1 (a)

$$\text{Gain, } G = \frac{\text{Max. power radiated per unit area}}{\text{power per unit area from isotropic ant.}}$$

Effective Aperture, A_e : power delivered to a matched load by
(Area) an antenna $= A_e \times$ power density in incident radio wave

Radiation Resistance, R_r : power radiated $= \frac{1}{2} I^2 R_r$, where I is current feed to antenna

Polarisation describes the plane in which the electric field is aligned, orthogonal to the direction of wave propagation

Note: antenna eqn. $G = \dfrac{4\pi A_e}{\lambda^2}$

[20%]

(b) $v = f\lambda$ with $v = c = 3\times10^8 \text{ ms}^{-1}$ $\therefore$ at 172 MHz

$$\lambda = \frac{3\times10^8}{172\times10^6} = 1{\cdot}744\text{m} \quad \therefore \text{ each element} = 0{\cdot}436\text{m}$$

Assuming gain of ¼-wave dipole to be 1·5, power density 2km away $= \dfrac{30\times10^{-3} . 1{\cdot}5}{4\pi\, 2000^2} = 8{\cdot}95\times10^{-10} \text{ Wm}^{-2}$

[20%]

(c) For receiving antenna, $A_e = G\lambda^2/4\pi$ with $G = 1{\cdot}5$, $\lambda = 1{\cdot}744$ m

$\therefore A_e = 0{\cdot}363 \text{ m}^2$

$\therefore$ Power received $= 8{\cdot}95\times10^{-10} . 0{\cdot}363 = 3{\cdot}25\times10^{-10}$ W

1(c) contd.

$$P_{rec} = V_r^2/R \quad \text{where } R = 50\,\Omega$$

$$\therefore \underline{V_r = 1.27 \times 10^{-4} \text{ V rms}}$$

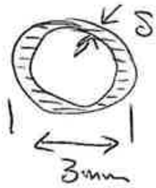
[20%]

(d) skin depth in stainless steel @ 172 MHz.

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} \qquad \begin{aligned} \rho &= 7.2 \times 10^{-7}\ \Omega m \\ \omega &= 2\pi f = 1.08 \times 10^{9} \text{ rad/s} \\ \mu &= 4\pi \times 10^{-7}\ \text{H/m} \end{aligned}$$

$$\therefore \delta = 3.26 \times 10^{-5} \text{ m}$$

$\therefore$ resistance given by conducting cross-section over antenna length



$$R_{ohmic} = \frac{\rho L}{A} = \frac{7.2 \times 10^{-7} \cdot 1.744/2}{2\pi \cdot 1.5 \times 10^{-3} \cdot 3.26 \times 10^{-5}} = 2.05\ \Omega$$

$$\text{Hence radiation efficiency} = \frac{R_r}{R_r + R_{ohmic}} = \frac{50}{52.05} = 0.96$$

$\underline{\text{or } 96\%}$

(taking R_r to be matched to $50\,\Omega$).

[25%]

(e) $\underset{\text{per unit area}}{P} = \frac{1}{2} E^2/\eta \quad$ where $\eta = 120\pi$ (impedance of free space)

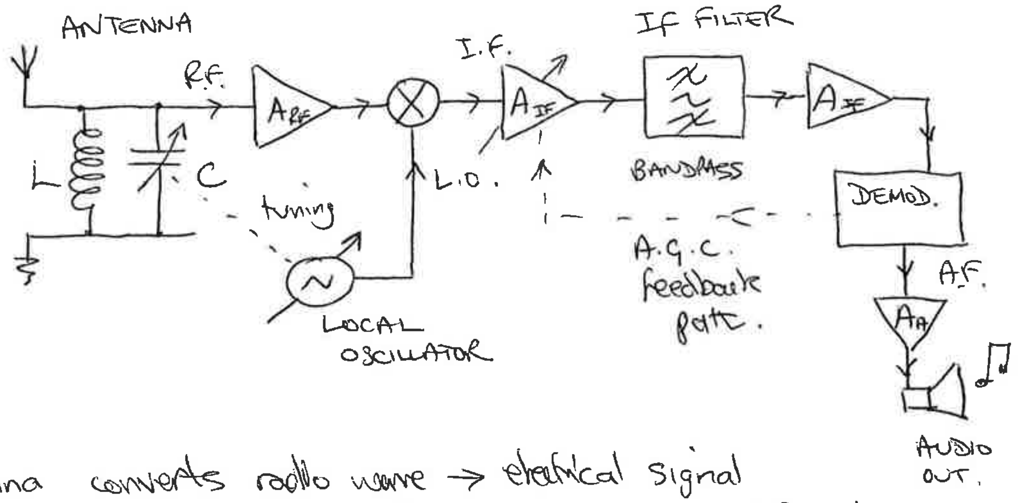
$$\therefore E = \left(120\pi \cdot 2 \cdot 8.95 \times 10^{-10}\right)^{1/2} = 8.21 \times 10^{-4}\ \text{Vm}^{-1}$$

$$\therefore V_{sig} = \frac{1.744}{2} \times 8.21 \times 10^{-4} = 7.16 \times 10^{-4} \text{ V} = \underline{5 \times 10^{-4} \text{ Vrms}}$$

This is higher than part(c) result as the antenna is unloaded (open cct.) and potential distribution is rather optimistic for a simple dipole.

[15%]

2(a)

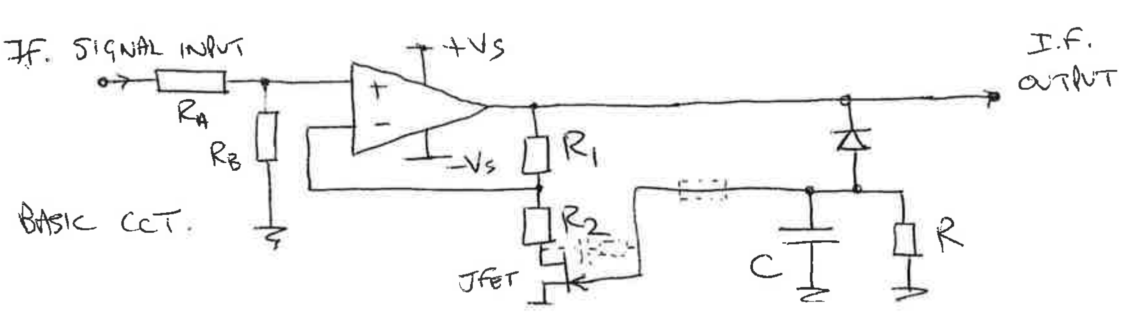


- Antenna converts radio wave → electrical signal
- LC front end selects signal freq. and magnifies it's amplitude by Q-factor
- A_{RF} boosts RF signal to feed into mixer
- Mixer multiplies RF signal by LO to give sum and difference frequencies (from cos x cos trig relation)
- A_{IF} amplifies the mixer output (and b/w can remove sum freq.)
- IF filter selects difference freq.: I.F. = (L.O. − R.F.)
- A IF amplifies IF signal before demodulation, which eg: with envelope detection for AM, recovers original signal, A.F.
- This is then amplified and output as audio.
- Tuning is effected by sweeping LO freq. and LC (RF). freq. together, maintaining the IF. fixed separation.
- AGC can be achieved by using dc from demodulator to control IF. gain.

[20%]

(b)

Use JFET as variable resistance element in op-amp circuit:



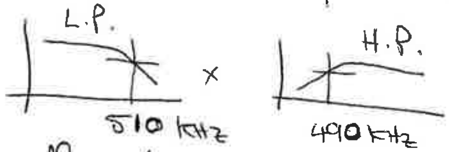
2(b) cont'd

to maximise JFET small sig. linearity, the dotted components can be added $R = 1M\Omega$, $C = 100nF$: adds $\frac{V_{DS}}{2}$ to V_{gs}.

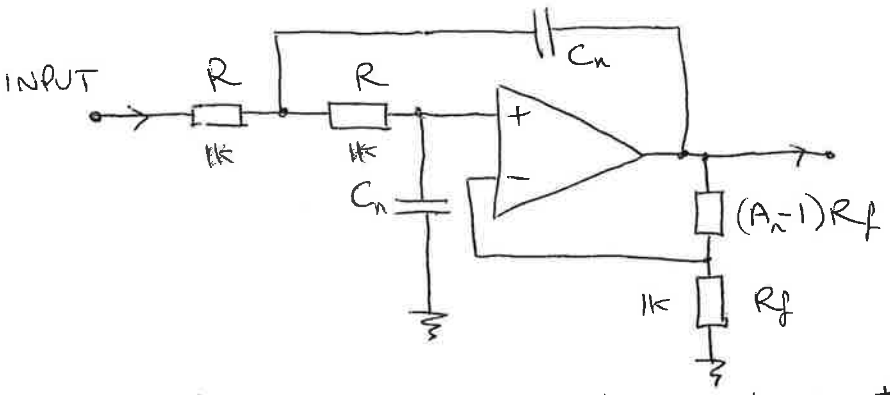
- 1st ly R_A and R_B divide signal by 10 $R_A = 9k\Omega$, $R_B = 1k\Omega$
  - this is because the op-amp cct has gain ≥ 1 and we need x0.1

- select R_2 and R_1 to give max gain of 200 for op-amp
  $\therefore$ $R_1 = 200k\Omega$, $R_2 = 1k\Omega$
- the conduction resistance of JFET (depletion mode) sets gain.
- the diode and RC circuit gives a smoothed -ve bias to the gate: choose $10 = \frac{1}{2\pi RC}$ for response time eg:
  $\therefore$ $R = 10k\Omega$
  $C = 1.6\mu F$ [35%]

(c) we want bandpass filter $\therefore$



choose chebyshev for sharpest cut-off (20kHz in 500 kHz is quite sharp) - and ripple in gain doesn't matter much here.



LOW PASS SECTIONS (x2) followed by HIGH PASS SECTIONS (x2)

For high pass, swap R's and C's at input / feedback and cascade.

2(c) contd.

For Low pass $\quad f_{-3dB} = 510 \times 10^3 = \dfrac{1}{2\pi f_n RC}$

$\therefore$ with $R = 1\,k\Omega$ throughout $= R_f \,100$

stage 1 : $\quad C_1 = 1\,nF \quad , \quad R_1 = 523\,\Omega$
LP1 $\quad (A_1 - 1)R_f = 582\,\Omega \quad , \quad R_f = 1\,k\Omega$

stage 2 : $\quad C_2 = 1\,nF \quad , \quad R_2 = 303\,\Omega$
LP2 $\quad (A_2 - 1)R_f = 1{\cdot}66\,k\Omega \quad , \quad R_f = 1\,k\Omega$

for high pass $\quad f_{-3dB} = 490\,kHz = \dfrac{f_n}{2\pi RC}$

stage 3 : $\quad C_3 = 1\,nF \quad , \quad R_3 = 194\,\Omega$
HP3 $\quad (A_3 - 1)R_f = 582\,\Omega \quad , \quad R_f = 1\,k\Omega$

stage 4 : $\quad C_4 = 1\,nF \quad , \quad R_4 = 335\,\Omega$
HP4 $\quad (A_4 - 1)R_f = 1{\cdot}66\,k\Omega \quad , \quad R_f = 1\,k\Omega$

[35%]

(d) 'Tracking' is the term which describes how well the RF resonant LC peak freq. moves with the L.O. freq. to give a constant I.F. If they misalign, then the I.F. signal freq. does not match the fixed I.F. filter – and the signal is attenuated. A low Q-factor gives a wider LC bandwidth, ~~&~~ so tracking is not so critical – but the signal magd. is lower.

[10%]

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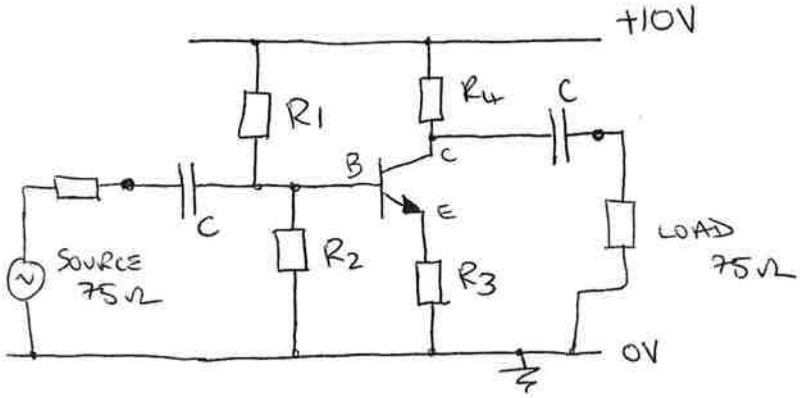
3(a)



- Choose $R_4 = 75\Omega$ for match impedance
- 10 dB gain = $\times 3$ linear gain loaded = $\times 6$ unloaded at output

  $\therefore \; R_3 \simeq \frac{75}{6} \; \therefore$ say 12Ω

- with collector biased to 5V dc., $I_C = \frac{5}{75} = 67\,mA$

  $\therefore \; r_e = \frac{0.025}{0.067} = 0.37\Omega$

  $\therefore \; |gain| = 75/12.4 = 6.06$ and $V_E = 0.8V$

- choose R_1 and R_2 to give $V_B = V_E + 0.65V \approx 1.5V$ dc.

  $\therefore$ with $R_2 = 100\Omega$ $(\approx 1.5 \times 75)$, $R_1 = 560\Omega$

  $$R_{in} = \left(\frac{1}{100} + \frac{1}{560} + \frac{1}{12 \times 250}\right)^{-1} = 82\Omega. \text{ ok.}$$

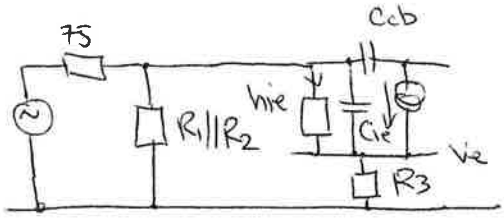
- Choose C to be large eg: 100nF. $\left(\frac{1}{2\pi f C} @ 172MHz = 0.01\Omega\right)$

[25%]

(b) with $f_t = 18\,GHz$ and $r_e = 0.38\Omega$, $C_{ie} = 23\,pF$

$$f_t = \frac{1}{2\pi r_e C_{ie}}$$

Input circuit in small sig. model



3(b) Contd.

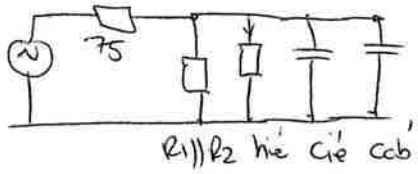
$$V_e = \frac{R_3}{R_3 + r_e} \cdot V_i = 0.97$$

From Miller effect $C_{cb} \rightarrow C_{cb} \times (1+G)$

when loaded $G = 3$ $\therefore$ C_{cb}' referred to ground $= 0.80\,pF$

and C_{ie} referred to ground is $\div \left[\frac{1}{(1-0.97)}\right]$ due to -ve feedback from emitter node. $\therefore$ $C_{ie}' = \frac{23}{33} = 0.70\,pF$.

So input s/sm. $\simeq$



$h_{ie}' = h_{fe} \times r_e \times 33 \approx 3k\Omega$.

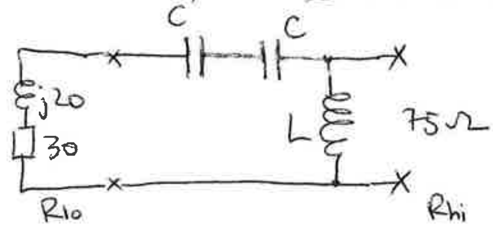
So, roll off at $\approx \frac{1}{2\pi R'C'} = f_{-3dB}$

$R' = 75 \| 82 = 39\,\Omega$
$C' = C_{ie}' + C_{cb}' = 1.5\,pF$

$$f_{-3dB} = 2.72\text{ GHz}$$

[35%]

(c)



$f = 172$ MHz

First, cancel $j20$ with C' giving $-j20$ $\Rightarrow$ $20 = \frac{1}{2\pi f C'}$

$\therefore C' = 46.3\,pF$

Then use filter design eqns:

$$Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{lo}} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1} = \sqrt{\frac{75}{30} - 1} = 1.22$$

$\therefore X_p = 75/1.22 = 61.5 = 2\pi f L$ $\therefore$ $L = 57\,nH$

$X_s = 1.22 \times 30 = 36.7 = \frac{1}{2\pi f C}$ $\therefore$ $C = 25\,pF$

So we can combine C and C' into 1 capacitor $\left(\frac{1}{25} + \frac{1}{46}\right)^{-1} \Rightarrow 16\,pF$

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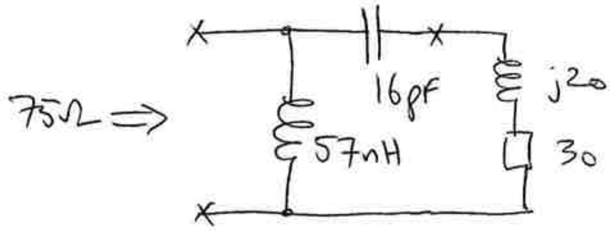
3(c) contd

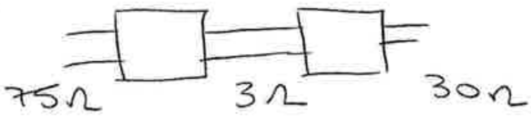
So, matching circuit is this :-



[30%]

(d) To act as a filter, we can transform the impedance to an intermediate value to get high Q values

eg:



Then, the match is quite sharp with freq. eg: Q ~ 5 to 10.

or can cascade more stages with even higher Q's. [10%]

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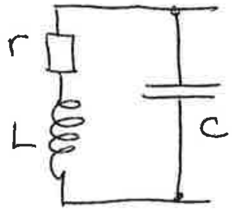
4 (a)



$$f_{res.} = \frac{1}{2\pi\sqrt{LC}}$$

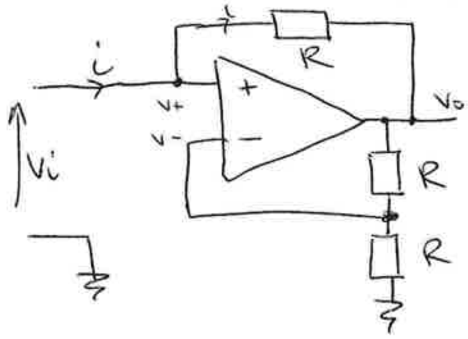
$$Q = \frac{\omega L}{r} = \frac{2\pi f L}{r}$$

with $f = 172 \times 10^{6}$ Hz and $C = 18 \times 10^{-12}$ F $\Rightarrow L = 47.6$ nH

$\therefore Q = 25.7$ [20%]

So the bandwidth $= \frac{172}{25.7}$ MHz $=$ 6.69 MHz.

(b)



$$V_+ = V_- = V_i$$

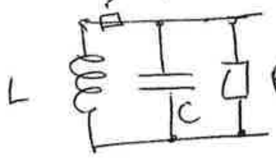
$$i = \frac{V_i - V_o}{R}$$

and $V_- = \frac{V_o}{2} = V_i$

$\therefore i = \frac{V_i - 2V_i}{R} = -\frac{V_i}{R}$ $\quad \therefore Z_{in} = \frac{V_i}{i_i} = -R$ negative resistance.

This can be used to increase Q-factor (but not too far or it will oscillate).

we can refer the series resistance r to a parallel resistance R



$Q = \frac{R}{\omega L} = \left(\frac{r}{\omega L}\right)^{-1}$ $\quad \therefore R = \frac{(\omega L)^2}{r}$

and with $R' \parallel -R$ we can increase the effective value of R'

eg: $5k\Omega \parallel -10k\Omega \Rightarrow 10k\Omega$

so doubles Q.

[15%]

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4(c) $33\Omega + 10nH$ @ 960MHz

$= 33 + j60\,\Omega \quad \div 50 \Rightarrow 0.66 + j1.2$

normalised. point 'A'

length of stripline: $(0.176\lambda - 0.153\lambda) = 0.026\lambda$

series capacitor to cancel $j1.5 \Rightarrow -j75\,\Omega$ at 960MHz

So $75 = \dfrac{1}{2\pi . 960\times10^6 . C}$ $\quad \therefore C = 2.21\,pF$

If the freq. changes to 1060 MHz, then the length of stripline $= 0.026\lambda \times \dfrac{1060}{960} = 0.029\lambda$

and C has a normalised impedance of $\dfrac{-j68}{50} = -j1.36$

Also, the input impedance will be $0.66 + j1.33$

'x' @ $0.159\lambda \rightarrow 0.159\lambda + 0.029\lambda = 0.188\lambda$.

$\therefore$ 'x' $\rightarrow 1.15 + j1.85 - j1.36$

$= 1.15 + j0.49$

$\therefore \rho = 0.22$ from radial scale [40%]

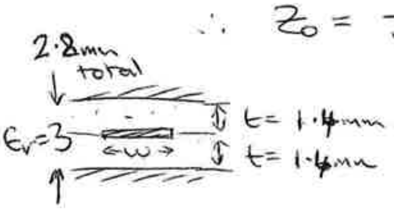
(d) If $\varepsilon_r = 3$ then $v_{line} = \dfrac{3\times10^8}{\sqrt{3}} = 1.73\times10^8\ ms^{-1}$

$\therefore \lambda$ @ 960MHz $= 0.18$ m

$\therefore 0.026\lambda = 4.7$ mm

For 50Ω track: $v_{line} = \dfrac{1}{\sqrt{L}\sqrt{C}}$ and $Z_0 = \dfrac{\sqrt{L}}{\sqrt{C}}$

$\therefore Z_0 = \dfrac{1}{v_{line} C}$ and $C = \dfrac{2(w + 2t)\,\varepsilon_0\varepsilon_r}{t}$ includes fringing fields



$\therefore 50 = \dfrac{1.4}{1.73\times10^8 . 2(w + 2.8) . 8.854\times10^{-12} . 3}$

$\therefore w = 0.24$ mm [25%]

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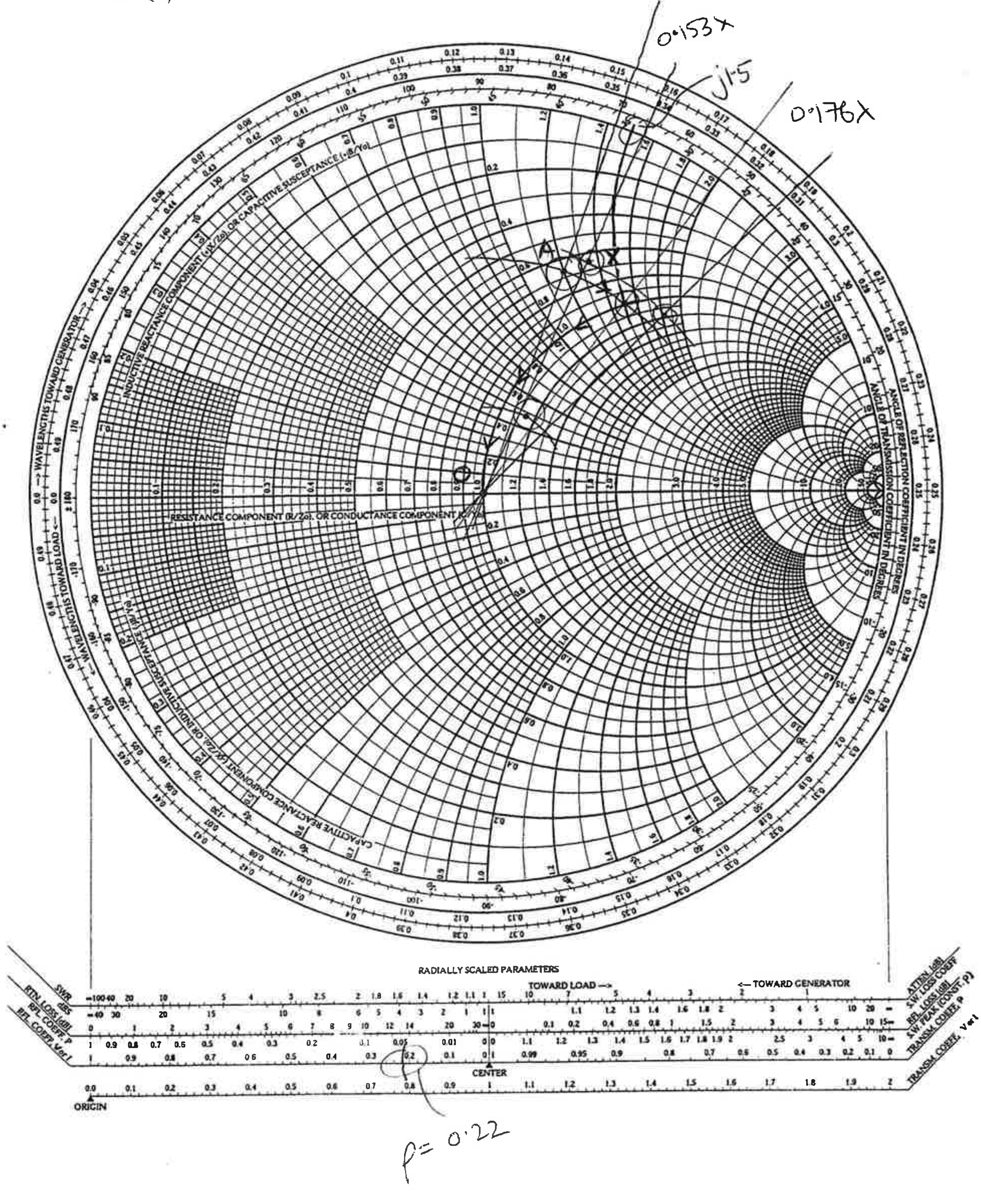
4(c)



3B1 2012 – Numerical answers

1 (b) 8.95×10^{-10} W m^{-2}
(c) 1.27×10^{-4} V rms
(d) 96 %
(e) 5×10^{-4} V rms

2 (c) 510 kHz, high pass, Rf = 1 kΩ
R1 = 523 Ω, (A-1)Rf = 582 Ω
R2 = 303 Ω, (A-1)Rf = 1.66 k Ω

490 kHz, low pass, Rf = 1 kΩ
R3 = 194 Ω, (A-1)Rf = 582 Ω
R4 = 335 Ω, (A-1)Rf = 1.66 k Ω

3 (a) R1 = 100 Ω , R2 = 560 Ω , R3 = 12 Ω , R4 = 75 Ω
(b) 2.72 GHz
(c) 57 nH, 16 pF

4 (a) Q = 25.7, B/w = 6.69 MHz
(c) 0.029 λ , 2.21 pF , ρ = 0.22
(d) w = 0.24 mm , l = 4.7 mm