -k \frac{\mu}{k}(\omega R)^2 \left.\left(\frac{1}{\delta} - \frac{y}{\delta}\right)\right|_{y=0}$$

$$q_s = -\mu \frac{(\omega R)^2}{\delta} = \mu \phi \delta = \mu \int_0^y \phi \, dy$$

since ϕ = const across y.

(c) Same equation, different boundary conditions:

$$T(y) = -\frac{\mu}{k}\left(\frac{\omega R}{\delta}\right)^2 \frac{y^2}{2} + Ay + B$$

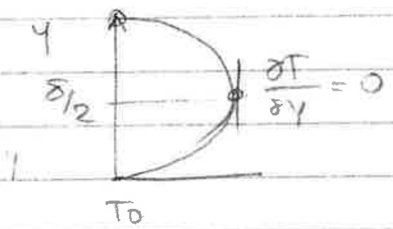
$$T(0) = T_0$$

By symmetry: $q_0 = q_\delta \rightarrow \left.\frac{\partial T}{\partial y}\right|_{y=\delta/2} = 0$

$$\left[-\frac{\mu}{k}\left(\frac{\omega R}{\delta}\right)^2 y + A\right]_{y=\delta/2} = 0$$

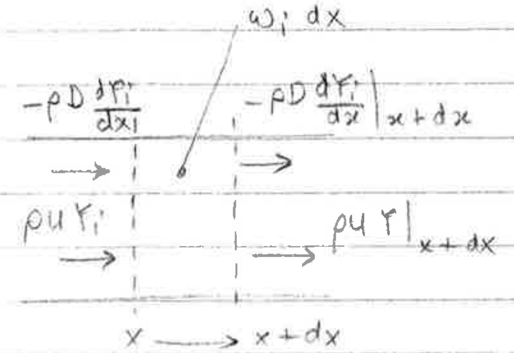
$$A = \frac{\mu}{k}\left(\frac{\omega R}{\delta}\right)^2 \frac{\delta}{2}$$

$$T(y) = T_0 + \frac{1}{2}\frac{\mu}{k}\left(\frac{\omega R}{\delta}\right)^2 \left(\delta y - y^2\right)$$



(d) Starting from ambient temperature T_0, the fluid will warm up due to dissipative heating. Initially, both shaft and outer wall will be cold and heat will be transferred uniformly to both. As the fluid heats up, the inner surface of the shaft will rise, and the boundary conditions for the fluid will depend on the conduction through the material of the shaft, via $k_s \left.\frac{\partial T}{\partial r}\right|_s = k \left.\frac{\partial T}{\partial r}\right|_F$.

(a)



$$j_i = -\rho D \frac{dY_i}{dx}$$

THE BALANCE ACROSS THE ELEMENT PER UNIT AREA NORMAL TO X UNDER STEADY STATE IS:

$$\rho u Y_i - \left(\rho u Y_i + \frac{d}{dx}(\rho u Y_i)\,dx\right)$$
$$-\left(\rho u \frac{\partial Y_i}{dx}\right) - \left(-\rho D \frac{dY_i}{dx_i} - \frac{d}{dx}\left(\rho D \frac{dY_i}{dx}\right) dx\right) +$$
$$+\ \dot{\omega}_i\, dx = 0$$

EXPANDING AND CANCELLING TERMS, WE HAVE;

$$\frac{d}{dx}(\rho u Y_i) = \frac{d}{dx}\left(\rho D \frac{dY_i}{dx}\right) + \dot{\omega}_i$$

For any species i.

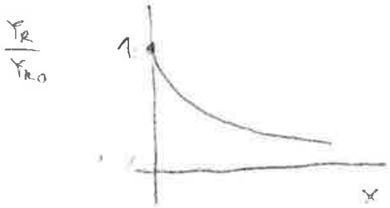
For constant ρ, u, D; FOR THE REACTANT

$$\rho u \frac{dY_R}{dx} = \rho D \frac{d^2 Y_R}{dx^2} - \omega_R$$

where $\omega_i = -\omega_R = -k\rho^2 Y_R Y_D$ FOR THE REACTANT (b)(i) ASSUME DIFFUSION IS NEGLIGIBLE:

$$\rho u \frac{dY_R}{dx} = \omega_R = -k\rho^2 Y_{D,0} Y_R$$

$$\ln \frac{Y_R}{Y_{R,0}} = -\frac{k\rho^2}{\rho u} Y_{D,0}\, x$$



(ii) All 3 TERMS:

$$\rho u \frac{dY_R}{dx} - \rho D \frac{d^2 Y_R}{dx^2} - k\rho^2 Y_{D,0} Y_R = 0$$

THIS IS A SECOND-ORDER EQUATION FOR Y_R WITH CONSTANT COEFFICIENTS

$a = \rho D$
$b = -\rho u$
$c = k\rho^2 Y_{D,0}$

WHICH CAN BE SOLVED USING $Y_R = A e^{-\lambda_1 x} + B e^{-\lambda_2 x}$

WHERE λ_1, λ_2 ARE THE ROOTS TO

$$a\lambda^2 + b\lambda + c = 0$$

AND BOUNDARY CONDITIONS $Y_R(0) = Y_{R,0}$

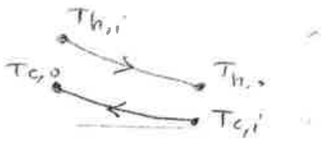
$$\rho u Y_R(0) - \rho D \left.\frac{dY_R}{dx}\right|_0 = \dot{m}_{R,0}$$

(d)

IF THE DYE CONCENTRATION CHANGES, WE MUST solve the equations for the dye and the REACTANT SIMULTANEOUSLY. THIS CAN USUALLY only be done numerically due to the non-linearity in the product $Y_a \cdot Y_b$.

Q4

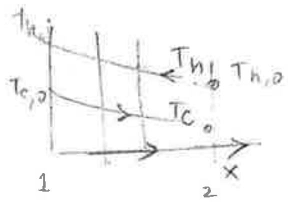
(a) MAXIMUM RATE OF HEAT TRANSFER

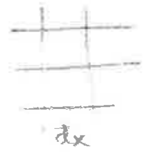


$$Q_{max} = C_{min}(T_{h,i} - T_{c,i}) = C_h (T_{h,i} - T_{c,i})$$ (ie possible for infinite HX area)

(b) $$\varepsilon = \frac{Q}{Q_{max}} = \frac{C_h (T_{h,i} - T_{h,o})}{C_h (T_{h,i} - T_{c,i})} = \frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{c,i})}$$

(c)





$$C_h \frac{dT_h}{dx} = -U(T_h - T_c)P \qquad dT_h = -\frac{U}{C_h}(T_h - T_c)\,dA$$

$$C_c \frac{dT_c}{dx} = -U(T_h - T_c)P \qquad dT_c = -\frac{U}{C_c}(T_h - T_c)\,dA$$

$$d(T_h - T_c) = -\left(\frac{1}{C_h} - \frac{1}{C_c}\right)(T_h - T_c)\,U\,dA$$

$$d\ln(T_h - T_c) = -\left(\frac{1}{C_h} - \frac{1}{C_c}\right)U\,dA$$

$$\ln\frac{(T_{h,o} - T_{c,i})}{(T_{h,i} - T_{c,o})} = -\frac{UA}{C_h}\left(1 - \frac{C_h}{C_c}\right) = -n(1 - C_r)$$

(d) $$T_{h,i} - T_{c,o} = (T_{h,i} - T_{c,i}) - (T_{c,o} - T_{c,i}) = \frac{(T_{h,i} - T_{h,o})}{\varepsilon} - (T_{c,o} - T_{c,i})$$

BUT $(T_{h,i} - T_{h,o})C_h = C_c (T_{c,o} - T_{c,i}) \rightarrow$

$$(T_{h,i} - T_{c,o}) = \frac{T_{h,i} - T_{h,o}}{\varepsilon} - (T_{h,i} - T_{h,o})C_r = (T_{h,i} - T_{h,o})\left(\frac{1}{\varepsilon} - C_r\right)$$

$$T_{h,o} - T_{c,i} = (T_{h,o} - T_{h,i}) + (T_{h,i} - T_{c,i}) = (T_{h,i} - T_{h,o})\left(-1 + \frac{1}{\varepsilon}\right)$$

$$\therefore \ln\left[\frac{1/\varepsilon - 1}{\frac{1}{\varepsilon} - C_r}\right] = \exp(-n(1 - C_r))$$

$$\frac{1 - \varepsilon}{1 - C_r\varepsilon} = \exp(-n(1 - C_r))$$

$$1-\varepsilon = \left(1 - C_r\,\varepsilon\right)\exp\left(-n(1-C_r)\right)$$

$$1 - \exp\left(-n(1-C_r)\right) = \varepsilon\left(1 - C_r\exp\left(-n(1-C_r)\right)\right)$$

$$\varepsilon = \frac{1-\exp\left(-n(1-C_r)\right)}{1-C_r\exp\left(-n(1-C_r)\right)}$$

AS $C_r \to 0$, THERE IS INFINITE cooling flow, so

$T_{c,o} \approx T_{c,i}$ AND $\varepsilon = 1 - \exp(-n)$ only

depends on QA/C_h

(d) we CAN USE THE MOMENTUM - HEAT TRANSFER ANALOGY ; FOR EITHER LAMINAR OR TURBULENT FLOWS

$$\frac{C_f}{2} Re = Nu \quad (LAM) \qquad OR \qquad \frac{C_f}{2} Re\, Pr^{1/3} = Nu$$

ACCEPTABLE ANSWERS ALSO $Nu = Nu\left(C_f(Re), Pr\right)$ OR WORDS TO THAT EFFECT.

(e) multi-pass heat exchangers

Turbulence - enhancing features

Fins

Help heat exchange, at the expense of extra pressure drop.